We thank Adrian Auclert, Refet Gurkaynak, Amir Kermani, Ricardo Reis, Victor Rios-Rull, Martin Schneider, and Stanley Zin for valuable comments and suggestions. We are also grateful to Anton Braun and Andreas Hornstein for their conference discussions (we thank Andreas for coming up with the MoNK acronym). We have also benefited from comments by seminar and conference participants at Vienna Macroeconomics Workshop, CERGE-EI, Economic Growth and Policy Conference in Durham, London School of Economics, GBS Summer Forum in Barcelona, St. Louis Fed, Reserve Bank of New Zealand, Federal Reserve System Meetings in San Francisco, Konstanz Seminar on Monetary Theory and Policy, Bank of Italy, Stockholm University, Bundesbank, and Swiss National Bank. The views expressed are those of the authors and not necessarily of the Federal Reserve Bank of St. Louis, the Federal Reserve System, or the National Bureau of Economic Research. The financial support of the Czech Science Foundation project No. P402/12/G097 (DYME Dynamic Models in Economics) is gratefully acknowledged.

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MoNK: Mortgages in a New-Keynesian Model
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NBER Working Paper No. 26427
November 2019
JEL No. E52,G21,R21

ABSTRACT

We propose a tractable framework for monetary policy analysis in which both short- and long-term debt affect equilibrium outcomes. This objective is motivated by observations from two literatures suggesting that monetary policy contains a dimension affecting expected future interest rates and thus the costs of long-term financing. In New-Keynesian models, however, long-term loans are redundant assets. We use the model to address three questions: what are the effects of statement vs. action policy shocks; how important are standard New-Keynesian vs. cash flow effects in their transmission; and what is the interaction between these two effects?

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1 Introduction

We propose a tractable framework for monetary policy analysis in which both short- and long-term debt affect equilibrium outcomes. Our objective is motivated by two observations on nominal interest rates, made in the literature, whose implications for the economy cannot be fully addressed by standard workhorse models used to study monetary policy.

The first observation concerns the nature of monetary policy surprises and their effects on interest rates. The traditional analysis of the transmission mechanism of monetary policy, based on structural vector autoregressions (SVAR), e.g., Christiano, Eichenbaum and Evans (1999), identifies policy surprises as unexpected changes in the current policy rate. A limitation of this analysis is that it focuses on the actions of the central bank, relative to what the empirical model or the agents in its theoretical counterpart expected. Such shocks are empirically found to have a temporary effect on short rates and almost no effect on long-term interest rates (e.g., Evans and Marshall, 1998). Market participants, however, pay attention not only to what the central bank does but also to what it says, or signals, it may do in the future. Unexpected changes in policy statements therefore lead to substantial movements of the entire yield curve, including the 10-year yield.\(^1\) This dimension of policy surprises has been established in high-frequency studies, using daily or intra-day data. Gürkaynak, Sack and Swanson (2005a) and Campbell, Evans, Fisher and Justiniano (2012), for instance, decompose monetary policy surprises into actions vs. statements about the likely future path of the policy rate and document that the latter component is strongly positively correlated with the observed changes in long-term interest rates on FOMC dates.\(^2\)

\(^1\)During the recent Great Recession such communication became known as “forward guidance”. However, policy communication had been used, and affected financial markets, long before this period. For instance, in the United States, on January 28, 2004, a slight change in the wording in the Federal Open Market Committee’s (FOMC) announcement from “policy accommodation can be maintained” to “the Committee can be patient in removing its policy accommodation” led to an increase in two- and five-year yields by about 25 basis points, even though the Federal Reserve left the policy rate, in line with expectations, unchanged.

\(^2\)Related studies include Hamilton (2008), Cieslak and Schrimpf (2018), Jarocinski and Karadi (2018), and Nakamura and Steinsson (2018). The responses of long-term interest rates can occur also due changes in term premia (e.g., Gertler and Karadi, 2015). Unfortunately, empirical decompositions into expected interest rates and term premia generally suffer from weak identification, large standard errors, and a small sample bias (Joslin, Singleton and Zhu, 2011; Bauer, Rudebusch and Wu, 2012; Hamilton and Wu, 2012; Kim and Orphanides, 2012). We proceed under the assumption that monetary policy statements at least
The second observation concerns the behavior of nominal interest rates over time. A large body of work extracts factors driving nominal interest rates from yield curve data at monthly or quarterly frequencies (e.g., Ang and Piazzesi, 2003; Diebold, Rudebusch and Aruoba, 2006; Rudebusch and Wu, 2008). A robust finding in this literature is that interest rates contain a highly persistent, latent, stochastic factor, with autocorrelation near the unit root, which has an approximately equal effect on both short and long rates. Importantly, the presence and dynamic properties of this factor are essentially unaffected by the change of measure from risk-neutral to physical, meaning that the factor drives predominantly expected future interest rates, as opposed to term premia (e.g., Cochrane and Piazzesi, 2008; Duffee, 2012; Bauer, 2018).  

While a consensus on the origins of this factor is yet to be reached, a long list of studies attribute a chunk of its movements to persistence in monetary policy, due to the factor’s strong positive correlation with inflation (e.g., Kozicki and Tinsley, 2001; Atkeson and Kehoe, 2009; Bekaert, Cho and Moreno, 2010).  

Both observations, while made in different literatures and at different frequencies, suggest that monetary policy has a dimension that affects expected future interest rates and, thus, the costs of long-term debt financing (Hamilton, 2008, establishes such a connection empirically). By facilitating various investment decisions, long-term debt plays an important role in the economy and is therefore frequently debated at policy meetings. However, in the standard workhorse model used for monetary policy analysis—the New-Keynesian model—long-term debt is a redundant asset. That is, it is priced but does partially affect expectations of future interest rates, as suggested by the studies cited.  

While the risk of the factor is priced—i.e., there is a risk premium attached to this factor—the risk premium does not vary with the factor itself and neither do risk premia attached to other factors. For instance, in a tightly identified decomposition by Cochrane and Piazzesi (2008), movements in term premia are set off by a hidden (unspanned by yields) factor, as in Duffee (2011) and Joslin, Priebusch and Singleton (2014), which is correlated with a factor driving the slope of the yield curve.  


Our focus on expected interest rates, rather than term premia, is grounded in practical considerations. Despite a large body of work, the economic drivers behind movements of term premia are still not well understood (e.g., a survey chapter by Duffee, 2012). Unsurprisingly, generating sufficiently large and time-varying term premia within structural general equilibrium models has so far proved unsuccessful (e.g., Rudebusch and Swanson, 2008; van Binsbergen, Fernandez-Villaverde, Koijen and Rubio-Ramirez, 2012).
not affect equilibrium outcomes (e.g., Hördahl, Tristani and Vestin, 2008; Bekaert et al., 2010; Rudebusch and Swanson, 2012). Only one-period debt has equilibrium consequences, through the interest rate in the standard Euler equation, in combination with nominal price rigidities. In the workhorse model, expected future policy rates affect current equilibrium allocations and prices through a sequence of such Euler equations, but the model misses any effects operating through the explicit role that long-term debt plays in the economy.

The contribution of this paper is to propose a tractable framework for monetary policy analysis that takes the explicit role of long-term debt into account, in addition to the standard New-Keynesian channel. To make this notion specific, we focus on mortgage debt.

There are a number of reasons why this is a sensible starting point. First, mortgages have one of the longest terms among the loans in the economy, 15-30 years in most countries (International Monetary Fund, 2011). Second, mortgages are the main financial liability of the household sector and the purchase of a house, the main asset for most households, is highly dependent on mortgage financing (Campbell and Cocco, 2003). Third, mortgage payments—interest and amortization—are sizable, equivalent to 15-30% of household disposable income, depending on the country and time period (see Garriga, Kydland and Šustek, 2017, and the references therein). And finally, the macro literature is increasingly turning attention to middle-class households, a segment of the population significantly exposed to mortgage debt.

A natural consequence of long-term debt in the economy is the distinction between the flow and the stock. An additional distinction in the case of mortgage debt is between a fixed-rate mortgage (FRM) and an adjustable-rate mortgage (ARM), the two basic types of mortgage contracts, each with a different exposure to the policy rate. FRM has a constant nominal interest rate set at origination, whereas the interest rate of ARM is linked to a

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6 An exception is Sheedy (2014). More recently, Sims and Wu (2019) develop a limited participation New-Keynesian model in which the value of long-term bonds held by the central bank affects the equilibrium.

7 The literature typically explores the role of housing debt in New-Keynesian settings by considering only one-period loans (see Iacoviello, 2010); Calza, Monacelli and Stracca (2013) consider two-period loans.

8 E.g., Kaplan and Violante (2014) and Di Magio, Kermani, Keys, Piskorski, Ramcharan, Seru and Yao (2017).
short-term nominal interest rate and can change during the life of the loan whenever the underlying interest rate changes. These distinctions have important implications for the pass through of the policy rate to the economy, illustrated in Figure 1, and captured by the model. The figure plots the movements in nominal mortgage interest rates (aggregate averages) in a number of euro area countries around the time of the ECB rate cuts in 2008. In the case of ARM countries, the interest rate on both new and outstanding debt declined more or less immediately in line with the ECB rate. In FRM countries, the interest rate on new loans also declined, though less than the ECB rate, but the interest rate on outstanding debt remained essentially unchanged.

In the model, long-term debt plays a role by facilitating purchases of housing. Specifically, a fraction of new housing is financed through new long-term nominal mortgage loans (either FRM or ARM), which are amortized over time in a way that mimics the amortization schedule of a typical mortgage. There are two types of representative agents: “homeowners”, representing middle-class households, and “capital owners”, representing the top quintile of the wealth distribution (Campbell and Cocco, 2003). Both agent types supply labor. Unlike homeowners, capital owners can invest in capital used in production. Capital owners also finance mortgages used by homeowners to purchase housing. Furthermore, the two agent types trade a noncontingent one-period nominal bond, albeit homeowners only at a cost. The key distinction between the two agents is thus their access to capital and bond markets. This has consequences for their valuation of mortgage payments (cash flows) over the life of a loan and for their marginal propensities to consume. The rest of the model has the standard New-Keynesian features: nominal price rigidities in product markets and a Taylor rule. The Taylor rule, however, includes shocks affecting expected future interest rates, in

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9 Typically, one of the two contracts dominates a country’s mortgage market (Scanlon and Whitehead, 2004; European Mortgage Federation, 2012; Badarinza, Campbell and Ramadorai, 2016). Unfortunately, there is still no theory of such cross-country differences (Campbell, 2013).

10 In principle, mortgages can be pre-paid or refinanced. The extent to which this is legally or economically feasible, however, varies across countries (Scanlon and Whitehead, 2004; Green and Wachter, 2005; European Mortgage Federation, 2012; Badarinza et al., 2016). The lack of responses of the interest rates on FRM outstanding debt in Figure 1 suggests that in the FRM euro area countries little refinancing took place in response to the ECB rate cuts.
addition to standard policy shocks. The two types of shocks capture different dimensions of policy surprises. Aggregate output can be used for consumption, investment in capital, and investment in housing. When the measure of homeowners is zero, the model boils down to a representative agent New-Keynesian (RANK) model with capital. When mortgages are removed, the model becomes akin to a two-agent New-Keynesian (TANK) model, e.g., Debortoli and Galí (2018). Introducing richer household-level heterogeneity would transform the model, at a significant computational cost, to a heterogenous-agent (HANK) model, e.g., Kaplan, Moll and Violante (2018), with mortgages.¹¹

We use the model to ask three questions. First, how do the real effects of monetary policy shocks that affect expected future interest rates differ from the effects of the standard temporary policy shocks? This question is motivated by the interest of policy makers in affecting long-term interest rates, e.g. by communication, and by the two empirical observations stated earlier.¹² Second, which channel—the standard New-Keynesian or mortgage cash flow channel—is more important for the transmission of monetary policy shocks? This question is related to the ongoing debate on the channels of monetary policy transmission: intertemporal substitution vs. household disposable income (e.g., Kaplan et al., 2018). We contribute to this literature by providing a direct link from monetary policy to disposable income operating through payments on outstanding mortgage debt. The long-term nominal nature of mortgages (both FRM and ARM), and the limited access of homeowners to bond markets, are critical for this linkage (the homeowners in the model can be thought of as the “rich hand-to-mouth” in the spirit of Kaplan and Violante, 2014).¹³ And third, we investigate potential interactions between the two channels and transparently document

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¹¹When labor supply by both agents is inelastic and prices are fully flexible, the model becomes a simplified version of the model developed by Garriga et al. (2017). The simplification lies in abstracting from optimal refinancing and mortgage choice, which, for plausible calibrations, turned out to have only secondary effects on the equilibrium dynamics in Garriga et al. (2017), with the responses of the economy under Refi FRMs still closer to those under pure FRMs than ARMs, when interest rates fall. Berger, Milbradt, Tourre and Vavra (2018) make the point that the fraction of debt that is refinanced depends on the historical path of interest rates, while Wong (2018) argues the fraction depends on demographics.

¹²Throughout the paper we abstract from the issue of the zero lower bound on nominal interest rates.

¹³Cloyne, Ferreira and Surico (2015), Di Maggio et al. (2017), and Flodén, Kilström, Sigurdsson and Vestman (2018) investigate empirically how changes in interest rates on outstanding mortgage debt affect consumption in household-level data, referring to such an effect as the “cash flow”, “income”, or “balance sheet” effect.
the mechanism of the model. An interesting question, for instance, is whether changes in disposable income, due to changes in mortgage payment cash flows, transmit through sticky prices to output. While such a task is relatively straightforward in the two-agent setting, we expect the key insights to carry over to New-Keynesian models with mortgage debt and richer household heterogeneity.¹⁴

As we model both sides of the trade in mortgages, monetary policy has naturally redistributive effects. Our paper is therefore also related to, e.g., Doepke and Schneider (2006), Meh, Rios-Rull and Terajima (2010), Doepke, Schneider and Selezneva (2015) Adam and Zhu (2016), and Auclert (2018). What distinguishes our debt channel from these studies is that it is the timing of the cash flows over the life of the loan, not just the real present value of the debt position (debt-revaluation/Fisher channel), that matters to households. Various empirical studies (e.g., Kaplan and Violante, 2014; Di Magio et al., 2017; Flodén et al., 2018) suggest cash flow effects are important for middle-class households.

