Mortgages are long-term loans with nominal payments. Consequently, in incomplete asset markets, monetary policy can affect housing investment and the economy through the cost of new mortgage borrowing and real payments on outstanding debt. These channels, distinct from the traditional real rate channel, are embedded in a general equilibrium model. The transmission mechanism is stronger under adjustable-rate mortgages compared with fixed-rate mortgages. Further, persistent monetary policy shocks affecting the level of the nominal yield curve have larger real effects compared with transitory shocks. Persistently higher inflation gradually benefits homeowners under FRMs, but hurts them immediately under ARMs. (JEL E32, E52, G21, R21)

Received October 26, 2015; editorial decision December 31, 2016 by Editor Itay Goldstein.

Mortgages are long-term loans with regular nominal payments, consisting of interest and amortization. The payments are set up to guarantee that, given the mortgage interest rate, the principal is gradually repaid in full by the end of the mortgage term, typically 15 to 30 years. A fixed-rate mortgage (FRM) has a...
fixed nominal interest rate and constant nominal payments, set at origination, for the entire term of the loan; an adjustable-rate mortgage (ARM), in contrast, sets nominal payments on a period-by-period basis so that, given the current short-term nominal interest rate, the loan is expected to be repaid in full during its remaining term. Various mortgage loans build on these two basic contracts, and in most countries, typically, one or the other type dominates.1

While the long-term nominal aspect of mortgages—a form of nominal rigidity—has been studied in the household finance literature (e.g., Campbell and Cocco 2003), a formal analysis of its consequences at the aggregate level has been missing (see Campbell 2013). This is despite the fact that mortgages, in the minds of policy makers, constitute an integral part of a monetary transmission mechanism (e.g., Bernanke and Gertler 1995; Bernanke 2007; Mishkin 2007).

In major economies, already the size of mortgage finance would suggest that its role in the monetary transmission mechanism must be important. Mortgage payments are equivalent to 15%-22% of homeowners’ pre-tax income in the United States (average for the past 30-40 years); 15%-20% in the United Kingdom (Hancock and Wood 2004); 27% in Germany (European Mortgage Federation 2012b); 36.5% in Denmark (first-time homeowners; European Mortgage Federation 2012c); and 30% in France (first-time homeowners; European Mortgage Federation 2009). And the mortgage debt to (annual) GDP ratio in developed economies has reached, on average, 70% in 2009, although cross-country variation is large (International Monetary Fund 2011, Chapter 3). In some countries outstanding mortgage debt is even larger than government debt and its maturity is longer.2

In light of these facts, this paper provides a conceptual and quantitative analysis of the role of the long-term nominal aspect of mortgages in the monetary transmission mechanism at the aggregate level. To accommodate the mortgage market structures of different countries, both FRM and ARM contracts are studied. First, in a simple partial equilibrium setup, the paper describes the channels through which this aspect of mortgages transmits nominal shocks to the real economy. These channels are then embedded in a calibrated general equilibrium model. The purpose of this exercise is to impose general equilibrium discipline on the problem, given its aggregate context, and

---

1 Countries in which FRMs—with interest rates fixed for at least ten years—have traditionally dominated the mortgage market include Belgium, Denmark, Germany, France, and the United States; in most other countries, either ARMs or FRMs with interest rates fixed for less than five years prevail; see Scanlon and Whitehead (2004) and European Mortgage Federation (2012a). Research is still inconclusive on the causes of the cross-country heterogeneity, but likely reasons include government policies, historical path dependence, sources of mortgage funding (capital markets vs. bank deposits), and inflation experience; see Miles (2004), Green and Wachter (2009), and European Mortgage Federation (2012b). Countries also differ in terms of prepayment penalties, costs of refinancing, recourse, prevalence of teaser rates, the frequency of ARM resets, and the ARM reference rate. Our analysis abstracts from these details.

2 Hilscher, Raviv, and Reis (2014) document that in the United States government debt has predominantly short-term maturity.
assess the quantitative importance of the mechanism. To isolate the effect of the long-term nominal aspect of mortgages, we abstract from all other nominal rigidities (sticky prices, wages, etc.), as well as other channels through which housing finance affects the macroeconomy (default, home equity lines of credit in providing liquidity, primary-secondary spread, etc.).

The nominal rigidity in question is studied in an environment in which homeowners do not trade a full set of state-contingent securities with mortgage investors. Consequently, the effective discount factors of the two agent types are not equalized state by state and risk sharing is limited. In this setup, we identify two channels through which mortgage contracts transmit nominal shocks into the real economy. Both channels are distinct from the traditional real rate channel of monetary policy transmission.

One channel works through the cost of new mortgage loans ("price effect"). In essence, it is a dynamic version of the tilt effect that previously has been studied in static settings (e.g., Schwab 1982). Expected future inflation, transmitted into nominal mortgage interest rates, redistributes real mortgage payments over the life of the loan so as to leave mortgage investors indifferent between new mortgages and other assets. If this results in real payments increasing in periods/states in which income has high value to homeowners, the effective cost of the mortgage to homeowners increases. This effect is qualitatively the same under both FRM and ARM.

