The Welfare Cost of Inflation in the Presence of Inside Money

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In this paper, we ask what role an endogenous money multiplier plays in the estimated welfare cost of inflation. The model is a variant of that used by Freeman and Kydland (2000) with inside and outside money in the spirit of Freeman and Huffman (1991). Unlike models in which the money–output link comes from either sticky prices or fixed money holdings, here prices and output are assumed to be fully flexible. Consumption goods are purchased using either currency or bank deposits. Two transaction costs affect these decisions: One is the cost of acquiring money balances, which is necessary to determine the demand for money and to make the velocity of money endogenous. The other is a fixed cost associated with using deposits. This cost is instrumental in determining the division of money balances into currency and interest-bearing deposits. Faced with these two costs and factors that may vary over time in equilibrium (such as over the business cycle), households make decisions that, in the aggregate, determine the velocity of money and the money multiplier.

The model is consistent with several features of U.S. data: (1) M1 is positively correlated with real output; (2) the money multiplier and deposit-to-currency ratio are positively correlated with output; (3) the price level is negatively correlated with output in spite of conditions (1) and (2); (4) the correlation of M1 with contemporaneous prices is substantially weaker than the correlation of M1 with real output; (5) correlations among real variables are essentially unchanged under different monetary policy regimes; and (6) real money balances are smoother than money-demand equations would predict.

A key feature of the model is that households purchase a continuum of types of goods indexed by their size. It comes from assuming a Leontief-type utility function over these types. One could argue that the distinction between nondurable and (usually larger) durable consumption goods should also be taken into account. We shall not take that step here. Instead, compared with Freeman and Kydland (2000), we consider a more flexible utility function than before, which, in equilibrium, permits the implication that households wish to consume large goods in relatively greater quantities.

With the model economy calibrated to the usual long-run relations in the data—including the selection of values for the two transaction-cost parameters so as to make the model consistent with the empirical average deposit-to-currency ratio and the fraction of capital that is intermediated—the estimated welfare cost of inflation turns out to be rather small. An interesting finding is that the welfare cost as a function of the steady-state inflation rate is very steep for low inflation rates (well under 10%) but quite flat for higher inflation rates. Moreover, we find that the welfare cost is sensitive to the values of the transaction-cost parameters.

Beginning with Bailey (1956) and Friedman (1969), a long line of research addresses the question of the cost of inflation. Among recent contributions, the estimated gain from reducing inflation from 10% to 0% range from a consumption equivalent of 0.38% by Cooley and Hansen (1989), who address the question within a cash-in-advance model, to a consumption equivalent of around 1% by Lucas (2000), who analyzes a representative agent model with shopping time.[1]

1. MODEL ECONOMY

1.1 The Household's Problem

There is a continuum of good types of measure c_t^*, ordered by size and indexed by j over [0,1]. The representative household has a Leontief-type instantaneous utility function over the continuum of good types,

$$\min \left[\frac{c_t(j)}{(1-\omega)j^{-\omega}} \right],$$

which gives us the parameterized distribution function for $c_t(j)$ over $[0,1]$

$$(1) \quad c_t(j) = (1-\omega)\, j^{-\omega} c_t^*.$$

The representative household has time-separable preferences over total consumption (c_t^*) and leisure (d_t),

$$(2) \quad \max E \sum_{t=0}^{\infty} \beta^t u(c_t^*, d_t),$$

[1] Other recent contributions include Bullard and Russell (2000), Dotsey and Ireland (1996), Gomme (1993), Imrohoroglu and Prescott (1991), Jones and Manuelli (1995), and Lacker and Schreft (1996).

where the instantaneous utility is given by

$$(3)\quad u(c_t^*, d_t) = \frac{1}{1-\upsilon}\left[(c_t^*)^{\zeta}(d_t)^{1-\zeta}\right]^{1-\upsilon}.$$

There are three vehicles of savings available to the household: nonintermediated capital (a_t), nominal bank deposits (h_t), and currency (m_t). Both bank deposits (h_t) and currency (m_t) can be used to purchase consumption goods, but the use of deposits incurs an extra fixed cost, denoted by γ. Because of this fixed cost of using deposits for purchases, the deposit rate of the return net of transaction costs goes to negative infinity as purchase size (j) goes to zero. Therefore, some j^* exists below which currency is a preferred means of payment and above which deposits are preferred.

The household's good budget constraint is given by

$$(4)\quad c_t^* + a_t + \frac{h_t}{p_t} + \frac{m_t}{p_t} + \gamma(1 - j_t^*) = w_t\, l_t + r_t\, a_{t-1} + \tilde{r}_t \frac{h_{t-1}}{p_{t-1}} + \frac{m_{t-1}}{p_t} + \frac{x_t}{p_t},$$

where p_t is the nominal price level, w_t is the wage rate, r_t is the real rate of return on capital, $\widetilde{r}_t$ is the real rate of return on deposits, and x_t is government lump-sum transfers.