The paper proceeds as follows. Section 2 develops the model, section 3 describes its calibration, section 4 presents benchmark findings for a simple formulation of the two shocks, section 5 explains the mechanism, and section 6 considers alternative formulations of the shocks. Section 7 summarizes the results, draws policy conclusions, and suggests avenues for future research. A supplementary material contains supporting and secondary derivations.

2 The model

The model intends to capture, in a parsimonious way, the pass-through effects illustrated in Figure 1, in addition to the standard New-Keynesian transmission of monetary policy. The model environment is motivated by some U.S. observations, with the hope that these apply more broadly, at least in the context of developed economies. In order to present the framework in the clearest possible way, we abstract from various features (habits, labor

¹⁴Hedlund, Karahan, Mitman and Ozkan (2017) is an example. As in Kaplan et al. (2018), their model includes a nontrivial interaction between monetary and fiscal policies, which are abstracted from in our setup. Another example of a HANK-type model with housing is Hergovich and Reiter (2018).
market frictions, backward indexation of prices, etc.) that are often required in the New-Keynesian literature to obtain a good fit to the data. Such extensions are left for future work.

2.1 Environment

There are two types of representative households, “homeowners” and “capital owners”, with measures $\Psi$ and $(1 - \Psi)$, respectively. Both agent types supply labor. Homeowners invest in housing whereas capital owners invest in productive capital. Capital owners also finance mortgages used by homeowners to purchase housing and the two agents trade a noncontingent one-period nominal bond, although homeowners only at a cost. The two types thus differ in their participation in capital and bond markets. This abstraction is motivated by the cross-sectional observations by, e.g., Campbell and Cocco (2003): the typical homeowner is a middle class household (in the third and fourth quintiles of the wealth distribution), with one major asset, a house, one major liability, a mortgage, and almost no corporate equity; in contrast, households in the top quintile of the wealth distribution own the entire corporate equity in the economy and housing makes up a small fraction of their assets. Further, the costly participation of homeowners in the bond market is in the spirit of Kaplan and Violante (2014) and limits the ability of homeowners to smooth out income shocks.\(^{15}\) The economy is studied under either only FRM or ARM, the two extreme cases of policy rate pass-through, and the loans are held until maturity.\(^{16}\)

The production side of the economy has the standard New-Keynesian features and monetary policy follows a Taylor rule. The Taylor rule has both temporary monetary policy shocks and shocks affecting expected future interest rates, in order to capture different dimensions of monetary policy. To achieve sensible calibration, labor supply by the two agent

\(^{15}\) The lowest two quintiles in the data are renters with little assets and little debt and are not included in the model. The framework can, however, be extended in the future to incorporate these agents.

\(^{16}\) See Garriga et al. (2017) for optimal mortgage choice and refinancing. The authors show that refinancing when interest rates decline is equivalent to having an ARM. However, for calibration replicating the historical frequency of refinancing in the United States, the equilibrium responses of their model are still closer to those with pure FRMs than ARMs.
types differs in efficiency units and the model also includes constant taxes and government expenditures. While these features have quantitative implications, they are unimportant for understanding the qualitative properties of the model.

2.2 Capital owners

The capital owner (indexed by “1”) chooses contingency plans for $c_{1t}, n_{1t}, x_{Kt}, k_{t+1}, b_{1,t+1},$ and $l_{1t}$ to maximize expected life-time utility

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{1t}, n_{1t}), \quad \beta \in (0, 1),$$

where $u(.,.)$ has the standard properties guaranteeing a unique interior solution, subject to a sequence of budget constraints and laws of motion for capital

$$c_{1t} + q_{Kt}x_{Kt} + \frac{b_{1,t+1}}{p_t} + \frac{l_{1t}}{p_t} = [(1 - \tau_K) r_t + \tau_K \delta_K] k_t + (1 + \tau_{i-1}) \frac{b_{1t}}{p_t} + \frac{m_{1t}}{p_t} + (1 - \tau_N) \epsilon_w w_t n_{1t} + \tau_{1t} + \Pi_t,$$

$$k_{t+1} = (1 - \delta_K) k_t + x_{Kt}. \tag{1}$$

We index by “1” only those variables of the capital owner that can pertain to both agent types. Here, $c_{1t}$ is consumption, $n_{1t}$ is labor, $x_{Kt}$ is investment in capital, $q_{Kt}$ is its relative price, $b_{1,t+1}$ is holdings of the one-period nominal bond between periods $t$ and $t + 1$, $p_t$ is the nominal price level, $l_{1t}$ is new nominal mortgage lending, $\tau_K \in (0, 1)$ is a capital income tax rate, $r_t$ is a real capital rental rate, $\delta_K \in (0, 1)$ is a capital depreciation rate, $k_t$ is capital, $\tau_{i-1}$ is the nominal interest rate on the one-period bond bought in the previous period, $m_{1t}$ is receipts of nominal payments from outstanding mortgages, $\tau_N \in (0, 1)$ is a labor income tax rate, $\epsilon_w > 0$ is the relative productivity of capital owners, $w_t$ is the aggregate real wage rate, $\tau_{1t}$ is government transfers, and $\Pi_t$ is profits of monopolistically competitive producers, assumed to be owned by the capital owner. The capital owner’s endogenous state variables are $k_t, b_{1t},$ and $m_{1t}$. The determination of the mortgage payments, $m_{1t}$, is described after
introducing the homeowner.\footnote{As capital owners are the sole holders of mortgage debt in the economy, and given that they are identical, outstanding mortgage debt is not traded in equilibrium. We therefore do not include adjustments in an individual capital owner’s holdings of outstanding mortgage debt in the budget constraint, as doing so would have no equilibrium consequences.}

2.3 Homeowners

A representative homeowner (indexed by “2”) chooses contingency plans for $c_{2t}$, $n_{2t}$, $x_{Ht}$, $h_{t+1}$, $b_{2,t+1}$, and $l_{2t}$ to maximize expected life-time utility

$$E_0 \sum_{t=0}^{\infty} \beta^t v(c_{2t}, h_t, n_{2t}),$$

where $v(.,.,.)$ also has the standard properties guaranteeing a unique interior solution, subject to a sequence of budget and financing constraints and laws of motion for housing

$$c_{2t} + q_{Ht}x_{Ht} + \frac{b_{2,t+1}}{p_t} = (1 - \tau_N)w_t n_{2t} + (1 + i_{t-1} + \Upsilon_{t-1}) \frac{b_{2t}}{p_t} + \frac{l_{2t}}{p_t} - \frac{m_{2t}}{p_t} + \tau_{2t}, \quad (2)$$

$$\frac{l_{2t}}{p_t} = \theta q_{Ht}x_{Ht}, \quad (3)$$

$$h_{t+1} = (1 - \delta_H)h_t + x_{Ht}.$$

The homeowner has the same discount factor as the capital owner. We index by “2” only those variables of the homeowner that can pertain to both agent types. Here, $c_{2t}$ is consumption, $h_t$ is the existing stock of housing, $n_{2t}$ is labor, $x_{Ht}$ is purchases of new housing, $q_{Ht}$ is the relative price of housing, $b_{2,t+1}$ is holdings of the one-period nominal bond between periods $t$ and $t + 1$, $l_{2t}$ is new nominal mortgage borrowing, $m_{2t}$ is nominal mortgage payments on outstanding debt, $\tau_{2t}$ is transfers, $\theta \in [0, 1)$ is a loan-to-value ratio, and $\delta_H \in (0, 1)$ is a housing depreciation rate. $\Upsilon_{t-1}$ is described below. The homeowner’s endogenous state variables are $h_t$, $b_{2t}$, and $m_{2t}$.

Observe that the financing constraint (3) applies to new housing, which is purchased with
new mortgages. That is, the homeowner purchases new housing with a new mortgage, at the loan-to-value ratio $\theta$, and then repays the loan over time according to a given amortization schedule, described below. In this sense, mortgages in the model are first mortgages, as opposed to home equity loans, which allow homeowners to draw new credit against the value of their existing housing stock.\textsuperscript{18,19}

Unlike mortgages, the one-period bond is not tied to housing. However, it entails a participation cost $\Upsilon_{t-1}$, taking the form of a spread over the short rate. The cost is governed by a function $\Upsilon(-\tilde{b}_{2t})$, where $\tilde{b}_{2t} \equiv b_{2t}/p_{t-1}$. The function $\Upsilon(.)$ is assumed to be increasing and convex and satisfy the following additional properties: $\Upsilon(.) = 0$ when $\tilde{b}_{2t} = 0$, $\Upsilon(.) > 0$ when $\tilde{b}_{2t} < 0$ (the homeowner is borrowing), and $\Upsilon(.) < 0$ when $\tilde{b}_{2t} > 0$ (the homeowner is saving). We think of $\Upsilon(.) > 0$ as capturing a premium for unsecured consumer credit, which is increasing in the amount borrowed; $\Upsilon(.) < 0$ can be interpreted as intermediation costs that reduce the homeowner’s returns on savings below those of the capital owner. At a technical level, the function $\Upsilon(.)$ controls the extent to which the homeowner can use the bond market to smooth out fluctuations in income. In equilibrium, however, the function also affects the extent to which the capital owner can use the bond market to keep consumption smooth, as the two agent types are counterparties in bond trades.

### 2.4 Mortgages

We adopt the representation of mortgages proposed by Kydland, Rupert and Šustek (2016), which has a recursive form that is convenient in models with infinitely lived agents. Under this representation, mortgage loans—like the agents—live forever, but their amortization

\textsuperscript{18}While widespread in the United States, home equity loans are less common in other countries and, to keep the model simple, are therefore abstracted from.

\textsuperscript{19}The financing constraint (3) is assumed to hold with equality. This assumption, which simplifies the model, has some empirical support at both aggregate and micro levels. First, over time there has been little variation in the aggregate loan-to-value ratio for newly built home first mortgages, despite large changes in interest rates and other macroeconomic conditions (Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10). Second, at the micro level, Greenwald (2018) documents that most newly originated mortgages are taken out at the maximum loan-to-value ratios. Micro-founded justifications for such findings may include life-cycle and tax considerations that our model abstracts from.
rates are chosen so as to approximate the amortization schedule, and thus the mortgage payments, of standard 30-year mortgages.

Denoting by $d_{2t}$ the outstanding nominal mortgage debt of the homeowner in period $t$, the nominal mortgage payments the homeowner has to make in period $t$ are

$$m_{2t} = (R_{2t} + \gamma_{2t})d_{2t}. \quad (4)$$

Here, $R_{2t}$ and $\gamma_{2t}$ are, respectively, the interest and amortization rates of outstanding debt. The variables determining $m_{2t}$ are state variables evolving as

$$d_{2,t+1} = (1 - \gamma_{2t})d_{2t} + l_{2t}, \quad (5)$$

$$\gamma_{2,t+1} = (1 - \phi_{2t}) (\gamma_{2t})^\alpha + \phi_{2t}\kappa, \quad (6)$$

$$R_{2,t+1} = \begin{cases} (1 - \phi_{2t})R_{2t} + \phi_{2ti_t^F}, & \text{if FRM,} \\ i_t, & \text{if ARM,} \end{cases} \quad (7)$$

where $i_t^F$ is the interest rate on new FRM loans and $\phi_{2t} \equiv l_{2t}/d_{2,t+1}$ is the fraction of new loans in outstanding debt next period. The parameters $\kappa, \alpha \in (0, 1)$ define the amortization schedule. Specifically, $\kappa$ is the initial amortization rate of a new loan and $\alpha$ governs the evolution of the amortization rate of outstanding debt. The amortization rate $\gamma_{2,t+1}$ thus evolves as a weighted average of the amortization rates of outstanding and new debt. The parameters $\kappa$ and $\alpha$ are chosen so that equation (6) satisfies the restriction that a single loan (i.e., no previous debt and no further loans in the future) gets effectively repaid in 30 years (120 periods in a quarterly model) and its nominal payments stay approximately constant during this period, provided the loan’s interest rate does not change (see Kydland et al., 2016, for details).\footnote{Essentially, the amortization rate needs to increase at a speed that just compensates for the fact that outstanding debt declines over the life of the loan, thus keeping mortgage payments roughly constant until the loan if effectively repaid.} The interest rate $R_{2,t+1}$ evolves in a similar way, as a weighted average of interest rates on outstanding and new debt. In the FRM case, the interest rates on the
stock and flow are potentially different, whereas in the ARM case they are the same, equal to
the short rate. An ARM, however, is still a long-term loan—the evolution of the amortization
rate is still dictated by the law of motion (6).

Notice that as long as $x_{Ht}$ is positive, $l_{2t}$ will also be positive due to the financing
constraint (3). As we do not observe negative housing investment in aggregate data, the
model will be calibrated so that $x_{Ht}$, and thus $l_{2t}$, are always positive. This, however,
does not mean that there cannot be deleveraging in the model. Such situation occurs when
$0 < l_{2t} < \gamma_{1}d_{2t}$.

The receipts of mortgage payments by the capital owner are determined analogously

\[ m_{1t} = (R_{1t} + \gamma_{1t})d_{1t}, \]  

\[ d_{1,t+1} = (1 - \gamma_{1t})d_{1t} + l_{1t}, \]  

\[ \gamma_{1,t+1} = (1 - \phi_{1t}) (\gamma_{1t})^{\alpha} + \phi_{1t} \kappa, \]  

\[ R_{1,t+1} = \begin{cases} 
(1 - \phi_{1t})R_{1t} + \phi_{1t}i_{t}^{F}, & \text{if FRM,} \\
i_{t}, & \text{if ARM,} 
\end{cases} \]  

where $\phi_{1t} \equiv l_{1t}/d_{1,t+1}$. Aggregate consistency requires: $(1 - \Psi)d_{1t} = \Psi d_{2t} \equiv D_{t}$, $\gamma_{1t} = \gamma_{2t} \equiv 
\gamma_{t}$, and $R_{1t} = R_{2t} \equiv R_{t}$. As a consequence, $(1 - \Psi)m_{1t} = \Psi m_{2t}$.