The other channel works through the effects of inflation on the real value of mortgage payments of outstanding debt ("income effect"). Under incomplete asset markets, mortgages translate inflation shocks into shocks to real disposable income. This channel is qualitatively different under the two contracts. Under FRM, higher inflation reduces real mortgage payments; under ARM, higher inflation increases real mortgage payments, if it sufficiently transmits into higher nominal interest rates. Under FRM, the effects gradually grow over time through accumulated inflation. Under ARM, they are immediate and resemble the effects of an increase in the real interest rate, even when the real rate itself does not change. This is because the effect, on real mortgage payments, of the immediate increase in the nominal interest rate dominates the offsetting gradual effect of higher inflation.

The general equilibrium model, which embeds the two channels, consists of homeowners and capital owners/mortgage investors. There is a representative agent of each type. Such a coarse split of the population is motivated by Campbell and Cocco (2003); in the data, the third and fourth quintiles of the wealth distribution represent typical homeowners; the 5th quintile represents

---

3 The real rate channel, whereby the central bank directly affects the ex ante short-term real interest rate, is described by, for example, Bernanke and Gertler (1995). In our model, the real interest rate responds to monetary policy shocks only indirectly through general equilibrium effects.

4 Bernanke and Gertler (1995) and Mishkin (2007) refer to this effect also as a "cash flow" or "household balance sheet" effect.
capital owners. As an approximation to the characteristics in the data, homeowners in the model derive income from labor and invest in housing capital, financing a given fraction of housing investment with mortgages. Capital owners do not work and invest in capital used in production, one-period nominal bonds, and mortgages, pricing the assets competitively by arbitrage. Homeowners also can access the one-period bond market, but at a cost. The production side is standard and monetary policy is characterized by an interest rate rule.

In equilibrium, the stochastic properties of short- and long-term nominal interest rates depend on the parameters of exogenous shock processes. By using the model, the persistence and standard deviations of two of these shocks—to the monetary policy rule (a nominal shock) and the one-period bond market (a real shock)—are estimated by matching the volatility and persistence of the long-term nominal interest rate and the long-short spread. Similarly to Atkeson and Kehoe (2009), the nominal shock turns out to be highly persistent, shifting the level of the nominal yield curve and inflation, the other shock fairly temporary, affecting the slope of the yield curve and the ex ante real interest rate. Given these estimates, the model is used as a laboratory to quantitatively assess the real effects of the nominal shock, which—like the level factor in the data, for example Piazzesi (2006)—is the more important of the two shocks for nominal interest rates (in contrast, existing money-macro literature mainly studies shocks affecting the real rate). The main focus is on housing investment, where the shock should matter the most, but responses of other variables are also studied.

The findings are summarized in three points. First, the real effects of the nominal shock turn out to be stronger under ARM than under FRM: a one-percentage-point (annualized) downward shift of the nominal yield curve and inflation generates, on impact, a 1.8% increase in housing investment under ARM, whereas a 0.7% decline under FRM (0.15% decline with refinancing). The magnitude of the response under ARM is similar to that occurring due to shocks affecting the real interest rate, typically studied in the literature. The finding that the effects are stronger under ARM than under FRM may seem surprising as, a priori, one may expect that a contract fixing nominal payments for the entire term will generate larger real effects. The failure to do so is because under FRM the increase in real mortgage payments due to

5 The first and second quintiles are essentially renters with no assets and little liabilities and are not included in the model.
6 In the model, risk premiums are constant and all effects of the shocks on the yield curve work through expectations hypothesis.
7 There are two additional shocks: total factor productivity and the marginal rate of transformation between housing and nonhousing use of output. Together, the four shocks produce standard deviations of endogenous variables, and their correlations with output, consistent with the cyclical moments of U.S. data.
8 The responses are symmetric for an upward shift of the yield curve, except in the case of refinancing.
lower inflation (income effect) is only gradual and is partially offset by less expensive new loans (price effect). In contrast, under ARM, the drop in the mortgage rate has an immediate effect on the real mortgage payments of the outstanding debt (income effect) and this is further supported by cheaper new loans (price effect). Refinancing, while in principle making FRM look like ARM when interest rates decline, does not overturn the result. This is because, once calibrated to the data, refinancing accounts, on average, for at most 2% of the outstanding debt per quarter.\footnote{Even though the refinancing fraction can as much as double in response to realistic drops in interest rates, it is still insufficient to overturn the findings. Of course, given the large size of the stock, the small variation in the refi fraction of the stock shows up as a large variation in the share of refi loans in new loans, periodically reported by Freddie Mac.} When FRMs and ARMs coexist, the real effects lie in-between the two separate cases.

Second, under both contracts, the size of the real effects declines with the persistence of the nominal shock (i.e., when the shock starts to affect mainly the slope of the yield curve, rather than its level). This is because what matters with long-term loans is the expected path of inflation, and nominal interest rates, over the entire term of the loan.