Available time for the households is normalized to 1, and the time available is spent on leisure (d_t), labor (l_t), and the number of times that money balances have to be replenished each period (n_t) multiplied by the time each replenishment takes (φ). The time constraint is

$$(5)\quad 1 = d_t + l_t + n_t\,\varphi.$$

1.2 Production

Output is given by a constant-returns-to-scale production function with two inputs, capital (k_t) and labor (l_t):

$$y_t = z_t\, f(k_t, l_t).$$

The law of motion for the technology level z_t is given by

$$z_t = p z_{t-1} + \varepsilon_t, \qquad z_t \sim N(\mu, \sigma^2),\ \mu > 0.$$

The depreciation rate is denoted by δ, so the law of motion for the capital stock is

$$k_{t+1} = (1-\delta)\, k_t + i_t,$$

where i_t is gross investment.

1.3 Government

The government controls the supply of intrinsically worthless fiat money. The law of motion for the money stock is

$$M_t = \xi M_{t-1}.$$

Net revenues from printing money are transferred to the household in a lump-sum fashion,

$$x_t = (\xi - 1)\, M_{t-1}.$$

1.4 Financial Intermediation

Banks accept deposits, hold the required-reserves fraction (θ) as cash, and invest the proceeds in capital. Free entry ensures zero profit, and the rate of return on deposits ($\tilde{r}$), therefore, is a linear combination of the real return on capital (r_{t+1}) and the return on holding currency (p_t / p_{t+1}):

$$\tilde{r}_{t+1} = (1-\theta)\, r_{t+1} + \theta \frac{p_t}{p_{t+1}}.$$

By definition, the total stock of fiat money (the monetary base) is equal to the combined stocks of currency and reserves,

$$M_t = m_t + \theta h_t,$$

whereas the total money stock (M1) is the sum of nominal deposits and currency, which can be rewritten as the product of the monetary base and the money multiplier:

$$\mathrm{M1}_t = m_t + h_t = M_t \left[1 + \frac{h_t (1-\theta)}{m_t + \theta h_t} \right].$$

For the representative household, the per-period holdings of real deposits (h_t / p_t) are

$$(6)\quad n_t \frac{h_t}{p_t} = \int_{j^*}^{1} c_t(j)\, \mathrm{d}j = \int_{j^*}^{1} (1-\omega) j^{-\omega} c_t^*\, \mathrm{d}j = [j^{1-\omega} c_t^*]_{j^*}^{1} = \left(1 - (j^*)^{1-\omega}\right) c_t^*,$$

and holdings of real fiat-money balances (m_t / p_t) are

$$(7)\quad n_t \frac{m_t}{p_t} = \int_0^{j^*} c_t(j)\, \mathrm{d}j = \int_0^{j^*} (1-\omega)\, j^{-\omega} c_t^* \mathrm{d}j = [\, j^{1-\omega} c_t^*\,]_0^{j^*} = (j^*)^{1-\omega} c_t^*.$$

2. CALIBRATION

In the steady state, investment is one-quarter of output and the annual capital–output ratio, 2.5. The depreciation rate is then calibrated to 0.025. The parameter α in the production function is calibrated such that the labor share of national income is 0.64. The autocorrelation coefficient ρ in the technology process is equal to 0.95, with a standard deviation of 0.0076.

Setting the average allocation of households' time (excluding sleep and personal care) to market activity equal to 0.30 restricts the value of the utility parameter ζ. The risk-aversion parameter, υ, is equal to 2, and the reserve-requirement ratio, θ, is 0.10.