Why not simply assume a constant amortization rate, $\gamma_{jt} = \gamma \forall t, j \in \{1, 2\}$, as in, for
instance, Woodford (2001)? This is because a constant amortization rate implies geometrically
decaying mortgage payments, $m_{jt}$, thus assigning most of the nominal cash flows to the
beginning of the life of a loan. This is inconsistent with the key feature of standard mortgage
contracts that nominal cash flows are distributed equally across the life of the loan, absent
any changes in the mortgage interest rate.

21The system (5)-(7) has a well-defined steady state given by: $l_{2} = \gamma_{2}d_{2}$, $(1 - \kappa) = (1 - \gamma_{2})^{\alpha-1}$, and
$R_{2} = i^{F}$ or $R_{2} = i$. It is straightforward to show that the steady-state non-linear equation in $\gamma_{2}$ has a unique
solution in $[0, 1]$. It is also straightforward to show that when linearized around the steady state, the highest
eigenvalue of the system is less than one in absolute value, rendering the system stationary. One period
loans result under $\kappa = 1$ and $\alpha = 0$. 

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2.5 Production

The production side of the economy has the standard New-Keynesian features resulting in the prototypical New-Keynesian Phillips Curve. Identical perfectly competitive final good producers, of which there is a measure one, produce output $Y_t$, using as inputs a continuum of intermediate goods $y_t(j), j \in [0, 1]$. The representative final good producer solves a static profit maximization problem

$$\max_{Y_t, \{y_t(j)\}_0^1} p_t Y_t - \int_0^1 p_t(j) y_t(j) dj \quad \text{subject to} \quad Y_t = \left[ \int_0^1 y_t(j)^\varepsilon dj \right]^{1/\varepsilon},$$

where $p_t(j)$ is the nominal price of an intermediate good $j$ and $\varepsilon \in (0, 1]$. As the measure of the producers is equal to one, $Y_t$ represents also aggregate output. A first-order condition of this problem gives a demand function for good $j$

$$y_t(j) = \left[ \frac{p_t}{p_t(j)} \right]^{\frac{1}{1-\varepsilon}} Y_t. \quad (12)$$

The producer of the intermediate good $j$ is a monopolist in market $j$. It faces the Calvo price stickiness, which stipulates that with probability $\psi \in [0, 1]$ the producer cannot change its price in a given period. If allowed to change its price in period $t$, the producer $j$ chooses $p_t(j)$, understanding it may not change the price in the future, to solve

$$\max_{p_t(j)} E_t \sum_{s=0}^{\infty} \psi^s Q_{1,t+s} \left[ \frac{p_t(j)}{p_{t+s}} y_{t+s}(j) - \chi_{t+s} y_{t+s}(j) - \Delta \right], \quad j \in [0, 1], \quad (13)$$

where $Q_{1,t+s} \equiv \beta u_{c,t+s}/u_{ct}$ is the stochastic discount factor of the capital owner, $\chi_{t+s}$ is the real marginal cost, the expression in the square brackets is the real per-period profit, and $y_{t+s}(j)$ is given by the demand function (12), with $p_{t+s}(j) = p_t(j) \forall s$. Further, $\Delta$ is a fixed cost, measured in terms of the final good, which is a common feature of New-

\footnote{Notation such as $u_{ct}$ means the first derivative of the function $u$ with respect to argument $c$, evaluated in period $t$.}
Keynesian models with capital, ensuring that profits in steady state are equal to zero.\textsuperscript{23} The discounted sum thus pertains to profits in all individual future states in which the producer cannot change its price.

The real marginal cost $\chi_t$ is given by a linear cost function derived from a static cost minimization problem of producer $j$

$$\chi_t y_t(j) = \min \_{k_t(j), n_t(j)} r_t k_t(j) + w_t n_t(j) \quad \text{subject to} \quad Ak_t(j)^\varsigma n_t(j)^{1-\varsigma} = y_t(j).$$

Here, $A$ is total factor productivity, which for most of the paper is constant, $\varsigma \in (0, 1)$, and $k_t(j)$ and $n_t(j)$ are capital and labor, respectively, rented by producer $j$ in spot markets. The first-order condition is

$$\frac{w_t}{r_t} = \left(\frac{1-\varsigma}{\varsigma}\right) \frac{k_t(j)}{n_t(j)}. \quad (14)$$

The value function of the cost minimization problem then yields $\chi_t \equiv A^{-1}(r_t/\varsigma)[w_t/(1-\varsigma)]^{1-\varsigma}$, which is independent of $j$, as already anticipated in the profit function (13).

2.6 Aggregate expenditures

Aggregate output $Y_t$, less the fixed costs incurred by the intermediate good producers, has four uses

$$C_t + q_{Kt}X_{Kt} + q_{Ht}X_{Ht} + G = Y_t - \Delta, \quad (15)$$

where $C_t \equiv (1-\Psi)c_{1t} + \Psi c_{2t}$, $X_{Kt} \equiv (1-\Psi)x_{Kt}$, $X_{Ht} \equiv \Psi x_{Ht}$, and $G$ is (constant) government expenditures.\textsuperscript{24} Further, $q_{Kt}$ is the marginal rate of transformation between consumption and capital investment and $q_{Ht}$ is the marginal rate of transformation between consumption and housing investment. The rates of transformation are given by strictly increasing convex functions $q^K(X_{Kt})$ and $q^H(X_{Ht})$, which make the economy’s production possibilities frontier

\textsuperscript{23}This is relevant only for mapping the parameter $\varsigma$ to capital share in National Income and Product Accounts in a straightforward way.

\textsuperscript{24}When we map the model into data, we take $Y_t - \Delta$ as the model counterpart to aggregate output in the data.
(PPF) concave. A normalization restriction is imposed on these functions so that the rates of transformation are equal to one in steady state. This specification of the PPF is akin to that of Fisher (1997) and Huffman and Wynne (1999). Such a concave PPF could be derived from, for instance, a multisectoral input-output environment (e.g., construction, manufacturing, services) with different factor shares or imperfect factor mobility, as in, e.g., Davis and Heathcote (2005). If the transformation of output into the respective uses is carried out by perfectly competitive firms, the rates of transformation are equal to the relative prices of capital and housing investment, as already anticipated in the budget constraints.\textsuperscript{25}

At a technical level, the nonlinear PPF plays a role similar to that of capital adjustment costs, controlling the size of the responses of $X_K$ and $X_H$ to shocks (to a first-order approximation, the two specifications are equivalent). The more curvature the PPF has, the more of the aggregate adjustment to shocks occurs through changes in relative prices, and the less through investment in capital or housing. The curvature of the PPF thus has implications for the ability of the agents to use capital and housing to keep consumption smooth in response to shocks.

\subsection*{2.7 Equilibrium}

Let us define the following aggregates, in addition to those already defined: $K_t \equiv (1 - \Psi)k_t$, $H_t \equiv \Psi h_t$, and $B_t \equiv \Psi b_{2t} = -(1 - \Psi)b_{1t}$. The aggregate state of the economy in period $t$ is characterized by the following endogenous state variables: $K_t$, $H_t$, $B_t$, $D_t$, $\gamma_t$, $R_t$, $i_{t-1}$, $p_{t-1}$, and the shocks introduced below.\textsuperscript{26} A (Markov) stochastic process for the shocks is known to all agents. The capital owner comes into period $t$ with the following individual endogenous state variables: $k_t$, $b_{1t}$, $d_{1t}$, $\gamma_{1t}$, and $R_{1t}$. The homeowner comes into the period with these individual endogenous state variables: $h_t$, $b_{2t}$, $d_{2t}$, $\gamma_{2t}$, and $R_{2t}$. The economy

\textsuperscript{25}Since old and new housing are perfect substitutes, $q^H_t$ represents a house price. As we are not interested in house prices per se, we abstract from the fact that a house consists of both a structure and a land; a house in the model is just a structure. A fixed plot of land works like an additional adjustment cost.

\textsuperscript{26}The variables $i_{t-1}$ and $p_{t-1}$ can be eliminated from the set of endogenous state variables if the budget constraints are written in terms of the price of the one-period bond, rather than its interest rate, and the model is rewritten in terms of the inflation rate, rather than the price level.
operates under either FRM or ARM.

In equilibrium, the following conditions hold: (i) the capital owner and the homeowner solve their respective maximization problems, choosing contingency plans for $c_{1t}$, $n_{1t}$, $x_{Kt}$, $b_{1,t+1}$, and $l_{1t}$ (capital owner) and $c_{2t}$, $n_{2t}$, $x_{Ht}$, $b_{2,t+1}$, and $l_{2t}$ (homeowner); (ii) the intermediate good producers choose $k_t(j)$ and $n_t(j)$ to solve their cost minimization problem and, if allowed, choose $p_t(j)$ to maximize the discounted profits, subject to their demand function; (iii) the relative prices $q_{Kt}$ and $q_{Ht}$ are given by the respective marginal rates of transformation; (iv) monetary policy follows a Taylor rule, specified in the next section; and (v) the mortgage, bond, labor, capital, and goods markets clear

$$(1 - \Psi)l_{1t} = \Psi l_{2t},$$

$$-(1 - \Psi)b_{1,t+1} = \Psi b_{2,t+1},$$

$$\int_0^1 n_t(j) = \epsilon w N_{1t} + N_{2t},$$

$$\int_0^1 k_t(j) = K_t,$$

$$C_t + q_{Kt}X_{Kt} + q_{Ht}X_{Ht} + G = Y_t - \Delta,$$

where $N_{1t} \equiv (1 - \Psi)n_{1t}$, $N_{2t} \equiv \Psi n_{2t}$, and $Y_t = \left[\int_0^1 y_t(j)^{\epsilon} dj\right]^{1/\epsilon}$. As capital owners’ and homeowners’ labor inputs are perfect substitutes, capital owners’ wage rate is $\epsilon w w_t$, whereas homeowners’ wage rate is $w_t$, as already anticipated in the respective budget constraints.

In the FRM case, the stochastic sequences of prices clearing these five markets are for $i_t^F$, $i_t$, $w_t$, $r_t$, and $p_t$. In the ARM case, as discussed below, the capital owner is indifferent between the one-period bond and a new ARM loan at any sequence for $i_t$ and supplies mortgages in the amount demanded by the homeowner. Under ARM, therefore, any sequence of $i_t$ that clears the bond market also clears the mortgage market. A complete list of the equations characterizing the equilibrium is contained in the supplementary material. For the reasons explained in the Introduction, we are not concerned with risk premia. The model is
therefore solved under certainty equivalence, in its log-linear form.\textsuperscript{27}

### 2.8 Benchmark policy shocks

The two strands of research described in the Introduction suggest monetary policy has a dimension affecting expected future interest rates. Our various specifications of the Taylor rule thus reflect the lessons from the two literatures. Here we start off with a benchmark specification, which provides useful insights into the inner workings of the model, and then expand on this specification in Section 6.

In a broad sense, the short-term nominal interest rate $i_t$ is typically considered in the aforementioned literatures to (linearly) depend on a number of factors, which can be both latent and observable. A standard formulation is

$$
\hat{i}_t = i + f_{1t} + \ldots + f_{Jt},
$$

where (without the loss of generality) the ‘loadings’ of all the factors on the short-term interest rate are normalized to equal to one and $i$ is the unconditional mean of the short rate (the factors are normalized to have an unconditional mean equal to zero). While the factors can be mutually correlated, a natural restriction typically imposed in the literature is that they are orthogonal to each other. Interest rates on bonds of longer maturities also depend on the $J$ factors, though potentially with different loadings. For a bond of maturity $s$

$$
\hat{i}^{(s)}_t = \hat{i}^{(s)} + A^{(s)}_{1} f_{1t} + \ldots + A^{(s)}_{J} f_{Jt},
$$

where $\hat{i}^{(s)}$ is the unconditional mean. The loadings, $A^{(s)}_{1}, \ldots, A^{(s)}_{J}$, can come from a purely statistical relationship, such as a principal component decomposition of the yield curve.

\textsuperscript{27}The government budget constraint holds by Walras’ law and is given by $G + (1 - \Psi)\tau_{1t} + \Psi\tau_{2} = \tau_{K}(\tau_{1} - \delta_{K})K_{t} + \tau_{N}w_{t}(\epsilon_{w}N_{1t} + N_{2t})$. In the government budget constraint, $\tau_{1t}$ adjusts so as to ensure that the budget constraint is satisfied state-by-state. The transfer to the homeowner, $\tau_{2t}$, is given by $\tau_{2t} = \tau_{2} - (b_{2t}/p_{t-1})Y_{t-1}$, where $\tau_{2}$, the ‘genuine’ transfer, is constant. The bond market participation cost is rebated back to the homeowner as a part of $\tau_{2t}$ in order not to affect the definition of aggregate output. In steady state, $b_{2t} = 0$ and the participation cost is equal to zero.
or can be derived by imposing theoretical no-arbitrage restrictions on bond prices. Duffee (2012) and Diebold, Piazzesi and Rudebusch (2005) provide brief overviews.28

Given the factor loadings, the $S$ yields can be used at any point in time to back out the $J$ factors ($J < S$). The high-frequency studies carry this out using daily data or data from a narrow window around FOMC announcements. The lower-frequency studies do so using monthly or quarterly data. A robust finding from both literatures is that two factors capture about 95% of the movements of yields across maturities. A parsimonious representation of nominal interest rates is thus: $i_t = i + f_{1t} + f_{2t}$ and $i_s^{(s)} = i^{(s)} + A_1^{(s)} f_{1t} + A_2^{(s)} f_{2t}$. The first factor turns out to affect all yields more or less equally (the loadings are close to one for all maturities) and is therefore commonly referred to as the ‘level’ factor. Its time series has high persistence, close to random walk, implying near one-for-one effects on expected future interest rates. The second factor affects the long-short spread and is therefore generally referred to as the ‘slope’ factor. Its time series is also less persistent, implying a small effect on expected future interest rates at the long end.29 While various macroeconomic shocks affect interest rates, the yield curve decomposition suggests that by far and large these shocks manifest themselves in the term structure by moving only its slope and/or its level. Monetary policy shocks that affect expected future interest rates at the long end are therefore reflected in the level factor.

