STABILITY OF DYNAMIC DOMINANT FIRM EQUILIBRIUM*

by

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1. Introduction

Many manufacturing industries consist of firms with highly unequal market shares. The rankings of these firms according to size have usually been fairly stable over time. There are several examples of industries in which one firm had all or most of the market near the beginning of this century, either because of patents or for other reasons, and this dominant firm is still substantially larger than its rivals. The market share of the dominant firm has typically declined over time, however. Evidence of this can be found in Burns (1936, pp. 77-140), who describes the histories of several industries over 3 or 4 decades. Scherer (1980, pp. 239-240) lists many industries with similar histories even up to the present time. They include steel (ingots and castings), rayon, tin cans, corn products refining, farm implements, synthetic fibers, aluminum extrusions, instant mashed potatoes, frozen orange juice, and, on the regional level, in the gasoline industry.[1]

Burns found substantial evidence of price-leadership behavior in some of the industries, and in the fifties and before dominant firm or price-leadership models attracted a lot of attention. As Cyert and March (1956) pointed out, however, the fact that the market share of the dominant firm has shown a steady downward trend is difficult to explain on the basis of the traditional price-leadership model. Worcester (1957), in an analysis of the dynamics of dominant firm pricing, concluded that the dominant firm case is a short-run phenomenon that will break down in the long run. Although he did not explain why this process might take close to a century, as is indicated by the examples above, his conclusion still appears to be the final word on this issue. Dominant firm or leader-follower models are discussed in textbooks and elsewhere, but they do not seem to be taken seriously as a description of long-run industry conduct. This is understandable, because in these models, there is typically no incentive for the passive firms or followers to continue acting in this way. This is certainly the case when industry demand and cost functions are linear. As Hause (1977, p. 85) showed, one has to resort to rather contrived examples, such as introducing extreme curvature on demand and/or cost functions, in order for there to be an incentive to act as a follower.

A possible reason for dismissing these models is that they are basically static. To the extent, however, that one is mainly interested in the long-run behavior of industries, one can often view static models as describing the steady states of inherently dynamic environments. Their dismissal, therefore, is certainly not immediate. In this paper, we shall demonstrate two important claims in this regard. Firstly, although the typical static model does correspond to the steady state of a dynamic solution concept for dominant firm models, this steady state is <u>not stable</u>. Secondly, using an alternative equilibrium concept that we justify by a stability argument, we demonstrate that there can be incentives to act as a follower even in the standard case of linear demand curve and long-run constant returns to scale. This issue cannot be dealt with only by looking at steady states. When there are costs associated with rapid changes in capacity, for example, it is essential to determine the equilibrium path towards a new steady state if a potential entrant decided to enter. Thus, when the inherent dynamics are considered, the dominant firm model emerges as a plausible description also of long-run conduct in many industries.[2]

In the model to follow, the industry produces a homogeneous good. Cost structures are identical across firms, and we assume long-run constant returns to scale in order to be consistent with the large body of evidence supporting this hypothesis in many industries. One firm is dominant in the sense that it can take into account the reactions of its rivals to its decisions in every period. One could imagine, although this is not necessary, that the dominant firm decides first, while the rivals do their best given this decision. Everyone assumes that similar behavior will prevail in future periods. The followers behave noncooperatively among themselves. This setup includes as extreme special cases the dynamic version of the Stackelberg (1934) leader-follower model on the one hand and the case of a dominant firm with a competitive fringe (where the number of rivals is very large) on the other. It is important to note that the definition of a dominant firm is behavioral or strategic in nature. This means that the dominant firm will not necessarily have a market share larger than some number like 50% which is sometimes used as a definition.[3]

The paper is organized as follows. In the next section, the model is presented, and our equilibrium concept is discussed in Section 3. In Section 4, two alternative solution concepts are examined which, we argue, are both unsuitable as an equilibrium concept for reasons of instability. In Section 5, we examine the stability of the dominant firm structure in the sense of whether it will persist over time. The possibility that the dominant firm and its rivals base their decisions on different information is considered in Section 6, and some concluding remarks are offered in the last section.