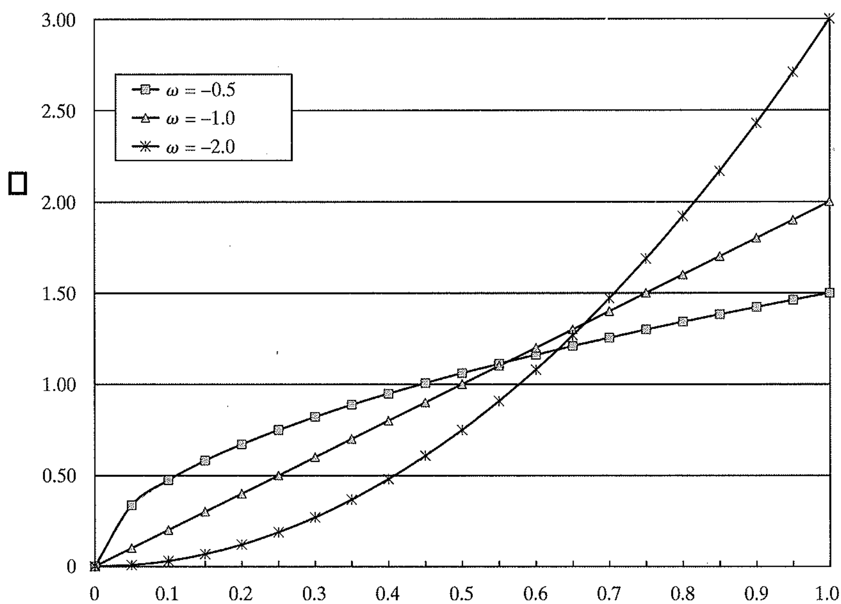
2.1 Utility Function

As an illustration, let the continuum of good types $c_t(j)$ be of measure c_t^*=1. Equation (1) can then be simplified as

$$c_t\,(j) = (1-\omega)j^{-\omega}.$$

In figure 1, $c_t\,(j)$ is plotted for three different values of ω. As is apparent from the expression and visualized in the figure, for ω>–1, the amount of a good that is consumed is a concave function of the size of the good, whereas for ω<–1, the amount of a good that is consumed is a convex function of the size of the good.

Figure 1: $c(j)$ for $0 \leq j \leq 1$, $c^* = 1$



Combining equations (6) and (7) gives us the cutoff size for purchase, above which deposits are preferred over currency:

$$(8) \quad j^* = \left(1 + \frac{h_t}{m_t}\right)^{\frac{1}{\omega - 1}}.$$

The derivative of j^* is negative, implying that, loosely speaking, the more convex $c_t(j)$, the higher j^*, or, conversely, the more concave $c_t(j)$, the lower j^*.

Note that equations (6) and (7) combined with (8) imply

$$\int_{j^*}^{1} c_t(j)\,\mathrm{d}j = \left(1 + \frac{m_t}{h_t}\right)^{-1} c_t^*$$

and

$$\int_{0}^{j^*} c_t(j)\,\mathrm{d}j = \left(1 + \frac{h_t}{m_t}\right)^{-1} c_t^*.$$

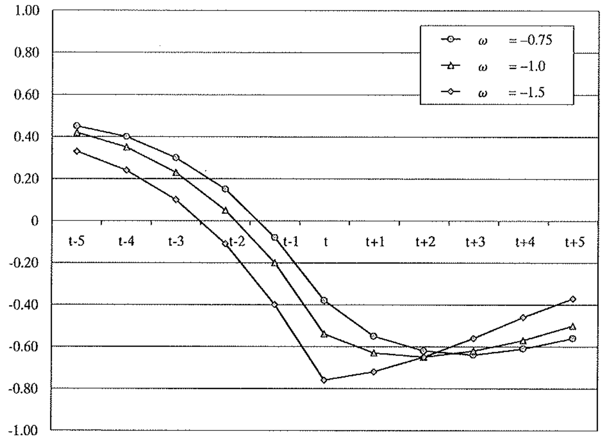
In other words, the cutoff size of purchases for which deposits are preferred over currency is a function of ω, whereas the share of total consumption (c_t^*) for which deposits are preferred over currency (and vice versa) depends only on the deposit-to-currency ratio.

2.2 Business Cycle Properties

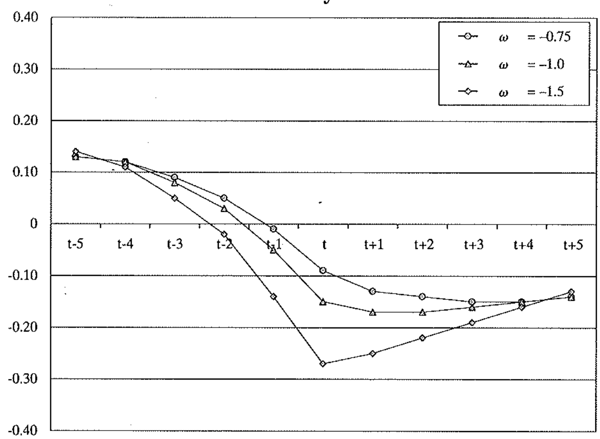
To get a sense of the reasonable values of ω, we start by reexamining the business cycle findings of Freeman and Kydland (2000) with this modification of the utility function. As in Freeman and Kydland (2000), we examine the model's behavior under three different policy regimes (see figure 2): Under the first, policy A, the growth rate of fiat money is fixed at 3% in every period. Under the second, policy B, serially uncorrelated shocks have been added to the supply of fiat money, with a standard deviation of 0.5%. And under the third, policy C, the shocks to the growth rate of the monetary base are serially correlated with an autoregressive parameter of 0.7 and a standard deviation of 0.2.