In our benchmark formulation, there are two policy shocks modeled as independent AR(1) processes. One shock is temporary and the other close to random walk. Following much of both macro and macro-finance literatures, the highly persistent shock is modeled as a shock to an inflation target (the central bank’s tolerance for inflation).30 The benchmark Taylor

28In the case of the popular no-arbitrage affine term structure models, a given coefficient $A_j^{(s)}$ is a sum of two parts. One captures the effect of the factor $j$ on yield $s$ working through expected future short rates and is purely determined by the factor’s persistence. The other captures the effect of the factor working through a term premium. For the reasons stated in the Introduction, we are only concerned with the expectations part.

29The level factor is largely unrelated to movements in term premia (Cochrane and Piazzesi, 2008; Duffee, 2012; Bauer, 2018), whereas the slope factor and term premia are strongly correlated (a long list of studies, starting with Fama and Bliss, 1987).

30Technically, the inflation target shock is just a label for a ‘standard’ but very persistent policy shock, as the Taylor rule can always be rewritten in such a way. Gürkaynak, Sack and Swanson (2005b) provide
rule, which closes the model, is therefore specified as

\[ i_t = i + \mu_t - \pi + \nu_\pi (\pi_t - \mu_t) + \eta_t, \quad \nu_\pi > 1. \]  

(16)

Here, \( \pi_t \equiv p_t/p_{t-1} - 1 \) is the inflation rate between periods \( t \) and \( t-1 \), \( \pi \) is its steady-state value, and \( \nu_\pi \) is a weight on deviations of the inflation rate from a stochastic inflation target \( \mu_t \). The inflation target has an unconditional mean equal to \( \pi \) and follows a stationary, though highly persistent, process \( \mu_{t+1} = (1 - \rho_\mu) \pi + \rho_\mu \mu_t + \xi_{\mu,t+1} \), where \( \xi_{\mu,t+1} \) is a mean-zero innovation and \( \rho_\mu \) is close to but less than one. The other shock, \( \eta_t \), is a temporary shock. It has an unconditional mean equal to zero and follows a less persistent process \( \eta_{t+1} = \rho_\eta \eta_t + \xi_{\eta,t+1} \), where \( \xi_{\eta,t+1} \) is a mean-zero innovation. The two innovations, \( \xi_{\mu,t} \) and \( \xi_{\eta,t} \), are assumed to be orthogonal to each other. Observe that when \( \mu_t \) is equal to its unconditional mean \( \pi \), the Taylor rule is standard.

The present assumption of modeling the shocks as two independent AR(1) processes has two useful implications. First, as shown in the next section, it allows a straightforward mapping from the equilibrium effects of the two shocks on interest rates into the orthogonal level and slope factors, thus connecting the model to the framework employed in the aforementioned literatures. Second, as shown in Section 5, it allows a simple exposition of the inner workings of the model. In Section 6 we explore formulations that provide more structural interpretations of the policy shocks. In particular, as action vs. statement policy shocks (e.g., Gürkaynak et al., 2005a) or pure policy vs. information shocks about the future state of the economy (e.g., Cieslak and Schrimpf, 2018; Nakamura and Steinsson, 2018). Throughout, however, we maintain the orthogonality assumption. While different policy shocks may in principle be correlated (e.g., the central bank may surprise markets through both an action and a statement), orthogonalization is a useful analytical restriction.\(^{31}\)

\(^{31}\)A possible formulation of correlated policy shocks would be by making the inflation target positively

reasons why a central bank’s tolerance for inflation may be time-varying. Ireland (2007) contains a long list of references in the macro literature, including the celebrated Smets and Wouters (2003) model, that employ Taylor rules with inflation target shocks. A number of the studies listed in the Introduction employ inflation target shocks in the macro-finance literature.
As a final remark, one may think that including interest rate smoothing into the Taylor rule (dependence on $i_{t-1}$), as is common in the macro literature, may affect the equilibrium persistence of nominal interest rates. While this is true to some extent, Gürkaynak et al. (2005b) demonstrate that without highly persistent policy shocks, Taylor rules are unable to generate in equilibrium as persistent nominal interest rates as in the data.\footnote{Hördahl et al. (2006), Rudebusch and Wu (2008), Bekaert et al. (2010) and others confirm this result.} Another common element of Taylor rules, the output gap, is dropped from our specification for an easier exposition of the transmission mechanism in the model. Experimentation with output gap did not change the main properties of the model in any significant way.

### 2.9 Equilibrium interest, mortgage, and inflation rates

This section prepares the ground for our discussion of the inner workings of the model later on by explaining how the short- and long-term nominal interest rates, and the inflation rate, are determined in equilibrium. It also relates the effects of the two shocks on interest rates to the above discussion on slope and level factors.

The capital owner invests in all three assets in the economy. His optimality conditions thus have to exclude arbitrage opportunities across capital, the nominal one-period bond, and the nominal long-term mortgage (without the loss of generality, it is useful in the following discussion to abstract from the capital income tax to simplify notation). His Euler equation for the one-period nominal bond, together with the Euler equation for capital and the Taylor rule, provide a convenient characterization of the equilibrium short rate. The first-order conditions for $b_{1,t+1}$ and $x_{Kt}$, respectively, are

\[
1 = E_t \left( Q_{1,t+1} \frac{1 + i_t}{1 + \pi_{t+1}} \right) \quad \text{and} \quad 1 = E_t \left[ Q_{1,t+1} \left( \frac{r_{t+1}}{q_{Kt}} + \frac{q_{K,t+1}(1 - \delta_K)}{q_{Kt}} \right) \right],
\]

where $Q_{1,t+s} \equiv \beta u_{c,t+s}/u_{ct}$ is the real pricing kernel of the capital owner. Once log-linearized respond to inflation. An interpretation of such a formulation is that the central bank surprises markets by increasing the current policy rate and issues a statement that once inflation declines, as a result of its action, it will do whatever necessary to keep it at the lower level, thus affecting inflation expectations.
around a deterministic steady state (for convenience of the exposition), the two equations yield the Fisher equation

\[ i_t - E_t \pi_{t+1} \approx E_t [ r_{t+1} + (1 - \delta_K) q_K, t+1 - q_K] \equiv r^*_t, \]  

(17)

where \( r^*_t \) is the ex-ante real interest rate and (abusing notation) all variables are in percentage point deviations from steady state.\(^{33}\) Combining equation (17) with the policy rule (16), assuming \( \rho_{\mu} \) is close to one and excluding explosive paths for inflation, yields

\[ i_t \approx \mu_t + \left[ \sum_{s=0}^{\infty} \left( \frac{1}{\nu_{\pi}} \right)^s E_t r^*_{t+s} - \frac{\rho_{\eta}}{\nu_{\pi} - \rho_{\eta}} \eta_t \right], \]  

(18)

where \( \mu_t \) is the effect of monetary policy on the level factor and the expression inside the square brackets is the effect on the slope factor. In the notation of the conceptual framework of the preceding section, equation (18) can be written as \( i_t \approx f_{1t} + f_{2t}. \(^{34}\) This is how it works: Observe that unless the effect of \( \mu_t \) is sufficiently offset by an endogenous response of the expected future path of the real rate, the \( \mu_t \) shock generates highly persistent, one-for-one, changes in \( i_t \). It thus affects not only the short rate but also the long rate (the FRM rate derived below) and, thus, works like a level factor. The \( \eta_t \) shock represents the standard temporary monetary policy shock, calibrated to generate the standard New-Keynesian responses. That is, through the New-Keynesian channel (discussed in Section 5), a positive \( \eta_t \) shock increases the ex-ante real interest rate to the extent that \( i_t \) increases as well. Because the \( \eta_t \) shock and this New-Keynesian effect are only temporary, \( i_t \) responds only temporarily, leaving the long rate unaffected.\(^{35}\) The \( \eta_t \) shock thus triggers a movement in the slope factor. Furthermore, as long as the persistent shock \( \mu_t \) leaves the real rate

\(^{33}\)As we are not concerned with term premia, we can work with log-linear expressions, in which certainty equivalence holds.

\(^{34}\)As noted earlier, other shocks can affect the level and slope factors but our focus is only on monetary policy shocks.

\(^{35}\)In principle, in a richer model, the \( \eta_t \) shock could affect the long rate by generating an increase or a decline in the risk premium. In the conceptual framework of the preceding section, the risk premium effect on yield \( s \) would show up as the part of the loading \( A^s_j, j = \eta \), that is not purely due to the shock’s persistence.
relatively unaffected, the movements in the level and slope factors due to the two monetary policy shocks will be approximately orthogonal to each other. The numerical findings in Section 4 confirm this to be the case.

The capital owner’s first-order condition for \( l_{1t} \) in the FRM case determines the long-term interest rate on a new FRM loan. Recall that according to equation (8) mortgage payments are determined by three state variables as \( m_{1t} = (R_{1t} + \gamma_{1t})d_{1t} \), where the state variables follow the laws of motion (9)-(11). In this representation, the first-order condition for \( l_{1t} \) consists of the marginal effects of \( l_{1t} \) on the capital owner’s expected life-time utility, working through the three state variables

\[
1 = E_t \left[ \beta \frac{U_{d,t+1}}{u_{ct}} + \beta \frac{U_{\gamma,t+1}}{u_{ct}} \zeta_{1t} (\kappa - \gamma_{1t}^\alpha) + \beta \frac{U_{R,t+1}}{u_{ct}} \zeta_{1t} (i_F^t - R_{1t}) \right].
\]

Here, \( U_{d,t+1} \), \( U_{\gamma,t+1} \), and \( U_{R,t+1} \) are the derivatives of the capital owner’s value function \( U \) in a recursive formulation of the problem and

\[
\zeta_{1t} \equiv \frac{1-\gamma_{1t}}{1+\pi_t} \frac{\tilde{d}_{1t}}{(1-\gamma_{1t}) \tilde{d}_{1t} + \tilde{l}_{1t}} \in (0,1),
\]

where \( \tilde{d}_{1t} \equiv d_{1t}/p_{t-1} \) and \( \tilde{l}_{1t} \equiv l_{1t}/p_t \). The supplementary material provides the recursive formulation of the capital owner’s problem.

An insight into the first-order condition is gained when there is no previous debt and no further loans beyond period \( t \). In this case \( \zeta_{1t} = 0 \) and the first-order condition simplifies to

\[
1 = E_t \beta \frac{U_{d,t+1}}{u_{ct}}, \quad \text{where} \quad U_{dt} = u_{ct} \frac{i_F^t + \gamma_{1t}}{1+\pi_t} + \beta \frac{1-\gamma_{1t}}{1+\pi_t} E_t U_{d,t+1}
\]

is obtained by the Benveniste-Scheinkman condition. Successive substitutions then yield an intuitive expression

\[
1 = E_t \left[ Q_{1,t+1} \frac{i_F^t + \gamma_{1,t+1}}{1+\pi_{t+1}} + Q_{1,t+2}(1-\gamma_{1,t+1}) \frac{i_F^t + \gamma_{1,t+2}}{(1+\pi_{t+1})(1+\pi_{t+2})} + \ldots \right],
\]
where $\gamma_{1t}$ evolves in a deterministic way according to the law of motion (10); specifically, as $\gamma_{1,t+s+1} = \gamma_{1,t+s}^\alpha$, starting with $\gamma_{1,t+1} = \kappa$. According to this condition, the real mortgage payments on a loan of one unit of consumption have to be worth in present value terms one unit of consumption, when discounted by the capital owner’s pricing kernel. The FRM interest rate $i^F_t$ has to be such that this condition holds. Using the first-order condition for the one-period nominal bond, and the law of iterated expectations, the FRM pricing equation can also be written in terms of future short-term nominal interest rates as

$$1 = E_t \left[ \frac{i^F_t + \gamma_{1,t+1}}{1 + i_t} + (1 - \gamma_{1,t+1}) \frac{i^F_t + \gamma_{1,t+2}}{(1 + i_t)(1 + i_{t+1})} + ... \right] + \Psi_t, \tag{20}$$

where $\Psi_t$ soaks up all the covariance terms between the pricing kernel and interest rates, and thus term premia. Using the property of the amortization rate that $\lim_{s \to \infty} \gamma_{1,t+s} = 1$ (i.e., the loan is eventually repaid), it is then straightforward to show that to a first-order effect, if all future short rates, $i_t, i_{t+1}, ...$, increase by $\lambda$ percentage points, due to an increase of the $\mu_t$ shock by $\lambda$, the FRM rate also increases by $\lambda$ percentage points.$^{36}$ The persistent policy shock thus affects the short and long rates equally, as claimed above. In terms of the conceptual framework of the preceding section, $\mu_t$ has loadings close to one on both the short and long rates. Changes in the short rate that are less persistent are also priced in $i^F_t$, but the impact is smaller.

In the ARM case, again using the property $\lim_{s \to \infty} \gamma_{1,t+s} = 1$, it is straightforward to verify that the discounted mortgage payment condition (again derived from the first-order condition for $l_{1t}$) always holds. That is,

$$1 = E_t \left[ \frac{i^F_t + \gamma_{1,t+1}}{1 + i_t} + (1 - \gamma_{1,t+1}) \frac{i_{t+1} + \gamma_{1,t+2}}{(1 + i_t)(1 + i_{t+1})} + ... \right]$$

$^{36}$A first-order Taylor-series expansion of equation (20) with respect to interest rates around a deterministic steady state yields $0 = \frac{1}{i^F_t}(i^F_t - E_t i_{t+1}) + \frac{1}{(1+i_t)^2}(i^F_t - E_t i_{t+2}) + ...$, where variables without a time subscript represent steady state and (abusing notation) variables with time subscript are in percentage point deviations from steady state (the first-order approximation essentially gets rid off nonlinearities arising due to Jensen’s inequality).
for any sequence of nominal interest rates. The capital owner is thus indifferent between the one period bond and a new ARM loan at any sequence for $i_t$, as claimed earlier.