And third, persistently higher inflation redistributes income from capital owners to homeowners under FRM, but (at least initially) from homeowners to capital owners under ARM. The redistribution is gradual under FRM, but immediate under ARM, working through a pass through to nominal interest rates.

### 1. Related Studies

The paper is related to different strands of a growing literature on monetary policy, housing, and debt. First, following Iacoviello (2005), a number of studies focus on the interaction between sticky prices and the collateral value of housing, whereby housing facilitates borrowing for general consumption purposes, similar to home equity lines of credit Iacoviello (2010, contains a brief summary of this line of research). To this end, loans in these models are short-term one-period loans.\footnote{Rubio (2011) extends the Iacoviello (2005) framework by considering one-period loans with interest rates evolving in a sluggish manner, as weighted averages of past interest rates, interpreting such loans as FRMs. Calza, Monacelli, and Stracca (2011) distinguish between ARMs, modeled as one-period loans, and FRMs, modeled as two-period loans.} We abstract from this channel, focusing instead on the role of mortgages as loans for house purchase (i.e., first mortgages) and stressing their long-term nominal aspect in transmitting nominal shocks to the real economy.\footnote{Ghent (2012) considers long-term FRM loans denominated in real, rather than nominal, terms.}

Second, following Doepke and Schneider (2006), there is work on the redistributive effects of inflation under incomplete markets and nominal debt...
This research focuses on debt contracts with fixed nominal payments, similar to FRM loans. However, we show that the redistributive consequences of inflation are quite opposite under ARM contracts. Policy recommendations based on this literature, such as nominal GDP targeting (Sheedy 2013), are thus inappropriate for countries with a large fraction of ARMs.

Third, a line of research investigates, in various contexts, the effects of inflation on housing: the tax code (Piazzesi and Schneider 2012), money illusion (Piazzesi and Schneider 2007; Brunnermeier and Julliard 2008), and substitution between market and home production (Aruoba, Davis, and Wright 2012). In our model, inflation transmits to housing investment through mortgage contracts.

Finally, in recent years, a few studies investigated empirically the connections between monetary policy and mortgage contracts. Villar Burke (2015) compares interest rates on outstanding mortgage debt and new mortgage loans in a sample of Eurozone countries in a period around the cut in the European Central Bank’s policy rate in 2008/2009. In countries in which FRMs dominate there was almost no change in the interest rate on the pool of outstanding mortgages. In contrast, in countries in which ARMs dominate, the interest rate on the pool declined almost in parallel with the policy rate. The responses of interest rates on new loans also differed across countries, declining more moderately in FRM countries than in ARM countries. Consistency with these empirical findings is at the core of the mechanism in our model.

Calza, Monacelli, and Stracca (2013), using a VAR model based on the usual identification strategy, find stronger negative responses of housing investment to positive monetary policy shocks in ARM than in FRM countries. The shock they identify is closer to the real interest rate shock in our model, than the nominal shock we focus on, in the sense that it increases the ex-ante real interest rate. Nevertheless, our model is consistent with their finding, producing stronger negative responses to the real interest rate shock under ARM than under FRM.

Di Maggio, Kermani, and Ramcharan (2014) and Keys et al. (2014) exploit the variation across U.S. counties in the use of FRM and ARM contracts to investigate the responses of consumption to the 2008 cut in the Fed funds rate using household-level data. Both studies find that counties with a larger share of ARMs in the existing pool of loans experienced a much smaller decline in consumption than counties with a larger share of FRMs.

12 Earlier studies include Meh, Rios-Rull, and Terajima (2010), Doepke, Schneider, and Selezevna (2015). Redistributive effects of monetary policy are also at the heart of the transmission mechanism proposed by Gertler and Gilchrist (1994).


14 Earlier VAR studies of housing investment include Bernanke and Gertler (1995) and Iacoviello and Minetti (2009).
larger boost in homeowners’ consumption than counties with a larger share of FRMs. While the authors are careful to control for other channels that could potentially lead to such an outcome, identification can always be an issue in reduced-form analysis. Our model provides a theoretical general-equilibrium underpinning for such empirical findings. The model also can replicate the empirical marginal propensities of consumption, documented by Di Maggio, Kermani, and Ramcharan (2014).\footnote{Cloyne, Ferreira, and Surrin (2012) also use household-level data. They attempt to identify changes in nominal interest rates in the United States and the United Kingdom due to monetary policy surprises. While they find substantial differences in the responses between outright homeowners and mortgagors, they do not find large differences between the responses of mortgagors in the two countries.}

2. A Simple Partial Equilibrium Model

The main ideas of the paper can be conveyed within a simple partial equilibrium model. We describe the two channels of transmission (price and income effects) under FRM and ARM, explain why the combined effect is stronger under ARM, and discuss under what conditions monetary policy is neutral and to what extent the results also apply to other forms of nominal debt. Refinancing and the choice between FRM and ARM are delayed until the full model, although we touch on these issues in Section 2.7, which provides numerical illustrations of the price and income effects.