Figure 2: Cross-Correlations: Output and Price Level

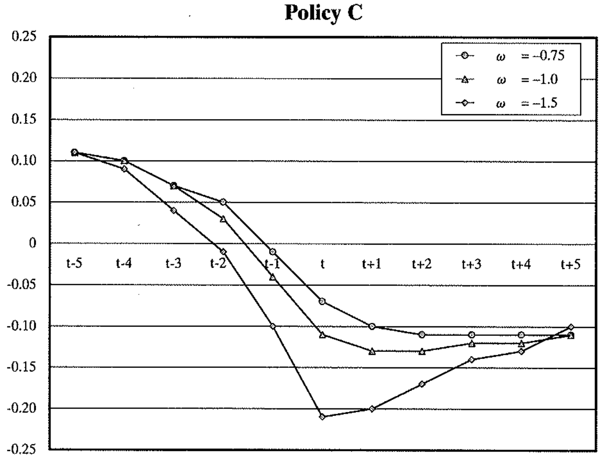
Policy A



Policy B



Policy C



For these three policies, we examine the business cycle properties for $\omega = \{-0.75, -1.0, -1.5\}$. Table 1 presents the contemporaneous correlations with output, which can be compared with actual data presented by Gavin and Kydland (1999).

Table 1: Contemporaneous Correlations with Output

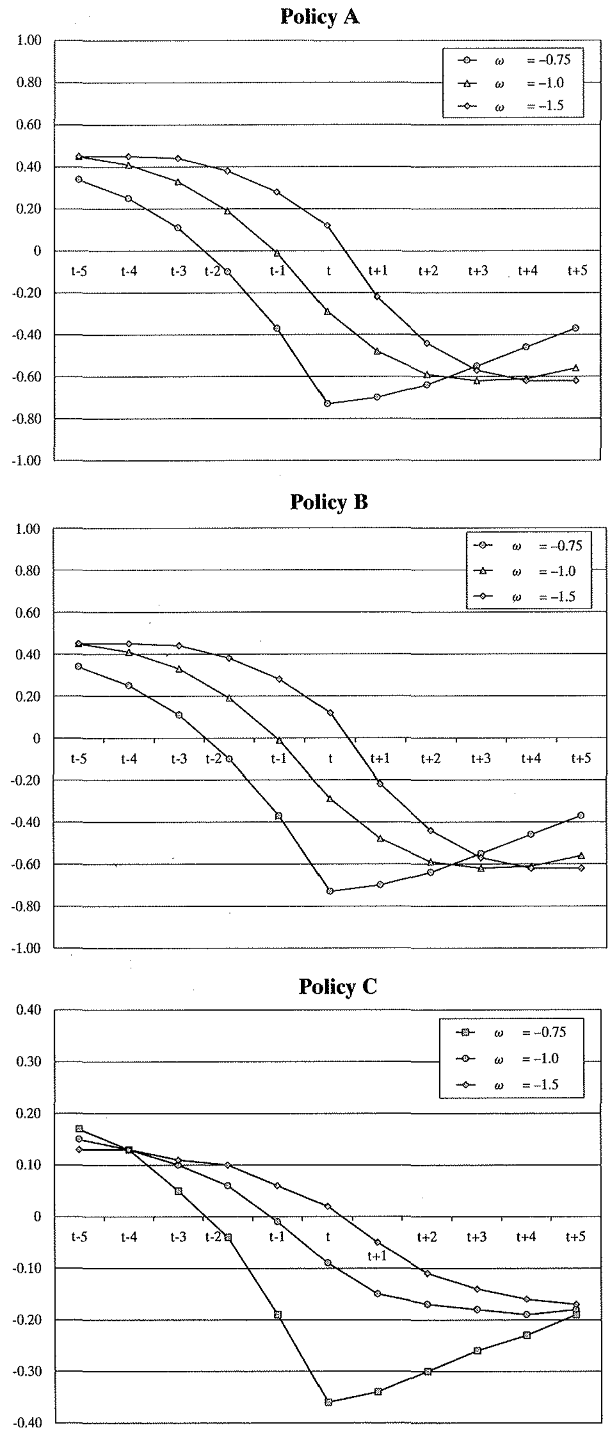
		M1	P	R_{nom}	C	I	L
Policy A:	$\omega = -0.75$	1	−0.38	−0.73	0.96	0.99	0.99
	$\omega = -1.00$	1	−0.54	−0.29	0.96	0.99	0.99
	$\omega = -1.50$	1	−0.76	0.12	0.96	0.99	0.99
Policy B:	$\omega = -0.75$	0.89	−0.09	−0.73	0.96	0.99	0.99
	$\omega = -1.00$	0.85	−0.15	−0.29	0.96	0.99	0.99
	$\omega = -1.50$	0.78	−0.27	0.12	0.96	0.99	0.99
Policy C:	$\omega = -0.75$	0.82	−0.07	−0.36	0.96	0.99	0.99
	$\omega = -1.00$	0.78	−0.11	−0.09	0.96	0.99	0.99
	$\omega = -1.50$	0.72	−0.21	0.02	0.96	0.99	0.99

Notice that the real variables—C, I, and L—are hardly affected by changes in monetary policy or the curvature of the utility function. We also see that M1 is strongly correlated with real output. Under policy A, in which there is no randomness, the correlation is 1. Under the two other policy regimes, M1 is slightly less tightly correlated but still highly correlated.