Finally, substituting the expression for the short rate (18) into the policy rule (16) gives an expression for the equilibrium inflation rate

$$\pi_t \approx \mu_t + \left[ \frac{1}{\nu_\pi} \sum_{s=0}^{\infty} \left( \frac{1}{\nu_\pi} \right)^s E_t r^s_{t+s} - \frac{1}{\nu_\pi - \rho_\eta} \eta_t \right]. \tag{21}$$

As long as $\mu_t$ leaves the real rate more or less unaffected and $\eta_t$ has only a temporary effect on the real rate, the inflation rate (21) is a sum of a near random walk and a temporary component, as in the model of inflation studied by Stock and Watson (2007). Furthermore, observe that $\mu_t$ moves the inflation rate approximately one-for-one with the short and long rates. The persistent shock thus makes the level factor positively correlated with inflation, as documented by the studies in the macro-finance literature noted in the Introduction.

### 2.10 Demand for mortgages

The preceding section explained how new mortgages are priced by no-arbitrage. Demand for new mortgages, $l_{2t}$, is determined by demand for new housing, $x_{ht}$, through the financing constraint (3). In our representation of mortgages, a new mortgage affects the homeowner’s expected life-time utility through the three state variables that make up mortgage payments. The first-order condition for $x_{ht}$ thus takes the form

$$(1 - \theta)q_{ht} + \theta q_{ht} \beta E_t \left[ -\frac{V_{d,t+1}}{v_{cl}} - \frac{V_{\gamma,t+1}}{v_{ct}} \zeta_2 t (\kappa - \gamma_{2t}^\alpha) - \frac{V_{R,t+1}}{v_{ct}} \zeta_2 t (i_{t+1}^M - R_{2t}) \right] = \beta E_t \frac{V_{h,t+1}}{v_{ct}},$$

where $V_{d,t+1}$, $V_{\gamma,t+1}$, $V_{R,t+1}$, and $V_{h,t+1}$ are the derivatives of the homeowner’s value function $V$ in a recursive formulation of the problem and $i_{t+1}^M$ is equal to either $i_{t+1}^F$ or $i_t$, depending on whether the contract is FRM or ARM, respectively. Further, $\zeta_2 t$ has the same form as in (19), except that the variables pertain to the homeowner. The first-order condition for $x_{ht}$ states that the marginal cost of new housing has to be equal to its marginal benefit. Further, the
marginal cost on the left-hand side is a sum of the current marginal cost of downpayment, the first expression, and the expected life-time marginal cost of debt financing, the second expression. The first-order condition can be conveniently re-written in terms of a wedge $\tau_{Ht}$ between the relative price of new housing and the marginal rate of substitution between housing and non-housing consumption

$$q_{Ht}(1 + \tau_{Ht}) = \beta E_t \frac{V_{h,t+1}}{v_{ct}}.$$ 

The wedge works like a tax/subsidy on new housing and the product $q_{Ht}(1 + \tau_{Ht})$ represents the effective price of new housing from the perspective of the homeowner.

An insight into the wedge is gained when, again, there is no previous debt and no further loans beyond period $t$. In this case $\zeta_{2t} = 0$ and the wedge becomes

$$\tau_{Ht} = -\theta \left\{ 1 - E_t \left[ Q_{2,t+1} \frac{i_{t+1}^M + \gamma_{2,t+1}}{1 + \pi_{t+1}} + Q_{2,t+2} \frac{(i_{t+2}^M + \gamma_{2,t+2})(1 - \gamma_{2,t+1})}{(1 + \pi_{t+1})(1 + \pi_{t+2})} + \ldots \right] \right\},$$

where $Q_{2,t+s} \equiv \beta v_{c,t+s}/v_{ct}$ is the stochastic discount factor of the homeowner and $\gamma_{2t}$ evolves in a deterministic way according to the law of motion (6). Observe that the expression inside the square brackets is the present value of future real mortgage payments, either on FRM or ARM, when discounted with the homeowner’s stochastic discount factor. If the stochastic discount factor of the homeowner is different from that of the capital owner (i.e., $Q_{2,t+s} \neq Q_{1,t+s}$), then the present value is in general different from one and the wedge is nonzero, its value being determined by the cash-flows of mortgage payments over the life of the loan. For instance, redistributing real cash flows from periods with low $Q_{2,t+s}$ to those with high $Q_{2,t+s}$ increases the wedge. Holding income constant, when the wedge increases (declines), demand for mortgages declines (increases). We refer to this effect as the ‘price effect’ of long-term mortgage debt, as it affects the effective price of new mortgages and, thus, housing investment.\(^{37}\)

\(^{37}\)In the ARM case, the wedge would always be equal to zero if the homeowner, like the capital owner, could trade the one-period nominal bond at zero cost. In the FRM case, the wedge would always be equal
3 Calibration

While the goal of the paper is to propose a framework for monetary policy analysis, without any specific country in mind, to demonstrate the quantitative properties of the model, we rely on U.S. calibration. Most of the parameter values are based on the New-Keynesian literature (e.g., Galí, 2015) and U.S. calibration targets described in detail in Garriga et al. (2017). One period in the model corresponds to one quarter. Given that the model has cross-sectional implications (even though the split of the population is quite coarse), the calibration of the model deserves some detailed discussion.

3.1 Functional forms

We consider utility functions that are standard in the New-Keynesian literature: 

\[ u(c_1, n_1) = \log c_1 - \left[ \frac{\omega_1}{1 + \sigma} \right] n_1^{(1+\sigma)} \]

and

\[ u(c_2, h, n_2) = \varrho \log c_2 + (1 - \varrho) \log h - \left[ \frac{\omega_2}{1 + \sigma} \right] n_2^{(1+\sigma)} , \]

where \( \omega_1 > 0, \omega_2 > 0, \sigma \geq 0, \varrho \in (0, 1) \). The production function has already been specified as Cobb-Douglas, with technology level \( A \) and capital share \( \varsigma \). The functions governing the curvature of PPF are

\[ q_H(X_{Ht}) = \exp(\zeta(X_{Ht} - X_H)) \]

and

\[ q_K(X_{Kt}) = \exp(\zeta(X_{Kt} - X_K)) \],

where \( \zeta > 0 \) and \( X_H \) and \( X_K \) are, respectively, the steady-state ratios of housing and capital investment to output (output, \( Y - \Delta \), is normalized to be equal to one in steady state).\(^{38}\)

Finally, \( \Upsilon(-B_t) = \exp(-\vartheta B_t) - 1 \), where \( \vartheta > 0 \) and in steady state \( B = 0 \) (\( p \) is normalized to be equal to one, so there is no need to distinguish between real and nominal quantities in steady state). All functional forms conform to the assumptions made in the description of the model.

\(^{38}\)For our calibration, \( A = 1.3712 \) ensures \( Y - \Delta = 1 \) in steady state.
3.2 Parameter values

The parameter values are listed in Table 1, organized into eight categories. Population: $\Psi$. Preferences: $\beta$, $\sigma$, $\omega_1$, $\omega_2$, $\varrho$. Technology: $\Delta$, $\varsigma$, $\delta_K$, $\delta_H$, $\epsilon_w$, $\zeta$. Fiscal: $G$, $\tau_N$, $\tau_K$, $\tau_2$. Price setting: $\varepsilon$, $\psi$. Mortgages: $\theta$, $\kappa$, $\alpha$. Bond market: $\vartheta$. And monetary policy: $\pi$, $\nu_{\pi}$, $\rho_{\mu}$, $\rho_\eta$.

Most parameters can be assigned values independently, without solving a system of steady-state equations. Six parameters ($\omega_1$, $\omega_2$, $\varrho$, $\epsilon_w$, $\tau_K$, $\tau_2$) have to be obtained jointly from such steady-state relations. The parameters $\zeta$ and $\vartheta$ are calibrated on the basis of the dynamic properties of the model, given all other parameter values.

We start by describing, in the order of the above categories, those parameters that are calibrated individually. The population parameter $\Psi$ is set equal to 2/3, so that homeowners correspond to the 3rd and 4th quintiles of the wealth distribution and capital owners to the 5th quintile. As is typical in the New-Keynesian literature, $\sigma = 1$. The discount factor is constrained by Euler equations. Data averages for the FRM and inflation rates imply $\beta = 0.9883$. Setting $\Delta = 0.2048$ implies zero steady-state profits, for the value of $\varepsilon$ noted below. The parameter $\varsigma$ then corresponds to the NIPA share of capital and is set equal to 0.283. The depreciation rates $\delta_K$ and $\delta_H$ are set equal to 0.02225 and 0.01021, respectively, on the basis of the average flow-stock ratios for capital and housing. Based on NIPA, $G$ is set equal to 0.138 and $\tau_N$ to 0.235. The price-setting parameters take on uncontroversial values, $\varepsilon = 0.83$ and $\psi = 0.75$. The loan-to-value ratio for new loans $\theta$ is set equal to 0.6, the long-run cross-sectional average for conventional newly-built home mortgages. The amortization parameters are taken from Kydland et al. (2016): $\kappa = 0.00162$ and $\alpha = 0.9946$. In the Taylor rule, $\pi = 0.0113$, the same value used in the calibration of $\beta$ and $\nu_{\pi} = 1.5$, a typical steady-state mortgage debt to quarterly GDP ratio equal to 1.61. This is somewhat lower than in the data, likely due to the fact that the model speaks only to first mortgages and does not include second mortgages and other forms of housing credit.

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39 In a deterministic steady state, FRM and ARM rates are equal. As a consequence, the steady-state ARM rate implied by the above value of $\beta$ is somewhat higher than its data average. The model dynamics, however, are quantitatively almost the same regardless of whether FRM or ARM rate (or return on capital) is used to calibrate $\beta$.

40 The average price duration is thus $(1 - \psi)^{-1} = 4$ quarters.

41 These parameters, together with the loan-to-value ratio, the depreciation rate for housing, and the steady-state housing stock imply a steady-state mortgage debt to quarterly GDP ratio equal to 1.61. This is somewhat lower than in the data, likely due to the fact that the model speaks only to first mortgages and does not include second mortgages and other forms of housing credit.
value in the New-Keynesian literature. Finally, \( \rho_\mu = 0.99 \) and \( \rho_\eta = 0.3 \). The persistent shock is thus close to random walk and the temporary shock has autocorrelation in the range of values typically found in the literature.

Given the above parameter values, six parameters \((\omega_1, \omega_2, \varrho, \epsilon_w, \tau_K, \overline{\tau}_2)\) are calibrated jointly by minimizing, in steady state, equally weighted distance between six targets in the data and the model: the observed average capital-to-output ratio \((K)\); the housing stock-to-output ratio \((H)\); the aggregate hours worked \((N)\); the ratio of mortgage payments to homeowner’s income \((m_2/income_2)\); homeowner’s income share from transfers \((\overline{\tau}_2/income_2)\); and capital owner’s income share from labor \((\epsilon_w wn_1/income_1)\). Here, \( income_1 = (rk + m_1) + \epsilon_w wn_1 + \tau_1 \) and \( income_2 = wn_2 + \overline{\tau}_2 \), which are constructed to be consistent with the way income is defined in the Survey of Consumer Finances (SCF). In particular, the capital owner’s income includes income from all assets, including those backed by mortgage payments. The steady-state relations between the six parameters and targets consist of four optimality conditions (for capital, housing, and labor supply by the two agents) and the model counterparts to individual incomes \((income_1 \text{ and } income_2)\). While the six parameters and targets are interdependent, they are loosely related as follows. \( K \) identifies \( \tau_K \), \( H \) identifies \( \varrho \), and the homeowner’s income share from transfers identifies \( \overline{\tau}_2 \). Further, the labor supply parameters \( \omega_1, \epsilon_w, \text{ and } \omega_2 \) are identified from the three labor-related variables: aggregate hours worked, capital owner’s income share from labor, and the ratio of mortgage payments to homeowner’s income, most of which is labor income. The values of the six targets are provided in Table 2 and the resulting parameter values in Table 1. Table 2 also reports the model’s implications for steady-state moments of the two agents not targeted in calibration, as well as some additional aggregate ratios. While the model’s cross-section is coarse, the model is consistent with the corresponding cross-sectional facts, alongside the standard aggregate ratios.

Given all other parameter values, the PPF curvature parameter \( \zeta \) and the parameter \( \vartheta \) governing the bond market participation cost are calibrated on the basis of dynamics.
In particular, the model is required to generate the standard New-Keynesian responses to the temporary policy shock.\textsuperscript{42} The logic behind this calibration is that there is much more information in the literature on the responses of key variables to temporary than persistent policy shocks (e.g., Christiano, Eichenbaum and Evans, 2005, among many others). Thus, $\zeta$ is calibrated so that in response to the temporary policy shock the model generates an increase in the short-term nominal interest rate, accompanied by a decline in both output and inflation of a smaller magnitude in absolute value than the increase in the short rate. Setting $\zeta = 3.2$ achieves this outcome.\textsuperscript{43} Further, $\vartheta$, which controls consumption smoothing by homeowners, is set equal to 0.15 so that, as in the data, aggregate consumption declines in response to the temporary shock by a little over half as much as output. Essentially the same values of $\zeta$ and $\vartheta$ are obtained regardless of which mortgage contract is used.

Finally, we note the role of the fiscal parameters ($G$, $\tau_N$, $\tau_K$, $\tau_2$). Government expenditures ensure that tax revenues can be sensibly distributed across the agents, without distorting the composition of their incomes. The tax on capital ensures that, given the calibrated $\beta$, the observed capital-to-output ratio is consistent with the Euler equation for $k$. The transfer represents a fixed part of homeowners’ income and thus determines the extent to which their income is subject to fluctuations in labor income. The labor income tax achieves realistic net income and thus a realistic ratio of mortgage payments to net income.

\section{Findings for the benchmark monetary policy shocks}

This section presents benchmark findings for the two policy shocks modeled as independent AR(1) processes. Here, we simply report the findings, highlight the main lessons, and defer the explanation of the mechanism behind the results to the next section.