Throughout this section, the real interest rate and real labor income are held constant, the one-period nominal interest rate is varied exogenously, and the current inflation rate is also exogenous. All these variables are endogenized in the general equilibrium model. The details of FRM and ARM loans differ across countries. We abstract from the details and focus on the key common features\footnote{For instance, in the United States, the ARM rate may not change at the same frequency as the monetary policy rate, as in our model. Typically, the ARM rate changes only once a year. In addition, many ARMs have an initial period for which the rate is fixed. In contrast, tracker mortgages, common, for instance, in Ireland, change the mortgage rate whenever the policy rate of the European Central Bank changes.}.

2.1 A mortgage-financed house purchase

There are three periods, with time denoted by \( t = 1, 2, 3 \). Each period a household is endowed with constant real income \( w \) and in \( t = 1 \) has no outstanding mortgage debt (outstanding debt is considered later). In \( t = 1 \), the household makes a once-and-for-all house purchase, financing a fraction \( \theta \) of the purchase with a loan and a fraction \( 1 - \theta \) with income. The loan can be used only for house purchase and the house lasts for \( t = 2, 3 \), then it fully depreciates. The lifetime utility function of the household is

\[
V = \sum_{t=1}^{3} \beta^{t-1} u(c_t) + \sum_{t=2}^{3} \beta^{t-1} g(h),
\]

where \( \beta \) is a discount factor, \( c_t \) is period-\( t \) nonhousing consumption, \( h \) is housing, and \( u(\cdot) \) and \( g(\cdot) \) have standard properties. The household maximizes utility with respect to \( c_1, c_2, c_3, \) and \( h \), subject to three per-period budget constraints:
\[ c_1 + h = w + l / p_1, \quad c_2 = w - m_2 / p_2, \quad \text{and} \quad c_3 = w - m_3 / p_3, \]

where \( l = \theta p_1 h \) is the nominal value of the loan, \( m_2 \) and \( m_3 \) are nominal mortgage payments, and \( p_i \) is the aggregate price level (i.e., the price of goods in terms of an abstract unit of account; this section abstracts from house prices).

Assume there is a financial market that prices assets by the no-arbitrage principle but in which the household does not participate due to, for instance, high entry costs. This exclusion is reflected in the sequence of the budget constraints above, which do not allow for financial instruments other than the mortgage (in the full model this assumption is partially relaxed, but maintaining it at this stage brings out the key aspects of the mechanism more clearly). Assume that monetary policy controls the one-period nominal interest rate \( i_t \).

No-arbitrage pricing implies the Fisher effect:

\[ 1 + \pi_{t+1} = (1 + \pi_t) \left( 1 + \frac{i_t}{1+r} \right), \]

where \( 1+r \) is a gross real interest rate, given by some exogenous effective discount factor (intertemporal rate of substitution) of investors, \( \mu^* = (1+r)^{-1} \), and \( \pi_{t+1} = p_{t+1} / p_t - 1 \) is the inflation rate between periods \( t \) and \( t+1 \). A given \( i_t \) thus pins down \( \pi_{t+1} \).

Mortgage payments have a general form, \( m_2 = (i_M^2 + \gamma)l \) and \( m_3 = (i_M^3 + 1)(1-\gamma)l \). Here, \( i_M^t \) denotes the mortgage interest rate, henceforth referred to as the ‘mortgage rate’. Under FRM, \( i_M^2 = i_M^3 = i^F \); under ARM, \( i_M^2 \) and \( i_M^3 \) may be different. Further, \( \gamma \) is the amortization rate in the first period of the life of the mortgage, when the outstanding nominal debt is \( l \). In the second period, the outstanding nominal debt is \( (1-\gamma)l \) and the amortization rate is equal to one (i.e., the mortgage is repaid in full). For a standard mortgage, \( \gamma \) is calculated so as to ensure \( m_2 = m_3 \), conditional on \( i_M^2 \).

In the FRM case, no-arbitrage pricing by investors means that \( i_F \) satisfies

\[ 1 = Q^{(1)}_1 (i_F + \gamma) + Q^{(2)}_1 (i^F + 1)(1 - \gamma), \]

where \( Q^{(1)}_1 = (1+i_1)^{-1} = (1 + \pi_2)^{-1} \mu^* \) and \( Q^{(2)}_1 = [(1+i_1)(1+i_2)]^{-1} = [(1+\pi_2)(1+\pi_3)]^{-1} \mu^* \) are the period-1 prices of one- and two-period zero-coupon bonds, determined according to the expectations hypothesis. The no-arbitrage condition states that the present value of payments from a mortgage of size one dollar is equal to one dollar. In the ARM case, \( i_M^2 = i_1 \) and \( i_M^3 = i_2 \) satisfy no-arbitrage pricing, as

\[ Q^{(1)}_1 (i_M^2 + \gamma) + Q^{(2)}_1 (i_M^3 + 1)(1 - \gamma) = \frac{i_1 + \gamma}{1+i_1} + \frac{(i_2 + 1)(1 - \gamma)}{1+i_1} = 1. \]