An interesting pattern is the countercyclical behavior of the price level. We see that, for all policies, the price level is more countercyclical for $\omega = -1.5$ than for the other two values, which is consistent with the business cycle statistics reported by Gavin and Kydland (1999).

We also notice that the cyclical behavior of the nominal interest rate is closer to what is observed in the data for $\omega = -1.5$ (figure 3). For the other two values of ω, the nominal rate of return (R_{nom}) is countercyclical, whereas for $\omega = -1.5$, the nominal interest rate is weakly procyclical. This is consistent with reported business cycle statistics.

Figure 3: Cross-Correlations: Output and Nominal *R*



Until we have data from which we can map more directly to ω, we choose $\omega = -1.5$ as our benchmark value because this value gives business cycle statistics closest to those observed.

3. QUANTITATIVE FINDINGS

We will begin by describing the steady-state properties of our economy under different inflation regimes. The economy is calibrated such that for an annual inflation rate equal to 0.03, the currency-to-deposit ratio is equal to 9 and the nonreserve portion of M1 divided by the capital stock is 0.05. This gives us calibrated values for $\gamma = 0.00529$ and $\varphi = 0.00060$, which implies that at this inflation rate, the fixed cost, $\gamma(1-j^*)$, is 0.36% of gross domestic product and φ corresponds to approximately 55 minutes per quarter.

3.1 Steady State

Figures 4 and 5 (figure 4 is just a subset of figure 5) plot the benchmark welfare cost function λ, defined such that

$$u[\lambda c(\pi), d(\pi)] = u[c(\tilde{\pi}), d(\tilde{\pi})],$$

where $\tilde{\pi}$ equals the average inflation rate over the last 15 years, about 3%.

Figure 4: Welfare Cost of Inflation Relative to Net Annual Inflation of 0.03

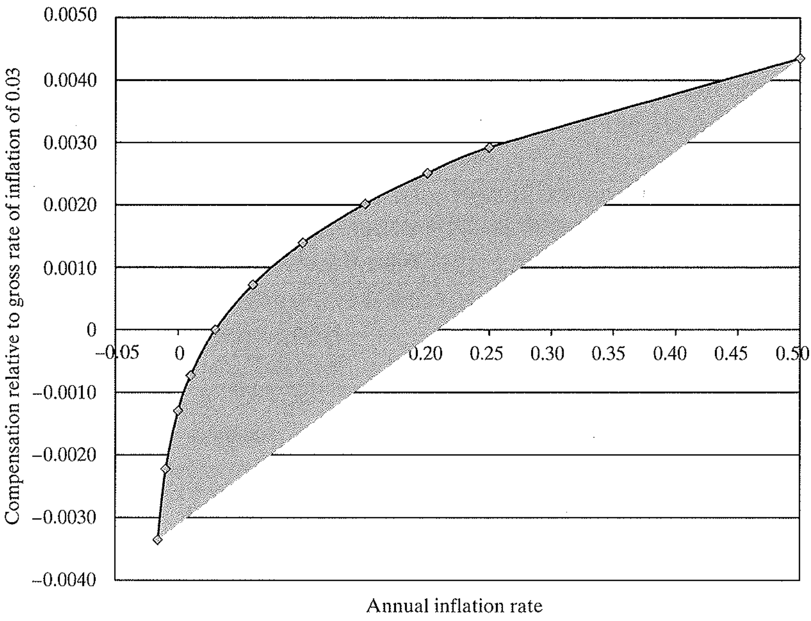
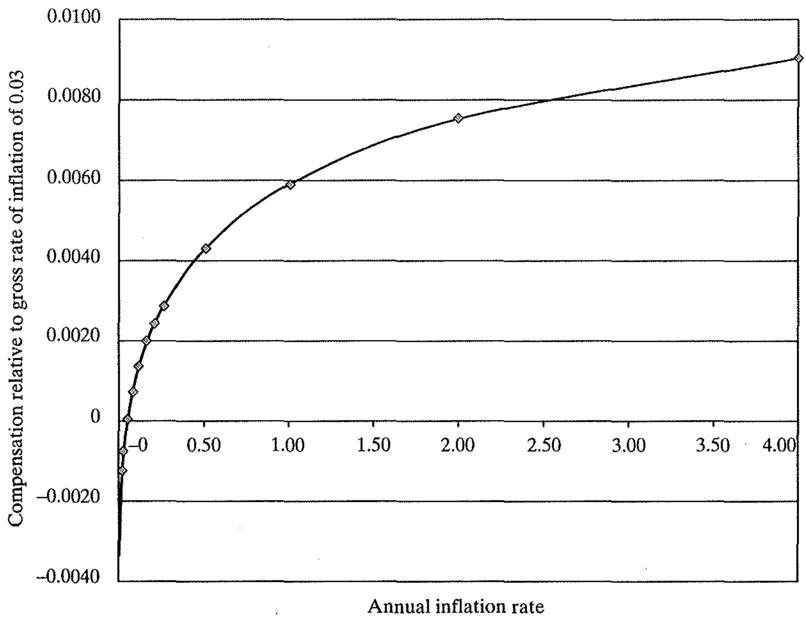