\textsuperscript{42}As noted earlier, we prefer to keep the model relatively simple to highlight its mechanism. As a result, the model does not reproduce the exact timing and shape of the New-Keynesian responses, for which various additional frictions are necessary. We simply aim to achieve the right direction and magnitude of the responses. Further research can fine tune their shape and timing.

\textsuperscript{43}Rupert and Šustek (2019) discuss why a parameter restricting capital adjustment is critical for generating the standard New-Keynesian impulse responses.
Figure 2 demonstrates the presence of the debt channel in the model. This is the model counterpart to Figure 1, conditional on shock persistence (here we consider an increase, rather than a cut, in the policy rate). The model generates the differences in the pass through of the policy rate to mortgage interest rates across the two mortgage contracts and across new and outstanding debt, for a given persistence of the policy rate.

4.1 The experiments

The next four figures, Figures 3-6, present the findings for some key variables, distinguishing between the two shocks (temporary and persistent) and contracts (ARM and FRM). Each figure contains responses of three versions of the model: with both mortgages and sticky prices (MoNK), with mortgages only (Mo), and with sticky prices only (NK). Specifically, Mo has $\psi = 0$ and NK has $\theta = 0$, implying $D_t = 0$ and $m_{jt} = 0$. All other parameter values are kept as in Table 1 (setting $\theta = 0$ affects the steady state, but the differences are minuscule). The purpose of the alternative specifications is to isolate the effects of the two frictions, as well as their interaction.44

For both contracts and shock types, the size of the shock is such that on impact, in MoNK, the short-term nominal interest rate increases by one percentage point (annualized). The same shock size is then used in the decomposition into Mo and NK. In the figures, the inflation rate is expressed as annualized percentage point deviations; quantities are expressed as percentage deviations.

4.2 Temporary policy shock

Figures 3 and 4 pertain to the temporary policy shock. Recall that the responses of the nominal interest rate, inflation, output, and aggregate consumption are engineered by an appropriate parameterization of $\zeta$ and $\vartheta$ to be consistent with the New-Keynesian channel.

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44Because the equilibrium needs to be recomputed when the specification changes, the below impulse responses from Mo and NK do not necessarily sum up to the one in MoNK.
There are two main, related, takeaways from Figures 3 and 4. First, the responses of all variables in MoNK (with the trivial exception of mortgage payments) are almost the same regardless of the contract used. Second, the responses of MoNK are similar to those of NK, with the exception of consumption of homeowners and, to a lesser extent, consumption of capital owners. Thus, except individual consumption, the debt channel is muted for the transmission of the temporary shock and this shock transmits mainly through sticky prices.

With regard to individual consumption, there are two features to note: (i) the differences in the responses across the agents and (ii) between MoNK and NK. Regarding the first feature, the model possesses the attractive property that consumption of homeowners declines more than consumption of capital owners. In MoNK, about twice as much under ARM and by two thirds more under FRM. The decline in homeowners’ consumption is also substantially more persistent. This property of the model is consistent with empirical evidence that homeowners are constrained in their ability to smooth out fluctuations in income (e.g., Kaplan and Violante, 2014). Regarding the second point, individual consumption is visibly sensitive to the presence of the debt channel. This is particularly the case under ARM, where the decline in homeowners’ consumption in MoNK is by a third as large as in NK, due to the temporary sharp increase in real mortgage payments, which provides an additional channel of monetary policy transmission interacting with sticky prices. The decline in homeowners’ consumption under ARM is also clearly more persistent in MoNK than in NK.

4.3 Persistent policy shock

Figures 5 and 6 pertain to the persistent policy shock. Due to higher persistence, the responses are plotted for 40 periods, instead of 20 as in the case of the temporary shock. Observe that in line with equations (18) and (21) the short rate and the inflation rate both increase by the same magnitude, leaving the ex-ante real interest rate essentially unaffected. Furthermore, as already observed in Figure 2, and in line with our earlier discussion, the FRM interest rate on new loans also increases by about the same magnitude as the short
rate. The persistent shock thus manifests itself in the level factor.

There are four main takeaways from Figures 5 and 6. First, the differences between MoNK and Mo are small, meaning that most of the transmission of the shock works through the debt channel, with sticky prices playing secondary role. Second, most of the effects are redistributive, between consumption of the two agents and the two types of investment, with only small effects on aggregates. The lack of sizable aggregate responses occurs despite the fact that there are technological costs of changing the composition of aggregate expenditures. Third, there are marked differences between ARM and FRM, with the responses quantitatively larger under ARM. And fourth, the real effects of monetary policy are present despite the fact that the real interest rate is essentially unaffected. In particular, the responses of individual consumption are not resulting from intertemporal substitution, a channel critiqued by Kaplan et al. (2018).

As in the case of the temporary shock, there are significant differences in the magnitude of the responses of consumption of the two agents. Here again, in absolute value, consumption of homeowners responds more than consumption of capital owners. This is especially visible under ARM, where the decline in consumption of homeowners is immediate, due to the sharp persistent increase in real mortgage payments, which is costly to smooth out. Under FRM, instead, consumption of homeowners increases, due to the gradual decline in real mortgage payments. But in contrast to the immediate decline under ARM, the increase under FRM is gradual. This again reflects the costly consumption smoothing of homeowners. Homeowners would like to increase consumption immediately by borrowing against the future savings on their mortgage payments on outstanding debt, but this is costly due to the bond market participation cost. As a result, consumption increases only gradually and homeowners substitute on impact towards housing consumption, which can be financed though mortgages without incurring the participation cost. This substitution occurs even though the FRM rate on new loans increases, as shown in Figure 2.

While there is ample evidence on the responses of macro variables to temporary policy
shocks, information on responses to persistent policy shocks is so far, at best, scattered, due to difficulties with identification. Nonetheless, a couple of studies that attempted such analysis give some support to the responses generated by the model. Diebold et al. (2006) estimate a bi-directional (macro to yields, yields to macro) term structure model. A level factor shock in their model increases the short and long rates, inflation, and to a smaller extent aggregate economic activity. A similar result is obtained also by Rudebusch and Wu (2008). Using a standard VAR empirical framework, but with novel identification, Uribe (2018) finds that a permanent nominal interest rate shock increases inflation nearly one-for-one, accompanied by a modest increase in output. The responses of interest rates, inflation, and output from these studies are consistent with the properties of the model.\footnote{A recent work by Inoue and Rossi (2018) suggests that the empirical effects of persistent shocks may be different when the economy is at the zero lower bound.}

5 The mechanism

This section explains the mechanism behind the findings reported above. In particular, it explains why in the above experiments the temporary shock in MoNK propagates mainly through the New-Keynesian channel, whereas the persistent shock propagates mainly through the long-term debt channel. This is again done by considering each channel in isolation. We also explain why the real effects of the temporary shock are aggregate whereas those of the persistent shock are predominantly redistributive.

5.1 New-Keynesian channel

While the New-Keynesian monetary transmission mechanism may be well understood, for completeness we briefly explain how it operates in our setting. As in the above experiments, when considering the role of the New-Keynesian mechanism in MoNK, we remove mortgages from the model (the NK specification: $\theta = 0$, implying $D_t = 0$ and $m_{jt} = 0$). In this case, housing is fully equity financed but the two agents still trade the one-period bond, with
homeowners at a cost. There is, however, no long-term debt in the economy. The model is thus akin to a TANK model, albeit with capital and housing, two assets that can be used to smooth out consumption in the aggregate.

The nominal rigidity in the New-Keynesian mechanism is the price stickiness contained in the optimization problem (13). As demonstrated in numerous texts (e.g., Galí, 2015), the log-linearized version of the first-order condition for this problem, once aggregation is imposed, yields the New-Keynesian Phillips curve (NKPC)

\[
\pi_t = \frac{(1-\psi)(1-\beta\psi)}{\psi} \hat{\chi}_t + \beta E_t \pi_{t+1},
\]

(22)

where \(\hat{\chi}_t\) is a deviation of the real marginal cost from steady state, or equivalently from the flexible-price level (like in steady-state, under flexible prices the marginal cost is constant).\(^{46}\)

For \(\beta\) close to one, the NKPC provides a negative relationship between an expected change in the inflation rate, \(E_t \pi_{t+1} - \pi_t\), and the real marginal cost, \(\hat{\chi}_t\). For a highly persistent inflation rate, \(E_t \pi_{t+1} - \pi_t\) is close to zero, implying \(\hat{\chi}_t \approx 0\). This results in the case of the persistent policy shock and monetary policy has almost no real effects (in the quantitative experiment the expected change in inflation is slightly negative, as inflation declines slowly back to its steady state following the shock, leading to small positive real effects). If, in contrast, the inflation rate is not very persistent, then \(E_t \pi_{t+1} - \pi_t \neq 0\) and \(\hat{\chi}_t \neq 0\). This results under the temporary policy shock, in which case monetary policy has real effects.

In the supplementary material we establish that percentage deviations in the real marginal cost are positively related to percentage deviations in aggregate output, \(\hat{Y}_t\).\(^{47}\) Equation (22) thus provides a negative relationship between \(E_t \pi_{t+1} - \pi_t\) and \(\hat{Y}_t\). As a result, the policy shock that temporarily reduces inflation, thus generating \(E_t \pi_{t+1} - \pi_t > 0\), produces a decline in output, \(\hat{Y}_t < 0\). As both agents’ incomes represent a claim on aggregate output

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\(^{46}\)Equation (22) is derived under the common assumption that the steady-state inflation rate is equal to zero, which leads to a more elegant expression for the linearized NKPC than would otherwise be the case. The model is computed under the calibrated non-zero steady-state inflation rate.

\(^{47}\)A positive relationship between \(\hat{\chi}_t\) and \(\hat{Y}_t\) is easier to establish in the textbook New-Keynesian model, in which \(\hat{C}_t = \hat{Y}_t\).
(in the form of labor and capital income), the temporary decline in output translates into a temporary decline in income, which the agents would like to smooth out. As a result, both capital and housing investment drop. However, as investment is costly to adjust, due to the nonlinear PPF, full consumption smoothing cannot be achieved and the declines in incomes must be partly reflected also in temporary declines in consumption.

Further, because consumption smoothing is costlier for the homeowner than the capital owner, $C_{2t}$ declines by more than $C_{1t}$. There are two reasons for this. First, housing is costlier to adjust than capital as it involves a direct utility loss. And second, the homeowner cannot easily tap (indirectly) into using capital to keep consumption smooth as borrowing from the capital owner through the bond market involves a cost in the form of the premium over the short rate.\footnote{An additional reason for a stronger response of homeowners’ consumption is that capital owners are partially hedged against the drop in aggregate output by an increase in monopoly profits, which are well known to be counter-cyclical in New-Keynesian models. This partially compensates them for the decline in labor and capital income. On the other hand, in our calibration, a larger fraction of homeowners’ income is derived from fixed transfers, which provide a buffer against shocks in the income of homeowners.} Homeowners nonetheless use the bond market to some extent, which explains the persistence in the decline in their consumption, as borrowing to mitigate the impact of the shock results in debt repayments, and thus lower consumption, over time.

Finally, the fall in capital investment leads to a decline in $q_{Kt}$ and thus positive expected capital gains, $E_t(1-\delta_K)q_{K,t+1}-q_t > 0$, generating the typical New-Keynesian increase in the ex-ante real interest rate $r^*_t$ (see equation (17)), following a monetary tightening. Effectively, the increase in the real interest rate ensures that, in equilibrium, capital owners are content with the temporary decline in their consumption, as required by their Euler equation.

\section*{5.2 Long-term debt channel}

Next, consider the case when housing is financed by mortgages and there are no New-Keynesian frictions in the economy (the Mo specification: $\psi = 0$, meaning flexible prices). In this case, the NKPC (22) implies $\hat{\chi}_t = 0$. The marginal cost is constant and $r_t$ and $w_t$ are equal to their respective marginal products, subject to a constant markup.
In this case, the only nominal rigidity in the model is the long-term nominal mortgage. Section 2.10 explained how demand for new mortgages is determined and that it depends on the cash flows of mortgage payments over the life of the loan. We have referred to that effect as the ‘price effect’. In this section we focus on how the equilibrium is affected by the outstanding stock of mortgage debt, referring to this effect as the ‘income effect’, since it affects disposable income. As in the case of the price effect, the income effect is underpinned by the timing of the cash flows of mortgage payments.

In the case of outstanding debt, what matters are the cash flows of mortgage payments over the remaining term of existing loans. It is convenient to write these payments in real terms: \( \tilde{m}_{j,t+s} \equiv m_{j,t+s}/p_{t+s} \), with \( j \in \{1, 2\} \) denoting the agent and \( s = 0, 1, 2, \ldots \) denoting the time period ahead. To focus squarely on outstanding debt, let us assume that \( l_{j,t+s} = 0 \), for \( s = 0, 1, 2, \ldots \). That is, there are no new loans originated after and including the current period \( t \). The sequence of real cash flows on outstanding debt, following and including period \( t \), is thus

\[
\tilde{m}_{jt} = \frac{R_{jt} + \gamma_{jt}}{1 + \pi_t} \tilde{d}_{jt},
\]

\[
\tilde{m}_{j,t+1} = \frac{R_{j,t+1} + \gamma_{j,t+1}}{(1 + \pi_{t+1})(1 + \pi_t)} (1 - \gamma_{jt}) \tilde{d}_{jt},
\]

\[
\tilde{m}_{j,t+2} = \frac{R_{j,t+2} + \gamma_{j,t+2}}{(1 + \pi_{t+2})(1 + \pi_{t+1})(1 + \pi_t)} (1 - \gamma_{j,t+1})(1 - \gamma_{jt}) \tilde{d}_{jt}, \quad \text{etc.}
\]

Here, \( \tilde{d}_{jt} \equiv d_{jt}/p_{t-1} \) is the stock of outstanding debt, as of period \( t \), in real terms. Recall that in period \( t \) the variables \( R_{jt}, \gamma_{jt}, \) and \( d_{jt} \) are pre-determined state variables, which in subsequent periods evolve according to the laws of motion (5)-(7), for the homeowner, and (9)-(11), for the capital owner (\( p_{t-1} \) is also pre-determined). Further, the evolution of the interest rate depends on whether the loans are FRM or ARM.