It is easy to verify that a higher \( i_M^2 \) implies higher mortgage payments, despite the fact that \( \gamma \) changes to equalize the payments between the two periods. Observe that to equalize the payments, \( \gamma = 1/(2+i_M^2) \). Therefore, \( d\gamma/di_M^2 = -1/(2+i_M^2)^2 \in (-0.25,0) \). That is, when the mortgage rate increases, the amortization rate needs to decline, but the decline is small. Overall, mortgage payments therefore increase. As a first pass, we can therefore abstract from changes in the amortization rate when studying the effects of interest rates on mortgage payments.

3344
Mortgages and Monetary Policy

2.2 Price effect

The price effect refers to the effect of monetary policy on the cost of new mortgages. By 'monetary policy' we mean a sequence of short-term nominal interest rates $i_1$ and $i_2$. The sequence is known to the household (and the investors). Generally speaking, different sequences of nominal interest rates (and the resultant inflation) lead to different sequences of real mortgage payments over the life of the loan, so as to ensure that the present value of the one-dollar loan is equal to one dollar, as dictated by the above no-arbitrage pricing by investors. This, however, affects the value of the loan from the household’s perspective, as the household does not have—due to the assumed market incompleteness—the same valuation of the real mortgage payments as the investors.

Specifically, after substituting the budget constraints in the utility function, the first-order condition for the utility maximization problem of the household with respect to $h$ gives

$$u'(c_1)(1 + \tau_H) = \beta(1 + \beta)g'(h),$$

where

$$\tau_H = -\theta \left\{ 1 - \left[ \frac{i_2^M + \gamma}{1 + \pi_2} + \mu_{12} \mu_{23} \frac{(i_2^M + 1)(1 - \gamma)}{(1 + \pi_2)(1 + \pi_3)} \right] \right\} \quad (3)$$

is a wedge between the marginal utility of period-1 nonhousing consumption and the marginal lifetime utility of housing. Further, $\mu_{t,t+1} \equiv \beta u'(c_{t+1})/u'(c_t)$ is the household’s effective discount factor. The wedge—working like a relative price of housing—captures the cost of mortgage finance from the household’s perspective. To see this, we observe that the expression in the square brackets is the present value of real mortgage payments, discounted with the household’s effective discount factor, $\mu_{t,t+1}$, rather than the effective discount factor of the investor, $\mu^*$. In the case of FRM

$$\tau_{\text{FRM}}^H = -\theta \left\{ 1 - \left[ \frac{i_F^M + \gamma}{1 + \pi_2(1 + \pi_3)} + \mu_{12} \mu_{23} \frac{(i_F^M + 1)(1 - \gamma)}{(1 + \pi_2)(1 + \pi_3)} \right] \right\},$$

in the case of ARM

$$\tau_{\text{ARM}}^H = -\theta \left\{ 1 - \left[ \frac{i_1 + \gamma}{1 + \pi_2} + \mu_{12} \mu_{23} (\mu^*)^{-1} \frac{(1 - \gamma)(1 + \pi_3)}{1 + \pi_2} \right] \right\},$$

where, in the second term, we have used the Fisher effect $(1 + i_2)/(1 + \pi_3) = (\mu^*)^{-1}$.

18 While the price effect is discussed in a deterministic setup, which is the simplest setup in which it can be demonstrated, it is straightforward to extend it to stochastic settings.
The Review of Financial Studies / v 30 n 10 2017

To explain the effect of monetary policy on the wedge, it is easier to start with the ARM case. A decline in $i_1$, for instance, reduces the real payments in the first period of the life of the loan, $(i_1 + \gamma)/(1 + \pi_2)$. To see this, observe that through the Fisher effect, $\pi_2$ declines one-for-one with $i_1$ but—as $\gamma$ is less than one—the effect of $i_1$ on the numerator is larger than the effect of $\pi_2$ on the denominator. The decline in $\pi_2$, however, increases the real payments in the second period of the life of the loan, $(\mu^{-1}(1 - \gamma))/(1 + \pi_2)$. If the household’s effective discount factor assigns a sufficiently large weight on payments in the first period of the life of the loan, mortgage finance becomes cheaper for the household and the wedge declines, inducing more housing investment. In the FRM case, the effect is similar, as long as $i_F$ sufficiently declines in response to the decline in the short rate. This will be the case if the decline in the short rate is persistent; that is, both $i_1$ and $i_2$ decline. The lower inflation rates, $\pi_2$ and $\pi_3$, increase real payments in the second period of the life of the FRM loan, but again, the wedge declines if the those payments are sufficiently discounted.

2.3 Income effect

The income effect refers to the effect of monetary policy on the real value of payments on outstanding mortgage debt. It concerns ex post changes in inflation and, in the case of ARM, also nominal interest rates. Effectively, mortgages translate nominal shocks to inflation and nominal interest rates into shocks to real mortgage payments and disposable income.