Figure 5: Welfare Cost of Inflation Relative to Net Annual Inflation of 0.03



We see from figures 4 and 5 that as steady-state inflation approaches an annual rate of 50%, the welfare cost is slightly less than 0.4% of consumption compared with the steady state associated with 3% inflation. As the steady-state inflation rate increases further, the associated welfare flattens out. At an annual inflation rate of 400%, the cost of inflation in terms of consumption compensation is still less than 0.8%.

The most striking feature of the graph is the predicted welfare gain from reducing inflation to below 3% annually. As we see from the graph, the effect of reducing inflation to its lower bound of –0.01644% gives a welfare improvement of almost the same magnitude as the welfare cost of increasing inflation from 3% to 50%.[2]

The main variables underlying these results are presented in table 2. As inflation increases, individuals become less and less willing to hold non-interest-bearing assets such as currency. The cutoff value of j^*, below which currency is preferred over deposits, is decreasing and eventually converges to zero as inflation increases towards infinity. Hence, the deposit-to-currency ratio

[2] In this model, there is a uniquely defined lower bound of inflation that is weakly greater than the inverse of the real rate of return on capital. At this lower bound, no one will hold deposits and the total money stock is equal to the monetary base (M1 = M).

Table 2: Steady-State Welfare Costs, Benchmark Calibration

Annual Inflation Rate

	–0.0164	0.00	0.01	0.03	0.06	0.10	0.25	0.50
n	1.320681	1.192505	1.171063	1.209201	1.309609	1.446476	1.600991	2.311321
l	0.299345	0.299722	0.299879	0.300000	0.300038	0.300018	0.299965	0.299597
a	9.713202	9.566608	9.516885	9.500000	9.521467	9.557323	9.593700	9.700938
h	0.000000	2.331871	3.358631	4.736842	5.927637	6.810877	7.441156	8.736977
m	1.000000	0.766813	0.664137	0.526316	0.407236	0.318912	0.255884	0.126302
h/m	0.000000	3.040992	5.057136	9.000000	14.555766	21.356580	29.080149	69.175101
j^*	1.000000	0.572012	0.486511	0.398107	0.333613	0.288561	0.256264	0.182611
p	3.327290	4.948311	6.309454	8.526316	11.118960	13.827927	16.529927	27.529942
c	0.746847	0.746759	0.746644	0.746420	0.746132	0.745815	0.745490	0.744131
d	0.699650	0.699370	0.699229	0.699079	0.698965	0.698880	0.698816	0.698643
u	–1.398807	–1.399235	–1.399496	–1.399835	–1.400167	–1.400477	–1.400765	–1.401841
k	9.978158	9.990729	9.995970	10.000000	10.001267	10.000614	9.998846	9.986565
l	0.299345	0.299722	0.299879	0.300000	0.300038	0.300018	0.299965	0.299597
y	0.997816	0.999073	0.999597	1.000000	1.000127	1.000061	0.999885	0.998656
$\lambda_{0.03}$	–0.0022	–0.0013	–0.0007	0.0000	0.0007	0.0014	0.0020	0.0043
$\lambda_{0.00}$	–0.0009	0.0000	0.0006	0.0013	0.0020	0.0027	0.0033	0.0057

increases and more resources in the economy are spent on facilitating transactions, both through the fixed cost for purchasing goods with deposits, $\lambda(1-j^*)$, and the time spent to withdraw currency $(n^*\varphi)$.

Steady states with lower inflation rates display the mirror image of the high-inflation regimes: Because the alternative value of holding deposits over holding currency diminishes, the cutoff value j^* increases and the deposit-to-currency ratio decreases. Henceforth, the welfare costs associated with individual liquidity management decrease.