### 5.2.1 FRM

Under FRM, \( R_{j,t+s} \) is constant for all \( s = 0, 1, 2, \ldots \). As a result, monetary policy affects the remaining sequence of real mortgage payments on outstanding debt only through inflation,
πt, πt+1, πt+2, .... An increase in inflation reduces the real value of the cash flows. What matters quantitatively, however, is the accumulated effect of the inflation rate. As is apparent from equations (23)-(25), the size of the real effects on the cash flows gradually increases over time in line with inflation persistence. The temporary policy shock affects inflation only in the short run and thus has a much smaller effect on the real value of mortgage payments in the later periods of the remaining term of the loan than the persistent shock, which affects inflation almost permanently.

Through inflation, a monetary policy shock thus generates a sequence of real transfers from one agent to the other. The responses of the agents are thus opposite to each other, with little effect on aggregate output (in the quantitative experiments, the effects on output are small despite the technological costs of changing the composition of aggregate expenditures embedded in the concave PPF). If homeowners had no means to smooth out fluctuations in mortgage payments, and so behaved as hand-to-mouth consumers, consumption would have to adjust each period in line with the cash flows. Conversely, if homeowners could costlessly trade the one-period bond, and so behaved as permanent income hypothesis consumers, all that would matter for their decisions would be the present value of the real cash flows following the shock, not their timing (higher inflation resulting in a lower real present value).

The present value effect, also known as the debt re-valuation or the Fisher channel, is the effect at work in, e.g., Doepke and Schneider (2006) and Auclert (2018). Our case lies in-between these two extremes: homeowners can adjust their bond holdings but this margin incurs a cost. As discussed above, the costs of keeping consumption smooth are higher for homeowners than capital owners. In addition, the cash flows are larger, relative to income, for homeowners than capital owners (see Table 2), which further contributes to the stronger response of homeowners’ consumption.

Observe from equations (23)-(25) that if mortgages were one-period loans (γjt = 1), the inflation effect would occur only in period t and inflation persistence would be irrelevant. Furthermore, if mortgages were written in real terms (inflation indexed), the effects described
The long-term nominal nature of the loan is thus critical for the real effects.

5.2.2 ARM

Under ARM, the income effect is dramatically different. In this case, the interest rate in the current period $t$, $R_{jt}$, is predetermined, equal to $i_{t-1}$, but subsequent interest rates are reset in line with the short rate: $R_{jt+1} = i_t$, $R_{jt+2} = i_{t+1}$, etc. After the current period, monetary policy thus affects both the denominator and the numerator in the expressions for the real cash flows (24) and (25). To highlight the combined effect, let us focus on the first period in which the mortgage rate is reset ($s = 1$). The ratio in equation (24) can be approximately written as

$$
\frac{\hat{i}_t + \gamma_{e,t+1}}{(1 + \pi_{t+1})(1 + \pi_t)} \approx \frac{\hat{i}_t + \gamma_{e,t+1}}{1 + \pi_{t+1} + \pi_t} \approx \hat{i}_t + \gamma_{e,t+1} = r^*_t + \pi_{t+1} + \gamma_{e,t+1}.
$$

Similar expressions can be derived for the cash flows in other periods, at least as long as the effect of accumulated inflation in the denominator can be safely ignored (discussed further below).

The key insight here is that an increase in the short rate has the same effect on the real mortgage payment regardless of whether the increase is due to an increase in the real rate, $r^*_t$, or the inflation rate, $\pi_{t+1}$. In contrast to the FRM case, an increase in $\pi_{t+1}$ under ARM increases the real value of the mortgage payment. Monetary policy can thus engineer the same effect on the real cash flows regardless of whether the effect comes from its ability to increase the real rate or expected inflation. All that matters is the increase in the short rate. But while its ability to affect the real rate may only be temporary (as under sticky prices

\[49\text{With inflation indexed loans, the numerator would be multiplied by the same sequence of inflation rates that appears in the denominator.}\]

\[50\text{Here, the first approximation step holds for sufficiently small inflation rates and the second step holds again for sufficiently small inflation rates and for } \gamma_{e,t+1} \text{ sufficiently smaller than one, which is generally the case, unless the stock is close to maturity. Further, in the final step, we have utilized the Fisher equation (17), assuming perfect foresight for easier exposition.}\]
and the temporary shock), expected inflation can be increased persistently in line with the
persistent policy shock. Like under FRM, under ARM the long-term mortgage debt channel
is again more powerful under the persistent than the temporary policy shock.

Over time, the effect of accumulated inflation in the denominator starts to dominate
and the overall effect of the persistent shock on real mortgage payments under ARM starts
to resemble that under FRM. This is why, in Figure 5, after period 30, the real value of
mortgage payments falls below the initial level.

As the effect of the persistent shock on real cash flows is now most powerful on impact, as
opposed to being gradual as in the FRM case, the responses of individual consumption are
immediate and stronger. Of course, because the shock generates transfers from one agent
to the other, the responses of the two agents are again opposite to each other (with the
responses of the homeowner again stronger for the reasons noted above), leaving aggregates
essentially unaffected.\textsuperscript{51}

Observe that if mortgages were one-period loans ($\gamma_{jt} = 1$), the above effects would be
eliminated. Again, the long-term nominal nature of mortgages is critical for the effects
coming from the debt channel.

\section*{5.3 Interaction between the channels}

One place where an interesting interaction between the two channels arises is consumption
of homeowners under the temporary shock, especially in the ARM case. In the Mo case,
there is almost no response of $C_{2t}$ on the impact of the shock but this variable responds
substantially more strongly in MoNK than in NK. The reason is that in the Mo case, the
temporary shock leads only to very temporary redistributive effects and thus, despite the
participation cost, homeowners can relatively easily use the bond market to borrow from

\textsuperscript{51}If the agents could costlessly trade the one-period bond, the present value of the remaining mortgage
payments would not be affected by changes in the short rate (a loan of one dollar would be for both agents
always worth one dollar in present value terms). An outstanding ARM can be thought of as a new ARM of
a shorter term and therefore the same argument of completing the market with a one-period bond, made in
the case of the price effect, applies.
capital owners, who have received a temporary income windfall, which they would like to smooth out. In the MoNK case, the redistributive effect is accompanied by an aggregate decline in income of both agents and an increase in the ex-ante real rate. Borrowing to smooth out the increase in mortgage payments is thus harder to achieve.

6 Alternative formulations of the policy shocks

To facilitate the understanding of the inner workings of the model, we have modeled the two policy shocks as independent AR(1) processes. This formulation also had the benefit that it allowed a clear mapping from the model to the standard level and slope factors. Here we consider two re-formulations of the Taylor rule that provide two alternative interpretations of the persistent shock, as a statement shock, in the sense of Gürkaynak et al. (2005a), and as an information shock about the future state of the economy, in the sense of Nakamura and Steinsson (2018). In both cases, the shock, like in the benchmark specification, affects expectations of future short rates.

6.1 Persistent shock as a statement shock

A statement shock is a policy shock whereby the central bank surprises the private sector by changing the expected path of future policy rates (e.g., by a press release) while leaving the current policy rate unchanged. To provide such interpretation of the persistent shock in the model, consider an orthogonal rotation of the two policy shocks. First, rewrite the Taylor rule (16) as

\[ i_t = i + \nu \pi_t (\pi_t - \pi) + v^\top z_t \]

where \( v^\top \equiv [1 - \nu, 1] \) and \( z_t^\top \equiv [z_{1t}, z_{2t}] \), with \( z_{1t} \equiv \mu_t - \pi \) being the persistent shock and \( z_{2t} \equiv \eta_t \) being the temporary shock. Next, consider an invertible two-by-two matrix \( M \) and define a new vector of shocks \( z_t^* = M z_t \). The Taylor rule (16) can then be written in terms
of the new shocks as
\[ i_t = i + \nu_\pi (\pi_t - \pi) + v^\top M^{-1} z_t^*. \] (26)

The stochastic process for \( z_t^* \) is derived from the stochastic process for \( z_t \) and the matrix \( M \). Consolidating the two AR(1) processes for \( z_{1t} \) and \( z_{2t} \) into a VAR(1), \( z_t \) evolves as
\[ z_{t+1} = \rho z_t + \xi_{t+1}, \] where \( \rho \) is a diagonal matrix with \( \rho_\mu \) and \( \rho_\eta \) on the diagonal. The process for \( z_t^* \) is then
\[ z_{t+1}^* = \lambda z_t^* + M \xi_{t+1}, \]
where \( \lambda = M \rho M^{-1} \). Observe that since \( z_t^* \) is just a linear combination of \( z_t \), \( \lambda \) has the same eigenvalues as \( \rho \) and, thus, the new process has the same persistence as the original process.

The four elements of \( M \) are identified from four restrictions. First, like the original shocks, we require the new shocks to be orthogonal to each other: \( E(z^*_1 z^*_2) = 0 \). Then, we normalize the variance of the shocks to be unity: \( E(z^*_1) = 1 \) and \( E(z^*_2) = 1 \). We are free to do so as we are not interested in data decomposition. Finally, we impose a restriction that allows \( z^*_1t \) to be interpreted as a statement shock. Specifically, combining the Taylor rule (26) with the Fisher equation (17), an equivalent expression to equation (18) can be derived for the new shocks
\[ i_t \approx i - v^\top M^{-1} \frac{1}{\nu_\pi} \lambda \left( I - \frac{1}{\nu_\pi} \lambda \right)^{-1} z_t^* + \sum_{s=0}^{\infty} \left( \frac{1}{\nu_\pi} \right)^s E_t r^*_{t+s}, \] (27)
where we have used the stochastic process for \( z_t^* \) to evaluate the expectations of future values of \( z_t^* \). Further, while \( r^*_{t+s} \) potentially depends on the \( t+s \) values of all state variables, suppose that a reasonable approximation is \( r^*_{t+s} \approx g^\top z^*_{t+s} \), where the vector \( g^\top \) is determined in equilibrium, as defined in Section 2. In this case, equation (27) can be written as
\[ i_t \approx i - v^\top M^{-1} \frac{1}{\nu_\pi} \lambda \left( I - \frac{1}{\nu_\pi} \lambda \right)^{-1} z_t^* + g^\top \left( I - \frac{1}{\nu_\pi} \lambda \right)^{-1} z_t^* \] (28)
or \[ i_t \approx i - a_1 z^*_1 + a_2 z^*_2, \] where \( a_1 \) and \( a_2 \) are the corresponding loadings from equation (28)
on $z_{1t}^*$ and $z_{2t}^*$, respectively. The final restriction on $M$, which makes $z_{1t}^*$ a statement shock, is $a_1 = 0$. That is, $z_{1t}^*$ has no effect on the equilibrium short rate but, through the transition matrix $\lambda$, it forecasts future values of $z_{2t}^*$ and therefore future values of the short rate. Thus, $z_{1t}^*$ is a statement shock, while $z_{2t}^*$ is an action shock. By communicating an expected path of future policy rates, the central bank implicitly specifies an expected sequence of deviations from the systematic part of the Taylor rule (responses to inflation). The statement shock is thus a pure policy shock, to be contrasted with the information shock in the next section.

Figure 7 reports the responses of the key variables in MoNK to $z_{1t}^*$ under ARM and FRM. The response of $i_t$ to $z_{1t}^*$ turns out to be about the same under both contracts (so only one line is plotted) and the size of the shock is chosen so that the increase in $i_t$ reaches its peak at one percentage point (annualized). This makes the responses of the economy as comparable to those in Figures 3-6 as possible. Figure 7 confirms that $z_{1t}^*$ works like a statement shock: the short rate does not move on the impact of the shock in period 0, but increases in subsequent periods, reaching its peak in period 4, which corresponds to an annual “hiking cycle”. Afterwards, it slowly converges back to its steady-state value. Except for the initial four-quarter hiking period, the response of $i_t$ resembles that in Figures 5 and 6 (the persistent shock $\mu_t$). Observe also that, under FRM, the long rate $i_F^t$ jumps on the impact of the shock by a little less that the full percentage point, in anticipation of the future persistent increase in the short rate.

Because the increase in $i_t$ is persistent, one may think that the dynamics in MoNK will be driven almost exclusively by long-term debt, as in Figures 5 and 6. The response of the economy in the first few periods after the shock (during the hiking cycle), however, turns out to be driven largely by sticky prices, even though the presence of mortgages affects the size of the responses of some variables (mainly individual consumption and housing investment). First, since $i_t$ stays above its steady-state persistently, once the hiking cycle is complete, inflation is ultimately determined by the Fisher effect. Because producers who can adjust

\[ \frac{p_t^*}{p_{t+s}} \approx (g^\top z_{t+s}^*) \]

is a reasonable approximation. The vector $g^\top$ is obtained by an iterative procedure: a given $g^\top$ yields a particular $M$, which in turn yields a new equilibrium $g^\top$. This iteration is repeated until $g^\top$ and $M$ are mutually consistent and $a_1 = 0$. 

\[ 52 \text{It turns out quantitatively that } p_t^*/p_{t+s} \approx g^\top z_{t+s}^*/z_{t+s}^* \]
prices understand this, inflation jumps up immediately on the impact of the shock. This makes the ex-ante real interest rate decline, which further increases inflation in the short run, above its ultimate increase and the ultimate increase in the short rate. As a result, output and all its components increase on impact. The announcement of a persistent future policy rate increase thus generates a short-lived boom.\textsuperscript{53} Once inflation lines up with the short rate, which happens around period 5, the responses of the economy start to be driven by the debt channel, with the effects similar to those in Figures 5 and 6, manifesting themselves mainly in individual consumption and housing investment.

### 6.2 Persistent shock as news about the future state of the economy

In the previous section, the statement shock was a pure policy shock. Nakamura and Steinsson (2018) instead argue that surprises in central bank statements about the future path of policy rates reflect revelation of information the central bank has about the likely future state of the economy (Cieslak and Schrimpf, 2018, document that different types of central bank statements have different content, in terms of pure policy shocks vs. information about the state of the economy).\textsuperscript{54} In this case, monetary policy communication is about the likely future path of some state variable affecting inflation, or output gap in a broader formulation of the Taylor rule, not about the central bank’s deviations from the rule.