2. The Structure of the Industry Model

Consider an industry producing a homogeneous commodity with inverse demand curve of the form

$$p_t = a_t - b \sum_{j=1}^{n} y_{jt},$$

where p_t is price, a_t is a stochastic demand shift variable, b is a fixed parameter, y_{jt} is output (equal to sales) by firm j in period t and n is the number of firms in the industry. Output by firm j is $Ak_{jt}^{\alpha} n_{jt}^{(1-\alpha)}$, where k_{jt} is the capital stock at the beginning of period t, and n_{jt} is labor input in period t. The supply of labor is assumed to be completely elastic at a constant real wage, say one. The capital stock is given by

$$k_{j,t+1} = (1 - \delta)k_{jt} + x_{jt}, \tag{1}$$

where x_{jt} is investment by firm j in period t, and δ is the depreciation rate. The unit cost of investment is assumed to be q as long as the capital stock is just maintained. For deviations from this investment rate, δk_{jt}, we assume a quadratic cost of adjustment, $\gamma(x_{jt} - \delta k_{jt})^2$. This insures that we have constant returns to scale in the long run. Total investment cost in period t is then

$$z_{jt} = qx_{jt} + \gamma(x_{jt} - \delta k_{jt})^2.$$

The net cash inflow for firm j in period t is

$$p_t y_{jt} - p_t n_{jt} - z_{jt}.$$

Maximizing over n_{jt} in each period, we can write net cash inflow as

$$w_j(k_t, a_t, x_{jt}) = (a_t - b\lambda \sum_{i=1}^{n} k_{it})\alpha\lambda k_{jt} - qx_{jt} - \gamma(x_{jt} - \delta k_{jt})^2 ,$$

where $\lambda = [A(1 - \alpha)]^{1/\alpha}/(1 - \alpha)$ is output per unit of capital. The firm's objective is to maximize

$$E[\sum_{t=1}^{T} \beta^{t-1} w_j(k_t, a_t, x_{jt})] ,$$

where $\beta = 1/(1 + r)$, r being the interest rate.

Finally, the stochastic demand shift variable, a_t, is here assumed to follow a first-order autoregressive process:

$$a_t = \rho a_{t-1} + \mu + \varepsilon_t , \tag{2}$$

where $-1 < \rho < 1, \mu > 0$, and ε_t are random disturbances uncorrelated over time with mean zero and variance σ_ε^2 .

We assume that firm n is dominant in the sense that it takes into account the rivals' reactions to its decision variable. Since the number of rivals is not necessarily very large, we assume that they behave noncooperatively among themselves, given the decision of the dominant firm.

3. Industry Equilibrium

An industry equilibrium will be defined in policy space, that is, in terms of decision rules that are functions of the current state. The equilibrium is defined for a given number of firms in the industry. This appears particularly reasonable for mature industries such as the ones listed in the introduction.

An equilibrium is defined as follows:

Definition: An equilibrium is a set of decision rules $X^*_{jt}(k_t,a_{t-1},x_{nt})$, $j = 1,\dots,n-1$, and $X^*_{nt}(k_t,a_{t-1})$ for all $t = 1,\dots,T$ such that

$$\max_{x_{jt}} E[w_j(k_t,a_t,x_{jt}) + \beta v_{j,t+1}(k_{t+1},a_t)|X^*_{it},\ i = 1,\dots,n,\ i \neq j]$$

$$= E[w_j(k_t,a_t,x_{jt}) + \beta v_{j,t+1}(k_{t+1},a_t)|X^*_{it},\ i = 1,\dots,n],\ j = 1,\dots,n,$$

subject to constraints (1) and (2), where

$$v_{jt}(k_t,a_{t-1}) = E[\sum_{s=t}^{T} \beta^{s-t} w_j(k_s,a_s,x_{js})|x_{is} = X^*_{is}(k_s,a_{s-1},x_{ns}),\ i = 1,\dots,n-1,$$

$$x_{ns} = X^*_{ns}(k_s,a_{s-1}),\ s = t,\dots,T].$$

The function v_{jt} gives the expected value of firm j when all firms behave according to the equilibrium decision rules from period t until the end of the horizon. In other words, the definition says that each firm chooses the best decision rule for period t, given the last observed state variables k_t and a_{t-1}, the decision rules of the other firms, and that decisions will be similarly selected in periods $t + 1,\dots,T$. The value functions and the equilibrium decision rules can be computed recursively (or by successive approximations in the case of infinite horizon) as described in Kydland (1977).