3.2 Sensitivity/Alternative Calibration

The qualitative results presented in the previous section turn out to be insensitive to the calibration of the model economy. The quantitative results, however, depend strongly on the way the data are mapped to the model and, in particular, on the calibration of the transaction parameters, γ and φ.

Both the deposit-to-currency ratio and the nonreserve portion of M1 divided by the capital stock are hard to measure. Our empirical deposit-to-currency ratio, which excludes an estimate of the ratio of currency held abroad, ranges from 12 early in the sample to 7 late in the sample. We have encountered estimates of the nonreserve portion of M1 divided by the capital stock as low as 0.03 and as high as 0.20.

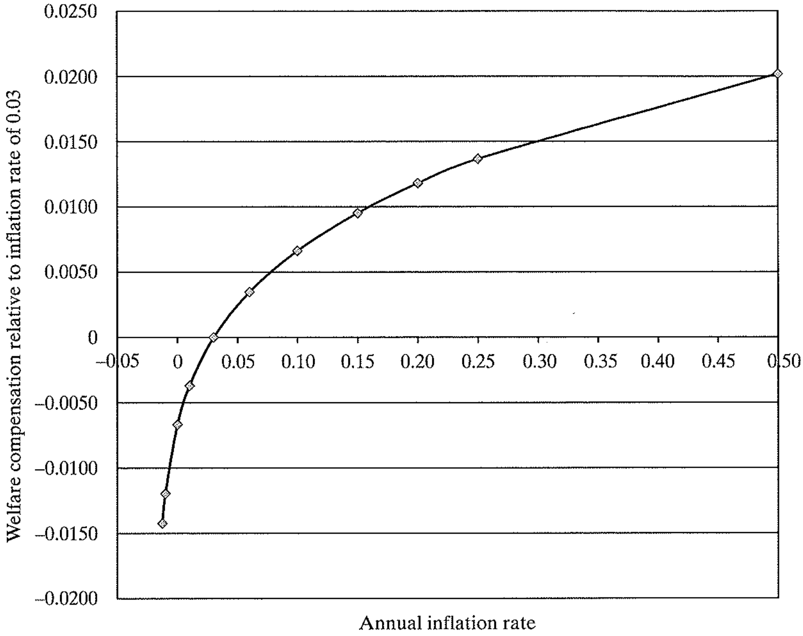
Table 3 shows how the values γ and φ vary as we change the deposit-to-currency ratio and the nonreserve portion of M1 divided by the capital stock. The last column of table 3 shows the welfare gain from reducing inflation from 0.03% to 0.00% annually. The values for φ and γ reported in the last row correspond to 19.7 hours per quarter and 1.58% of output, respectively.

As we see from the last row of this table and from figure 6, the steady-state welfare gains increase by a factor of about five with this alternative calibration. The welfare gain from reducing inflation from 3% annually to the lower bound is about 1.5%, whereas the cost of increasing inflation from 3% to 50% is about 2%.

Table 3: Alternative Calibrations of φ and γ

	$\frac{h}{m}$	$\frac{M1-\theta h}{k}$	φ	γ	$\gamma_{0.03\to0.00}$
Benchmark	9	0.05	0.0007614	0.005948	0.0013
Alt. 1	7	0.05	0.0009014	0.006993	0.0017
Alt. 2	9	0.20	0.01236	0.02379	0.0052
Alt. 3	7	0.20	0.01466	0.02798	0.0067

Figure 6: Welfare Costs, Alternative Calibration



3.3 Transition

Restricting the comparison to steady states ignores some important aspects that are relevant to answering our question. Therefore, we conduct a series of policy experiments in which we reduce inflation from moderate levels (0.03%, 0.06%, 0.10%, and 0.25%) to zero. When conducting these policy experiments, we calibrate the economy to the benchmark case for γ and φ.

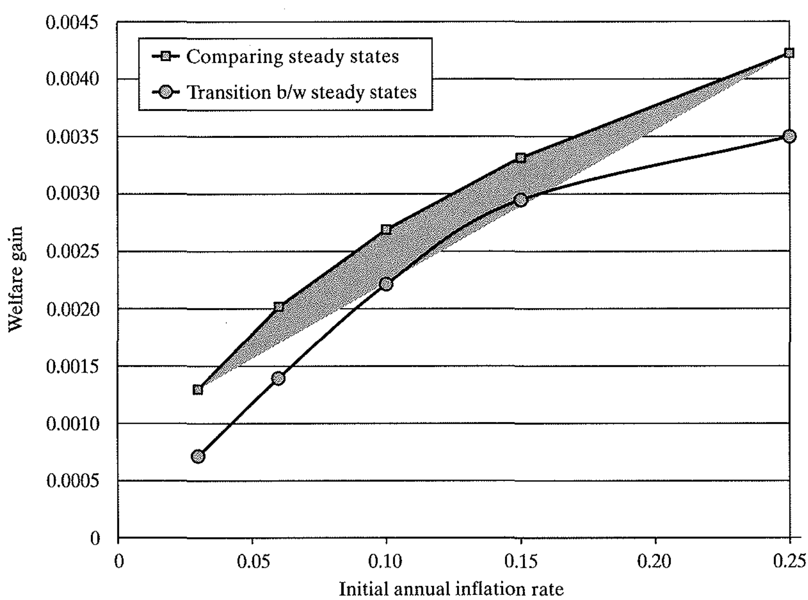
Table 4 and figure 7 present the results of these experiments. The welfare gains from reducing inflation are smaller than when comparing the steady states and range from 0.07% for an initial inflation rate of 3% to 0.35% for an initial inflation rate of 25%.