To make the discussion concrete, consider a household in period $t = 1$, that took out a mortgage in period $t = 0$. The mortgage matures in $t = 3$ (to show the income effect under ARM, the minimum length of the loan has to be three periods). By “monetary policy,” we mean a sequence $\pi_1$, $i_1$, and $i_2$, which becomes known to the household in period $t = 1$; that is, one period after the household took out the loan (in this sense it is a monetary policy “surprise”). Specifically, suppose that monetary policy reduces $\pi_1$, $i_1$, and $i_2$. Through the Fisher effect, $i_1$ and $i_2$ again pin down $\pi_2$ and $\pi_3$, respectively.

---

19 The intuition extends to multiperiod loans. Holding the nominal payments to income ratio constant across loans of different maturities (i.e., adjusting the size of the loan accordingly), we can show that the front- and back-end effects become stronger the longer is the term of the loan. At the front end, for a sufficiently small inflation rate, $i_M + \gamma + \pi = i_M + \gamma + \pi$, which holds since $\gamma \to 0$, as the term of the loan increases. Changes in real mortgage payments at the front end are thus approximately equal to changes in the nominal interest rate.

At the back end, for a small enough nominal interest rate, $\gamma + \pi + (1 + \pi + \gamma + \pi) \approx \gamma + \pi + (1 + \pi + \gamma + \pi)$, which holds since $\gamma \to 1$ as the end of the term approaches. The denominator becomes larger with the term of the loan, increasing the accumulated inflation effect at the back end. In contrast, when the loan is a one-period loan ($(\gamma + \pi + \gamma + \pi + \gamma + \pi) = 1 + \pi$, as $\gamma = 1$, and there is no price effect.

20 In the general equilibrium model, such a joint decline in nominal interest rates and current inflation is an equilibrium outcome.
The real mortgage payments that the homeowner has to make on the outstanding loan are

\[ m_1 = \frac{i^M_1 + \gamma_1}{1 + \pi_1}, \quad \text{in } t = 1, \]

\[ m_2 = \frac{i^M_2 + \gamma_2}{(1 + \pi_1)(1 + \pi_2)}(1 - \gamma_1), \quad \text{in } t = 2, \]

\[ m_3 = \frac{i^M_3 + 1}{(1 + \pi_1)(1 + \pi_2)(1 + \pi_3)}(1 - \gamma_2)(1 - \gamma_1), \quad \text{in } t = 3, \]

where \( \gamma_1 \) and \( \gamma_2 \) are the amortization rates in the first and second periods of the life of the loan and \( \tilde{l}_0 = l_0/p_0 \) is the real size of the loan in period \( t = 0 \). Under FRM, \( i^M_1 = i^M_2 = i^M_3 = i^F_0 \). Under ARM, \( i^M_1 = i_0, i^M_2 = i_1, \) and \( i^M_3 = i_2 \).

Unlike the price effect, the income effect works in opposite directions under FRM and ARM. Broadly speaking, in the FRM case, accumulated inflation affects the real value of mortgage payments over the life of the loan. In the ARM case, however, at least in the near term, changes in the nominal interest rate have the dominating effect on the real payments, as in the case of the price effect. The following paragraphs provide the details, describing the above three equations.

Starting with the payments in \( t = 1 \), the mortgage rate \( i^M_1 \) is predetermined; it is equal to some \( i^F_0 \) under FRM and to \( i_0 \), the period-0 short rate, under ARM, both determined in period \( t = 0 \). Clearly, a decline in \( \pi_1 \) generates a negative income effect for the household in \( t = 1 \).

Regarding payments in \( t = 2 \), there is still the lingering effect of the decline in \( \pi_1 \). More importantly, however, the payments are affected by \( i_1 \). In the FRM case, \( i^M_1 = i^F_1 \) and a decline in \( i_1 \) increases the real payments further due to the resultant decline in \( \pi_2 \). In the ARM case, the effects are different. Here, \( i^M_2 = i_1 \) and a decline in \( i_1 \) reduces the real mortgage payments, even though \( \pi_2 \) declines one-for-one with \( i_1 \) (again, as \( \gamma_2 \in (0, 1) \), the effect of \( i_1 \) on the numerator is larger than the effect of \( \pi_2 \) on the denominator). The decline in the nominal interest rate thus works like a decline in the real interest rate, reducing real interest payments on the loan.

Finally, the real payments in \( t = 3 \) increase under both contracts, due to the accumulated effect of persistently lower inflation.

2.4 Both effects

What happens when both effects are taken into account? Under FRM, a decline in the nominal interest rate reduces the cost of new housing investment, but increases real payments on outstanding debt. Under ARM, both the cost of new housing investment and real payments on outstanding debt decline (the latter at least in the near term). Furthermore, the increase in real mortgage payments on outstanding debt under FRM is gradual, whereas the decline under
ARM is immediate. Potential general equilibrium adjustments aside, one would therefore expect monetary policy to be more potent under ARM than FRM loans.