For the purpose of the discussion in the rest of this section, we assume that the horizon is infinite, so that the decision rules $X_i^*(k,a_{-1},x_n)$, $i = 1, \ldots, n-1$, and $X_n^*(k,a_{-1})$ are time stationary. A property of the equilibrium is that if firm j, say, takes as given the equilibrium decision rules of the other firms, X_i^*, $i = 1,\ldots,n$, $i \neq j$, then the firm-j part of the equilibrium, X_j^*, is the optimal solution to the resulting standard dynamic maximization problem. This is the case for the dominant firm as well as for its rivals. This observation suggests that a reasonable way to look at stability of the equilibrium is to consider what would happen if, at some stage, each firm j expects the other firms to follow decision rules X_i^j, $i \neq j$, that are different from the equilibrium X_i^* (say, because it does not know for sure the parameters of the other firms' technologies). If the equilibrium is stable, the industry will tend towards the equilibrium decision rules.

No attempt will be made here at providing a realistic model for how the expectations X_i^j and the resulting actual decision rules X_i would tend towards the equilibrium X_i^* over time. Instead, we shall consider an example of a simple class of such adaptive processes which, except for being in terms of decision rules, are comparable to the ones used by Telser (1972) and others in analyzing the stability of static equilibria in various versions of the Cournot model. For simplicity, assume there are two firms (firm 2 being dominant) whose expectations are adjusted according to the following processes in terms of decision rules (the arguments of these functions are ommitted):

(3) $$X_{i,m+1}^j = X_{im}^j + \eta_j(X_{im} - X_{im}^j),\ j = 1,2,\ i \neq j;\ m = 0,1,2,\ldots$$

Here, $X_{1m}^2(k,a_{-1},x_2)$ and $X_{2m}^1(k,a_{-1})$ are the expectations of firm 2 and firm 1, respectively, at stage m of this process, the initial beliefs being given by $X_{2,0}^1$ and $X_{1,0}^2$. The functions X_{jm} without superscripts are the actual decision rules at stage m, and $0 < \eta_j \leq 1$ are adjustment coefficients. At each stage, given the decision rule that the other firm is expected to follow for the entire future, the decision rule of each firm j is determined from its infinite-horizon maximization problem. If such processes converge to the equilibrium X_i^*, $i = 1,2$, then this provides a justification for using this equilibrium concept in analyzing the industry.

In Table 1, the outcome of a process like (3) is presented for an example with $\eta_1 = \eta_2 = 1$ in which the following parameter values have been chosen for the model of Section 2: $A = 1.1$, $\alpha = 0.3$, $q = \gamma = 0.5$, $\delta = 0.1$, $b = 1$, $r = 0.1$, $\mu = 0.2$, $\rho = 0.8$, and $\sigma_\varepsilon^2 = 0.01$. The values of A and α give an output-capital ratio, λ, of 0.6, and the values of μ and ρ imply that a_t is equal to one on average. Since the process converges toward the same equilibrium regardless of initial expectations, we chose the extreme case of $X_{1,0}^2 = 0$ and $X_{2,0}^1 = 0.037$, that is, constants. An interpretation is that firm 1 is entering an industry in which firm 2 has had a monopoly which here corresponds to an average capital stock of 0.37. Even for this example in which current expectations are the actual decision rules from the preceding stage, coefficients of the decision rules are seen to converge very rapidly. The equilibrium decision rules correspond to stage infinity.