Comparing steady states, the welfare gains come solely from the reduction in resources spent on facilitating transactions. In addition, we have the effect of changes in expectations of monetary policy. If the rate of money growth (ξ) is decreased, anticipated inflation decreases, demand for real money balances increases, and the price level must decrease in equilibrium. This is known as the *Friedman surge effect*.

Table 4: Transition between Steady States

	0.03→0.00	0.06→0.00	0.10→0.00	0.15→0.00	0.25→0.00
h/m initial	9.0000	14.5558	21.3566	29.0801	42.6042
h/m new st. st.	3.0419	3.0419	3.0419	3.0419	3.0419
j^* initial	0.3981	0.3336	0.2886	0.2563	0.2209
j^* new st. st.	0.5720	0.5720	0.5720	0.5720	0.5720
c initial st. st.	0.7464	0.7461	0.7458	0.7455	0.7450
c new st. st.	0.7468	0.7468	0.7468	0.7468	0.7468
c change (net)	0.0005	0.0008	0.0013	0.0017	0.0024
d initial st. st.	0.6991	0.6990	0.6989	0.6988	0.6987
d new st. st.	0.6994	0.6994	0.6994	0.6994	0.6994
d change (net)	0.0004	0.0006	0.0007	0.0008	0.0009
k initial st. st.	10.0000	10.0013	10.0006	9.9988	9.9969
k new st. st.	9.9906	9.9906	9.9906	9.9906	9.9906
k change (net)	–0.0009	–0.0011	–0.0010	–0.0008	–0.0006
output initial st.st.	1.0000	1.0001	1.0001	0.9999	0.9997
output new st. st.	0.9991	0.9991	0.9991	0.9991	0.9991
output change (net)	–0.0009	–0.0011	–0.0010	–0.0008	–0.0006
welfare gain	0.0007	0.0014	0.0022	0.0029	0.0035

Figure 7: Welfare Comparisons



4. CONCLUDING REMARKS

Compared to the existing literature on the welfare cost of inflation, the model in this paper contains some novel features. For one, people make purchases using both inside and outside money. The proportions of purchases made by either are determined by economic decisions in which the liquid assets' relative returns play an important role. The model allows for two transactions costs, one associated with using deposits (checks) when making purchases, and one incurred when liquid balances are replenished during the period. In equilibrium, people make small purchases with currency and large purchases with deposits. The transaction-cost parameters are calibrated to the average currency–deposit ratio and to the fraction of the economy's total capital that is intermediated. The extent to which these costs kick in for various inflation rates plays a major role in our quantitative estimates. Moreover, because banks invest individuals' deposits in capital, another interesting feature is an effect of steady-state inflation on the total capital stock, which in our model may go in the reverse direction of what is commonly called the *Tobin effect.*

Our welfare-cost estimates are somewhat lower than those that Cooley and Hansen (1989) and Lucas (2000) report. An interesting finding is that the welfare-cost curve is quite concave, meaning that the cost goes up steeply with steady-state inflation for low inflation rates (especially around 5% and lower) before flattening out considerably.

We have not taken into account fiscal considerations, such as replacing lost seigniorage revenue using proportional taxes on labor and/or capital income rather than lump-sum taxes; we believe our model has little new to say in that regard. It would probably replicate Lucas's (2000) finding that fiscal considerations affect the welfare-cost estimates noticeably only for very low inflation rates.

We consider the estimate's sensitivity to several features. In particular, it is quite sensitive to the magnitudes of the two quantities used to calibrate the transaction-cost parameters. This is an interesting finding because these quantities have changed over the decades. We also study the transition paths from one steady-state inflation rate to another. These converge very quickly and make little difference to the welfare costs. Initially, they do contain a sizable amount of the so-called Friedman surge effect, which a central bank might wish to avoid for some reason or another that is not included in this model. An interesting but not trivial extension, then, could be to evaluate the effect on the welfare-cost estimate of combining the current model with a price-smoothing rule.

ACKNOWLEDGMENTS

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