To make this notion concrete, we follow Nakamura and Steinsson (2018) and consider shocks affecting the real interest rate that the policy rule of the central bank needs to accommodate, in order to stabilize inflation. In contrast to their model, where the real rate is shocked directly, the fundamental driving force here is total factor productivity, to which

\textsuperscript{53}This short-term response of the economy is driven by similar effects that are responsible for the so-called forward-guidance puzzle (Del Negro, Giannoni and Patterson, 2015; McKay, Nakamura and Steinsson, 2016). Here, however, they lead to a short-lived expansion, rather than a prolonged recession. The main difference in the experiment is that the announcement is about a gradual increase to a persistently higher future policy rate, as opposed to an announcement of a temporary future increase.

\textsuperscript{54}See also Hamilton (2008) and Jarocinski and Karadi (2018).
the real rate endogenously responds. The Taylor rule is modified as

\[ i_t = r_t^* + \pi + \nu(\pi_t - \pi) + \eta_t, \quad \nu > 1, \]

where \( \eta_t \) is the standard temporary shock. In contrast to Taylor rules (16) and (26), there are no persistent policy shocks per se, but the central bank allows the policy rate to move one-for-one with the ex-ante one-period real rate \( r_t^* \), defined in equation (17). Such a Taylor rule is common in the literature (see, e.g., Galí, 2015). Combining the Taylor rule (29) with the Fisher equation (17) implies that, absent \( \eta_t \) shocks, this policy fully stabilizes inflation; i.e., \( \pi_t = \pi \forall t \). As a result, movements in the policy rate reflect one-for-one movements in the real rate.

A news about the future state of the economy is an exogenous random variable \( S_t \), which is informative about the future path of total factor productivity \( A_t \) (which we now allow to be time-varying). We capture this information by a VAR(1) process

\[
\begin{bmatrix}
A_{t+1} \\
S_{t+1}
\end{bmatrix} = \begin{bmatrix}
\rho_A & 1 \\
0 & \rho_S
\end{bmatrix} \begin{bmatrix}
A_t \\
S_t
\end{bmatrix} + \begin{bmatrix}
\xi_{A,t+1} \\
\xi_{S,t+1}
\end{bmatrix},
\]

where \( \rho_A, \rho_S \in (0, 1) \) and \( \xi_{A,t} \) and \( \xi_{S,t+1} \) are orthogonal innovations. We set \( \rho_A = 0.985 \), so that, if \( \xi_{A,t} \) is the dominating shock, \( A_t \) has autocorrelation in the range of the values typically estimated for Solow residuals, normalize the spillover effect from \( S_t \) to \( A_{t+1} \) to one, and then choose \( \rho_S \) so as to generate the same equilibrium persistence of \( i_t \) as in Figures 5 and 6. Such calibration yields \( \rho_S = 0.999 \). Observe that the above process implies

\[
\Delta A_{t+1} = (\rho_A - 1)A_t + S_t + \xi_{A,t+1},
\]

where \( S_t \) is close to random walk. The specification of the VAR(1) process in productivity level that is consistent with the required persistence of the nominal short rate is thus equivalent to a process for productivity growth that contains a random slow-moving component,"
$S_t$, as in Bansal and Yaron (2004). From this perspective, a realization of $S_t$ is a news about a very persistent change in the productivity growth rate. As higher productivity growth leads to higher output and consumption growth, a realization of a higher $S_t$, through the Euler equation for bonds, leads to a persistent increase in the ex-ante real rate and, through the Taylor rule (29), in the policy rate. The $S_t$ shock thus works as a persistent shock in the Taylor rule.

Figure 8 reports the responses of the key variables to the $S_t$ shock, where the size of the shock is normalized so as to make the results quantitatively comparable to the results in the previous experiments. As argued above, inflation is fully stabilized and sticky prices (the New-Keynesian Phillips Curve) have no effect on the economy. The realization of the news that productivity growth has persistently increased leads to expectations of higher output and incomes in the future. These expectations dominate the dynamics of the economy and generate a very strong incentive for households to bring consumption forward. Consequently, consumption of both agents increases on impact and investment in both capital and housing decline. There is some difference in the paths of consumption of homeowners under ARM vs. FRM, due to the initial increase in real mortgage payments on ARM outstanding debt, reflecting the increase in the real rate. Where mortgages have a marked impact is the response of housing investment. By increasing expected real cash flows over the life of new loans, the persistent increase in the real rate makes mortgage financing expensive through the price effect discussed in Section 2. As the increase in the real rate is highly persistent, the size of the price effect is essentially the same under ARM and FRM. Under both contracts, housing investment declines almost twice as much as in the absence of mortgages (full equity finance, $\theta = 0$). This additional decline reflects the price effect, as even under equity finance the opportunity cost of funds has increased, due to the increase in the real rate.\footnote{All responses eventually converge back to steady state but this process takes longer than 40 periods.}

Thus, under a policy that accommodates persistent changes in the real rate brought about by news about future productivity, inflation is stabilized and the debt channel affects consumption of homeowners through the payments on outstanding ARM debt and housing
investment through the price effect on new ARM and FRM loans. As inflation is stabilized, the debt channel operates as if mortgage loans were set in real, rather than nominal, terms.

7 Conclusions

Long-term debt facilitates important investment decisions and affects disposable income through its contracted cash flows for many years after origination. Because the cash-flows are specified in nominal terms, debt affordability, both ex-ante and ex-post, depends on two variables directly or indirectly affected by monetary policy: current and expected future nominal interest rates and inflation. Recent research, using both intra-day and monthly or quarterly data, argues that unexpected changes in monetary policy contain an important dimension affecting expectations of future nominal interest rates, in addition to standard temporary policy shocks. In light of these developments in the literature, this paper proposes a tractable framework that enriches the standard sticky-price model used for monetary policy analysis by taking the explicit role of long-term debt into account. Our specific focus was on mortgage debt, which affects a large fraction of the population. The model contains a key feature, highlighted in recent research, that homeowners are constrained in their ability to smooth out fluctuations in income. We have showed how the model can be calibrated to both aggregate and cross-sectional moments, documented and explained its inner workings, and provided two applications.

The lessons learnt from the model, and the answers to the questions posed in the Introduction, can be summarized as follows. The New-Keynesian channel is the dominating channel of transmission for policy shocks that affect the nominal interest rate only temporarily. In contrast, the long-term debt channel is dominating for policy shocks that have a persistent effect on the interest rate. The New-Keynesian channel generates short-lived aggregate effects that are essentially the same under ARM and FRM, with the exception of homeowners’ consumption. In contrast, the long-term debt channel generates prolonged redistributive effects, which are markedly different across ARM and FRM. Therefore, as a
first pass, there is a decoupling between the two channels in the transmission of monetary policy, each transmitting different types of policy surprises, with different consequences for the economy. The two channels, however, interact in affecting homeowners’ consumption under ARM and a temporary shock, making homeowners more exposed to the policy shock than in an economy with FRM or no mortgage debt. Nonetheless, in all experiments considered, homeowners’ consumption and housing investment are the variables most affected by the presence of long-term mortgage debt in the monetary transmission mechanism.

For analytical purposes, we have imposed orthogonality on the two types of shocks. In practice both policy shocks likely occur at the same time, making the two channels operative concurrently. Furthermore, the persistent and temporary shocks can be intertwined, forming shocks with interesting economic interpretations, such as a statement and an action shock. The persistent shock can also have different origins, as a pure policy shock or an information shock about the future state of the economy. Which source is quantitatively more important is a subject of an ongoing debate in the literature. The model is flexible to accommodate both cases. With small modifications of the Taylor rule and the underlying state space, we have used the model to analyze the effects of two alternative interpretations of the persistent shock, a statement vs. an information shock. As a statement shock is a linear combination of the baseline temporary and persistent shocks, both the New-Keynesian and the long-term debt channel are operative under such a shock. In contrast, an information shock is related to an expected persistent change in macroeconomic fundamentals (future productivity in our case). If such a change is accommodated by monetary policy, as a standard Taylor rule would require, the New-Keynesinan channel is neutralized but the debt channel is still operative, affecting mainly housing investment, through the ex-ante cost of mortgage financing.

There are various ways in which the present model could be extended in future work. First, a natural extension is to incorporate various features that have been found necessary in the literature to obtain a close fit to the data along the dimensions well established in empirical studies, such as the responses to the temporary shock. Next, while we have listed
a few reduced-form studies that give some support to the responses in the model to the persistent shock, this area clearly requires further empirical research. In some sense, the theory is ahead of measurement and more empirical work is needed to establish widely accepted ‘facts’. Third, long-term nominal debt adds an extra nominal rigidity to the standard sticky-price framework, with nontrivial implications for optimal monetary policy. What makes the design of optimal policy even more interesting is that both ARM and FRM loans may coexist in a single monetary area, such as the eurozone. Finally, the big open question regards the role of term premia in the transmission mechanism. Existing research suggests that the persistent shock, as reflected in the level factor, is an unlikely source of time variation in term premia. It is, however, feasible that the present framework underestimates the role of the temporary shock if temporary shocks cause movements in term premia, and thus in the cost of long-term loans.
References


Figure 1: Illustration of the debt channel. ECB policy rate and nominal interest rates on mortgages in the euro area. Data source: European Central Bank.
Table 1: Parameter values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
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<td>Disutility from labor (homeowner)</td>
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Table 2: Nonstochastic steady state vs. long-run averages of U.S. data

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<tr>
<td>$H$</td>
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<td>5.28</td>
<td>Housing stock</td>
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<td>0.15</td>
<td>Mortgage payments to income</td>
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<td>$\tau_2/(w_n + \tau_2)$</td>
<td>0.12</td>
<td>0.12</td>
<td>Transfers in homeowner’s income</td>
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<tr>
<td>$\epsilon_w w_n/income_1$</td>
<td>0.53</td>
<td>0.53</td>
<td>Labor income in cap. owner’s income</td>
</tr>
</tbody>
</table>

Not targeted:

**A.** Capital owner’s variables

$(rk + m_1)/income_1$ | 0.42 | 0.39$^\S$ | Income from assets in total income |

$\tau_1/income_1$ | 0.05 | 0.08 | Transfers in total income |

$m_1/netincome_1$ | 0.07 | N/A | Mortg. income to post-tax income |

**B.** Homeowner’s variables

$wn_2/(wn_2 + \tau_2)$ | 0.88 | 0.82 | Labor income in total income |

$m_2/[(1 - \tau_N)wn_2 + \tau_2]$ | 0.18 | N/A | Mortgage payments to post-tax income |

**C.** Earnings distribution

$\epsilon_w w N_1/(\epsilon_w w N_1 + w N_2)$ | 0.59 | 0.54 | Capital owners’ share |

$w N_2/(\epsilon_w w N_1 + w N_2)$ | 0.41 | 0.46 | Homeowners’ share |

**D.** Income distribution

$Income_1/[Income_1 + (w N_2 + \Psi \tau_2)]$ | 0.70 | 0.61 | Capital owners’ share |

$(w N_2 + \Psi \tau_2)/[Income_1 + (w N_2 + \Psi \tau_2)]$ | 0.30 | 0.39 | Homeowners’ share |

**E.** Aggregate housing consumption

$(r_H + \delta_H)H/[C1 + C2 + (r_H + \delta_H)H]$ | 0.15 | 0.17$^\S\S$ | Housing services in aggr. consumption |

Notes

Normalizations in steady state: $Y - \Delta = 1$ and $p = 1$.

Income and earnings data come from SCF. The model counterparts are constructed to be consistent with SEF definitions.

income$_1 = (rk + m_1) + \epsilon_w w n_1 + \tau_1$ (capital owner’s income).  
Income$_1 = (1 - \Psi)income_1$ (aggregate income of capital owners).  
netincome$_1 = [(1 - \tau_K)rk + \tau_K \delta_K k + m_1] + (1 - \tau_N)\epsilon_w w n_1 + \tau_1$ (after tax income of capital owners).  
$^\S$ The sum of capital and business income in SCF, where capital income is income from all financial assets.  
$^\S\S$ NIPA-based estimate; the model counterpart is defined in line with NIPA (the numerator = imputed rents, where $r_H = 1/\beta - 1$ is the net rate of return on housing).
Figure 2: The debt channel in the model. Policy and mortgage interest rates, expressed as percentage point (annualized) deviations from steady state.
Figure 3: Temporary monetary policy shock: ARM. Interest rates and the inflation rate are expressed as percentage point (annualized) deviations from steady state, quantities are in percentage deviations. One period = one quarter. Output is $Y - \Delta$. 
Figure 4: Temporary monetary policy shock: FRM. Interest rates and the inflation rate are expressed as percentage point (annualized) deviations from steady state, quantities are in percentage deviations. One period = one quarter. Output is $Y - \Delta$. 
Figure 5: Persistent monetary policy shock: ARM. Interest rates and the inflation rate are expressed as percentage point (annualized) deviations from steady state, quantities are in percentage deviations. One period = one quarter. Output is $Y - \Delta$. All responses eventually converge back to zero.
Figure 6: Persistent monetary policy shock: FRM. Interest rates and the inflation rate are expressed as percentage point (annualized) deviations from steady state, quantities are in percentage deviations. One period = one quarter. Output is $Y - \Delta$. All responses eventually converge back to zero.
Figure 7: MoNK: responses to a statement shock under ARM and FRM (based on an orthogonal rotation of the original shocks). Interest rates and the inflation rate are expressed as percentage point (annualized) deviations from steady state, quantities are in percentage deviations. One period = one quarter. Output is $Y - \Delta$. All responses eventually converge back to zero.
Figure 8: MoNK: responses to an information shock about an increase in future productivity, under ARM and FRM, based on a policy rule that fully stabilizes inflation. Interest rates and the inflation rate are expressed as percentage point (annualized) deviations from steady state, quantities are in percentage deviations. One period = one quarter. Output is $Y - \Delta$. All responses eventually converge back to zero.