For future reference some results are given in Table 2 for the same example with the number of firms varying from 2 to 5. The steady state capacities are of course stochastic in the sense that capital stocks fluctuate as demand fluctuates. Expected values of the firms have been computed both for the last firm entering the industry with no capital stock, and for all the firms after they have reached the new stationary state. In the former case, the initial state is assumed to be the steady-state capital stocks with one firm less, with the demand shift variable, a_t, starting at its average of one. The movement toward the new steady state is then determined by the equilibrium decision rules. While building up the capital stock, the entering firm will have to incur substantial negative net cash inflows for the first few periods which, with a positive interest rate, may take a long time to recover, if ever. That is of course the reason why the value of the entering firm is very small compared with the value starting from the stationary point.

4. Alternative Solution Concepts

At a given point in time, the optimal decision rules for a dominant firm could be viewed as being the policies $X^o_{nt}(k_t, a_{t-1})$, $t = 1,\ldots,T$, that give the highest value of the firm after properly taking account of the effects on the decision rules of the rivals, including the effects of each X^o_{nt} on decisions in periods 1 to t-1. Thus, the reason the policies X^*_{nt}, $t = 1,\ldots,T$, of the preceding section can theoretically be improved upon from the point of view of the dominant firm is that they do not take into account these effects on earlier-period decisions. This feature of the optimal policy, however, also implies that it is inconsistent under replanning. At the beginning of period one, the optimal dominant-firm plans for periods 2,...,T take into account, among other things, the effects on rivals' decisions in period one. After period one is history, these effects are no longer of any concern, and the optimal plan for periods 2,...,T is therefore different at the start of period two. Thus, unless it is possible for the dominant firm to commit itself, there would always be a temptation to change in the future when the optimal policy from then on is different. Furthermore, even in period one the rivals would not behave as implied in the determination of the optimal dominant-firm decision rules unless they firmly believed that the dominant firm would stick to the initial plan in future periods.

It is interesting to note that the nature of the optimal policy of a former monopolist when a second firm is just entering the industry, is to meet any capacity increases by the entering firm with much larger increases in its own capacity. If the entering firm were to take this policy as given, its capacity would remain virtually at zero, while the dominant firm would have almost the same capital stock as when it was a monopolist.

A third solution concept, which in general is different from the two already mentioned, is obtained if the variational approach is used rather than policy space methods. In the deterministic case, this solution can be viewed as a sequence of decisions $\{x_{jt}\}_{t=1}^{T}$, $j = 1,\ldots,n$, depending on the initial state only. We can write them as

$$(4)\qquad \begin{aligned} x_{jt} &= f_{jt}(k_1,a_0,x_{n1},\ldots,x_{nT}),\ j = 1,\ldots,n-1, \\ x_{nt} &= f_{nt}(k_1,a_0),\ t = 1,\ldots,T. \end{aligned}$$

Thus, the decisions for all future periods would be determined regardless of future states. If the problem is solved again at time 2, however, the decision points from then on are different from the original plan at time 1 for those periods. In the presence of uncertainty, future decisions could formally be made conditional on exogenous disturbances observed up to that time, but would obviously still be time inconsistent.

When the variational approach is used in infinite-horizon stationary problems, rest-point analysis is often quite easy. For the model of Section 2 it is easy to see that the steady-state solution can be obtained by writing the problems as

$$\max_{k_i}[(\bar{a} - b\lambda \sum_{j=1}^{n} k_j)\alpha\lambda - q(\delta + r)]k_i,$$

where $\bar{a}$ is the average of a_t (given by $\mu/(1 - \rho)$), and $q(\delta + r)$ is the user cost of capital. This model corresponds to the standard linear textbook version of a leader-follower model. The first-order conditions for the non-dominant firms are:

$$2\alpha\lambda^2 bk_i + \alpha\lambda^2 b \sum_{\substack{j=1 \\ j\neq i}}^{n-1} k_j = \alpha\lambda\bar{a} - q(\delta + r) - \alpha\lambda^2 bk_n,\quad i = 1,\ldots,n-1.$$

Writing this system of n - 1 equations on matrix form, we get

$$Ck^{(n)} = d - gk_n ,$$

where each diagonal element of C is $2\alpha\lambda^2 b$, and each off-diagonal element $\alpha\lambda^2 b$. The vector $k^{(n)}$ is the (n - 1)-dimensional vector of capital stocks for all but the dominant firm. Each element of d is $\alpha\lambda\bar{a} - q(\delta + r)$, and in g each element is $-\alpha\lambda^2 b$.