2.5 Monetary policy neutrality

Generally speaking, if asset markets were complete, the Modigliani-Miller theorem would apply in our setting and debt finance (including its form, FRM, or ARM) would be irrelevant. As a result, monetary policy would be neutral.

Specifically, start by observing that the mortgage pricing conditions (1) and (2) can be rewritten as

\[
1 = \mu t + \gamma + \mu^* \mu t (i_M^2 + 1)(1 - \gamma) (1 + \pi^2) (1 + \pi^3).
\]

Thus, if asset markets were complete—implying \( \mu_{t+1} = \mu^* \)—the expression in the square brackets in equation (3) would be equal to one and the wedge \( \tau_H \) would be equal to zero. Intuitively, the household’s valuation of mortgage payments in every period (and more generally in every state) would be the same as that of the investor. And because the investor’s no-arbitrage pricing implies that the present value of mortgage payments is equal to one, the household’s valuation would also be equal to one. This holds regardless of what monetary policy does. Furthermore, under complete markets, the agents would be mutually insured against redistributive income shocks, making their income immune to monetary policy surprises. The income effect would therefore be eliminated.

Monetary policy would also be neutral if asset markets were generally incomplete, but complete with respect to inflation. That is, if the nominal rigidity in mortgages could be effectively removed from the economy. Suppose \( \mu_{t+1} \neq \mu^* \), but consider, for instance, mortgages that are index-linked, adjusting the principal for changes in the price level. The nominal payments of such mortgages are

\[
m_2 = (i_M^2 + \gamma)(1 + \pi^2), \quad m_3 = (i_M^3 + 1)(1 - \gamma)(1 + \pi^2)(1 + \pi^3)
\]

and no-arbitrage pricing implies \( i_M^2 = i_M^3 = r \). Converting the nominal payments to real, \( m_2 / p^2 = (r + \gamma) l \) and \( m_3 / p^3 = (r + 1)(1 - \gamma) l \), shows that the real payments do not depend on nominal variables. The income effect is thus absent. The wedge also does not depend on nominal variables, but is generally nonzero:

\[
\tau_H = -\theta \{1 - \mu_{12}(\gamma + r) + \mu_{12} \mu_{23}(r + 1)(1 - \gamma)\}.
\]

2.6 Other types of nominal debt

The focus of the paper is on mortgages, as opposed to corporate debt, as long-term corporate assets are less debt-dependent than housing and presumably corporations operate closer to complete asset markets than do households.\(^{21}\)

---

\(^{21}\): Long-term corporate assets are typically more than 75% financed through retained earnings and other forms of equity (Rajan and Zingales 1995).
Nevertheless, the two channels of transmission in principle apply also to the corporate sector. Long-term corporate debt usually takes the form of coupon bonds, which result in our setup when \( \gamma_1 = \gamma_2 = 0 \). With such amortization schedule, the nominal payments are concentrated in the final period of the loan and the real effects of monetary policy work predominantly by affecting the real value of those payments through accumulated inflation.\(^{22}\)

Within the household sector, our analysis applies also to auto loans, which have a similar payment structure as FRMs. We abstract from auto loans as mortgage debt has a longer term (the usual term of auto loans is only five years) and makes up a much larger fraction of household debt than auto loans.

2.7 Numerical illustrations

The three-period model has limitations to illustrate how the price and income effects vary with inflation persistence and what is the realistic size of the effects for a typical household. We therefore conclude this section with numerical examples, based on a standard 30-year mortgage, to illustrate these effects quantitatively.

Figure 1 plots debt servicing costs—the ratio of real mortgage payments \( (m_t/p_t) \) to real income \( (w) \)—over the term of the loan (120 quarters) under two alternative paths of \( i_t \); a constant “steady-state” \( i_t = 4\% \) and a mean-reverting decline of \( i_t \) to 1\% in period 1, which we refer to as “monetary policy easing.” The persistence of the decline is 0.95, which is the average quarterly autocorrelation of the short rate in the data. All the assumptions of the simple model—constant \( r \) and \( w \) and no-arbitrage pricing, with Equation (1) extended to 120 quarters—are maintained here. The parameterization is \( r = 1\% \) per annum and \( \tilde{l} = 16w \), that is, four times annual income. In the full model of the next section, the household chooses \( \tilde{l} \) optimally (by choosing \( h \)). The point here is simply to illustrate the size of these effects for one particular loan size.\(^{23}\)

At the steady-state interest rate, debt servicing costs are front-loaded and decline monotonically over the life of the mortgage from 29\% to 6.5\%. This is the standard “tilt effect” (e.g., Schwab 1982), occurring due to a positive inflation rate (in this case 3\%). This path is a baseline against which to compare the debt servicing costs under the monetary policy easing.

Starting with the case of a new loan, under both FRM and ARM, monetary policy easing reduces debt servicing costs at the front end, where they are

\(^{22}\) Gomes, Jermann, and Schmid (2013) study nominal corporate debt in a model in which monetary policy affects its default value.