The inverse, C^{-1}, of the coefficient matrix can be written as $\frac{1}{\alpha\lambda^2 b}[I_{n-1} - \frac{1}{n} J_{n-1}]$, where I_{n-1} is the (n - 1)-dimensional identity matrix, and J_{n-1} is a matrix of the same dimension with each element being equal to one. We then easily get

$$k_i = \frac{1}{\lambda bn}[\bar{a} - \frac{q(\delta+r)}{\alpha\lambda}] - \frac{1}{n} k_n , \quad i = 1,\ldots,n-1 .$$

The first-order condition for the dominant firm is

$$\alpha\lambda^2 b[2 + \sum_{j=1}^{n-1} \frac{\partial k_j}{\partial k_n}]k_n + \alpha\lambda^2 b \sum_{j=1}^{n-1} k_j = \alpha\lambda\bar{a} - q(\delta + r) .$$

But $\partial k_j/\partial k_n = -1/n$, j = 1,...,n-1, and

$$\sum_{j=1}^{n-1} k_j = \frac{n-1}{n}[\bar{a} - \frac{q(\delta+r)}{\alpha\lambda}] - \frac{n-1}{n} k_n .$$

We therefore obtain

$$k_n = \frac{1}{2\lambda b}[\bar{a} - \frac{q(\delta+r)}{\alpha\lambda}] ,$$

and, after substitution,

$$k_i = \frac{1}{2\lambda bn}[\bar{a} - \frac{q(\delta+r)}{\alpha\lambda}] , \quad i=1,\ldots,n-1 .$$

We thus see that the steady-state capacity of the dominant firm does not depend on the number of firms in the industry, and that its market share approaches 50 percent as the number of firms increases.

Using the same parameter values as in the example in Section 3 the steady-state capital stock will always be 0.37 (also for the case in which it is a monopolist), while the capital stocks for the rivals are 0.185 in a two-firm industry, 0.1233 in a three-firm industry, and so on.

Abstracting from uncertainty for the moment, one might think that if the firms happened to start out at the steady state resulting from the variational approach, then they would want to stay there. However, this turns out not to be the case. In a two-firm industry, for instance, if the initial capital stocks are $k_1 = (0.185, 0.370)$, then, using this solution concept, the capital stocks for the next three periods are $k_2 = (0.188, 0.355)$, $k_3 = (0.190, 0.349)$, and $k_4 = (0.192, 0.347)$. From then on the dominant firm capital stock starts to move back toward the steady state again. Every time the dominant firm decides to recompute its plan, a similar disruption will be optimal from then on. In fact, if the plan was reconsidered every period, with the rival every time being fooled into believing that the dominant firm capacity would eventually move back toward 0.37, then a different steady state would be approached, namely the one resulting when the first-period decision was made in every period. This state would be $k = (0.210, 0.312)$. Making such a decision in every period would be indistinguishable, however, from using the policy rule $x_{2t} = f_{21}(k_t, a_{t-1})$, where f_{21} is defined in equation (4), and eventually the rival would have to take this rule into account. His optimal decision, given that policy rule, would itself be a policy rule, which the dominant firm would have to take into account. Thus one might end up in a process similar to (3), the result of which would be the equilibrium decision rules with stationary capital stocks $k = (0.252, 0.290)$

as in Table 2. We can of course only speculate as to what would happen in the situation described in this paragraph, but the main point is that the steady state corresponding to the standard leader-follower model is not stable.

5. Stability of the Dominant Firm Structure

If the steady-state profits in a dominant firm industry with a given number of firms are compared with the steady-state profits in a noncooperative industry with the same number of firms, but with no firm being dominant, profits are clearly higher for each firm in the latter case than for each nondominant firm in the former case. In fact, it seems that the demand curve has to be convex with relatively large second derivative in order for it to be possible for the follower to make higher profits in a leader-follower situation than in the symmetric equilibrium. Worcester's finding that the dominant firm structure is a short-run phenomenon which would break down in the long run and presumably turn the industry into a noncooperative or competitive one, therefore seems to be valid in all but very extreme cases.

The purpose of this section is to show that explicit consideration of the dynamics of the present model may lead to new insights into this problem. Unfortunately, since our results will depend on equilibrium behavior of the industry in the transition from one steady state to another, we are not able to obtain general conditions under which the dominant firm structure will persist. Instead, we shall have to rely on the computation of equilibria for a representative set of numerical examples. The demonstration that the results are valid at least for a reasonable range of parameters within a noncontrived model is thought to be important. In view of what was said earlier, it seems particularly interesting to examine the case of a linear demand curve. The experience from static models suggests that our results are even more likely to hold when there is some curvature on industry demand.

Consider a dominant firm industry with n existing firms in which the steady-state expected value of each of the n-1 nondominant firms is $\Pi^{(n)}$, and the expected value of a potential entrant starting from zero capacity is $V^{(n)}$, where $V^{(n)}$ is determined as described in the last paragraph of Section 3. Similarly, in a noncooperative n-firm industry without a dominant firm, the steady-state expected value of each firm is $P^{(n)}$, and the expected value of a potential entrant is $W^{(n)}$. Tables 2 and 3 display these two alternatives for the example presented in Section 3. We see that $P^{(n)} > \Pi^{(n)}$ for all n. This means that if the rivals could somehow break up the dominance of the dominant firm and end up in a symmetric Nash equilibrium with n firms, each of them would have higher steady-state profits.

We also see that $W^{(n)} > V^{(n)}$, that is, the profit incentive to enter the industry will increase. Imagine now the dominant firm industry in a situation in which $V^{(n)}$ is just low enough to keep potential entrants out of the industry. Without a dominant firm, a new firm will typically find it profitable to enter the industry. The relevant comparison to make in deciding whether rational rival firms would accept the dominance of one firm, is whether $P^{(n+1)}$, rather than $P^{(n)}$, is larger than $\Pi^{(n)}$. From Tables 2 and 3 it is clear that for all n from 2 to 5 we have $P^{(n+1)} < \Pi^{(n)}$.

The same exercise was repeated for several combinations of parameter values of the model. In particular, it is not obvious what are reasonable relative values of the cost-of-adjustment factor γ and the long-run unit investment cost q. Various combinations of the two parameters with both in the range of 0.25 to 1.0 were attempted, as well as some alternative values of the slope of the demand curve, b, the output elasticity of capital, α, and the interest rate, r. In all cases, the inequalities went the same way as reported above. Thus, the existence of a dominant firm may provide an entry barrier which is acceptable to all firms. Essentially, this represents a form of limit pricing, and in particular when demand fluctuates a lot and there are more than just a few firms in the industry, this type of entry barrier may be more agreeable and easily enforceable than alternative forms of limit pricing that require more cooperation among existing firms.

6. Informational Considerations

[Revision not yet completed]

7. Concluding Remarks

We have argued that an equilibrium concept for a dynamic dominant firm model based on the sequential solution is likely to be stable in the sense that decision rules will tend toward the equilibrium decision rules. We also examined alternative solutions, namely the best dominant-firm solution using policy-space methods (or the closed-loop solution) and the solution using the variational approach (or open-loop solution), both of which are in general different from the equilibrium. They were both found to be unstable in the sense mentioned above.

In static oligopoly models with linear demand curves, it is easy to see that if one firm is dominant, the profits of the rivals will be lower than in the symmetric Nash equilibrium. It could be argued, then, that the rivals would not accept the dominance of one firm, and the industry would eventually approach the symmetric equilibrium. In this paper, we found that, in a dynamic model with linear demand curve and costly adjustment of capacity, the dominance of one firm may be acceptable to the rivals and in effect provide an entry barrier against potential new firms. Also, informational considerations may lead to a reduction in the difference in profits between the dominant firm and its rivals.

Some properties of dynamic dominant firm equilibria have already been presented in Kydland (1979). There a dominant firm structure was assumed without considering the question of whether it could be expected to persist over time. The number of firms in the industry was determined by whether or not expected discounted profits of an additional entering firm are positive. The incentive to enter would change with permanent changes in average demand. While corresponding static (or steady-state) models would predict the dominant firm output to remain unchanged as more firms enter the industry, and the market share to approach a fixed percentage, it was found that the equilibrium steady-state output for the dominant firm declined substantially as more rivals entered the industry. Also, the market share of the dominant firm declined when the cost of adjustment increased relative to the long-run unit cost of investment, and when the elasticity of demand increased. Thus, the observations mentioned in the introduction are consistent with the existence of a dominant firm whose dominance, as we define it, is not disappearing.

Stage	Firm 1					Firm 2			
	const.	k_{1t}	k_{2t}	a_{t-1}	x_{2t}	const.	k_{1t}	k_{2t}	a_{t-1}
0	0	0	0	0	0	.37	0	0	0
1	-.0211	-.2318	-.1293	.1356	-.1437	-.0129	-.1293	-.2318	.1356
2	-.0150	-.2091	-.1074	.1332	-.1193	-.0099	-.0831	-.1901	.1221
3	-.0164	-.2173	-.1094	.1299	-.1216	-.0132	-.0865	-.1972	.1192
4	-.0152	-.2168	-.1087	.1307	-.1208	-.0122	-.0847	-.1968	.1205
5	-.0157	-.2171	-.1086	.1304	-.1206	-.0128	-.0847	-.1971	.1202
∞	-.0156	-.2171	-.1085	.1304	-.1206	-.0127	-.0846	-.1971	.1202

TABLE 1. Coefficients of expected decision rules with $\eta_1 = \eta_2 = 1$

Number of firms in industry	2	3	4	5
Steady state:				
Average capacity of dominant firm	.2904	.2443	.2159	.1972
Average capacity per rival	.2516	.1858	.1459	.1196
Market share of dominant firm	.5357	.3966	.3304	.2920
Expected value of dominant firm	.2294	.1714	.1417	.1242
Expected value per rival	.1990	.1306	.0959	.0755
Starting from zero capacity:				
Expected value of potential entrant	.0186	.0100	.0061	.0040

TABLE 2. Results for dominant-firm industry

Number of firms	3	4	5	6
Steady state:				
Average capacity per firm	.2011	.1608	.1334	.1137
Expected value per firm	.1443	.1077	.0856	.0708
Starting from zero capacity:				
Expected value of potential entrant	.0123	.0076	.0051	.0032

TABLE 3. Results for noncooperative industry

Footnotes

[1]For example, the U.S. Steel production of ingots and castings declined from 65 percent of total U.S. production in 1902 to 52 percent in 1915, 39 percent in 1931, 28 percent in 1961, and 21 percent in 1968. In 1919, the American Viscose Company controlled 100 percent of the domestic rayon market. Once key patents expired, new entry caused its share to fall to 42 percent in 1930 and 26 percent in 1949. American Can controlled 90 percent of all tin can output in 1901. Its high pricing policy encouraged new entrants, and its market share fell to 63 percent in 1913 and 40 percent by 1960.

[2]The literature contains a large number of articles in which the formation of expectations is the only dynamic element. A typical example is the assumption of static expectations as in the Cournot model. More sophisticated reaction strategies are considered in Cyert and DeGroot (1970, 1971, 1973), introducing learning over time, and in Friedman (1977). Some recent dynamic models of oligopoly are Duchatelet (1977), Flaherty (1978), and Spence (1977). All of these models are game-theoretic. The connection between oligopoly and game theory is stressed in the two important books by Telser (1972) and Friedman (1977).

[3]Williamson (1975, Chapter 11) defines a dominant firm as one that has a market share of at least sixty percent. He emphasizes market failure as a possible explanation of dominant firms. His work can therefore be regarded as complementary to ours.

[More footnotes and references to be added.]