\(^{23}\) The parameterization of the loan size is based on the average ratio, 1975-2010, of the median price of a new home (assuming a loan-to-value ratio of 76\%) to the median household net income (assuming an income tax rate of 23.5\%). The data on house prices and gross incomes are from the U.S. Census Bureau. The loan-to-value ratio is the average ratio for single family newly built home mortgages (Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10); the tax rate is a NIPA-based estimate. A historical 2\% markup is added to the interest rate.
Monetary policy easing
Persistence = 0.95

Figure 1
Illustration of price and income effects
Debt servicing costs over a term of a new and an existing 30-year mortgage under alternative paths of the short-term nominal interest rate. The label “steady-state” refers to the case in which the short rate is at its steady-state level of 4%. The mortgage is equal to four times the household’s annual income; the real interest rate is held constant at 1% per annum.

the highest, and somewhat increases them at the back end, where they are the smallest. The decline under FRM is smaller than under ARM because the FRM interest rate, due to the mean-reverting nature of the short rate in this example, declines by less than the short rate itself. The flattening of the path of debt servicing costs results in a smoother consumption profile and thus a decline in $\tau_H$ under a concave utility function (and/or sufficiently small $\beta$). Using a log utility function and $\beta=0.9883$, a parameterization of the model of the next section, $\tau_H$ declines by 1.66 percentage points in the case of FRM and by 3.83 percentage points in the case of ARM. Recall that $(1+\tau_H)$ can be interpreted as the effective price of new housing and, thus, these numbers represent quarterly percentage declines in this price.24

24 As the decline in the wedge is larger under ARM than under FRM, given the choice, the household in this setup would prefer the ARM loan over the FRM loan.
For the case of an existing loan, we consider a loan with 119 periods remaining (the magnitudes of the income effect decline as the remaining term of the loan gets shorter for reasons analogous to those in footnote 19). In the case of ARM, as the loan is only one period into its life, the expected path of debt servicing costs is essentially the same as that for the new loan. Under FRM, however, the persistently low inflation leads to a gradual increase in debt servicing costs for the remainder of the term of the loan. The income effect under the two contracts thus goes in opposite directions.\footnote{25}

Figure 2 plots the results of the same experiment, but for two alternative degrees of persistence: 0.99 and 0.5. In the 0.99 case, the magnitudes are much larger than in the 0.95 case. Furthermore, for new loans, the results under FRM and ARM are more similar to each other than in the 0.95 case, as the long rate drops almost as much as the short rate. For existing loans, however, the effects under FRM and ARM diverge further apart. When the persistence is 0.5, the effects on both new and existing loans are small, in fact hardly noticeable in the FRM case.

3. General Equilibrium Model

The general equilibrium model extends the model of the previous section to infinite horizon, formally introduces shocks, and endogenizes the variables that were either held constant (real labor income and the real interest rate) or were treated as exogenous (the short-term nominal interest rate and current inflation).

3.1 Environment

The economy’s population is split into two groups, “homeowners” and “capital owners,” with measures $\Psi$ and $(1 - \Psi)$, respectively. Within each group, agents are identical. An aggregate production function combines capital and labor to produce a single good, which can be used for consumption, capital investment, or as new housing structures; new homes consist of new structures and new land. Capital owners own the economy’s capital stock, homeowners supply labor. Capital owners play the role of mortgage investors, kept outside of the simple model.\footnote{26} Where applicable, the notation is the same as in Section 2. Only new variables and functions are therefore defined. When a variable’s notation is the same for both agent types, an asterisk (*) denotes the variable

\footnote{25} With refinancing, in the aggregate, a fraction of the outstanding FRM debt would get refinanced (the whole stock would get refinanced if refinancing was costless). This would not be the case in the opposite experiment of monetary policy tightening.

\footnote{26} The split of the population is motivated by Campbell and Cocco [1996]. In the data, homeowners (corresponding to the third and fourth quintiles of wealth distribution) have one major asset, a house, and one major liability, a mortgage. Their main source of income is labor income. In contrast, capital owners (the fifth quintile) hold almost the entire corporate equity in the economy and housing is a less important component of their asset composition; labor income is also a less important source of their income. The first and second quintiles are essentially renters with no assets and little liabilities and are not included in the model.
pertaining to capital owners. Throughout the model, fiscal variables—taxes, transfers, and government spending—are included only to ensure sensible calibration.

3.1.1 Homeowners. The representative homeowner’s problem is an extended version of the problem in Section 2. The homeowner maximizes expected lifetime utility

$$E_t \sum_{r=0}^{\infty} \beta^r \{ v(c_t, h_t) - \chi_t \}, \quad \beta \in (0, 1),$$

(4)

where $v(\ldots)$ has standard properties and $\chi_t$ is a utility cost related to refinancing and mortgage choice, described below. The maximization is subject to a

A. High persistence (0.99) of the short rate decline

NEW LOAN (120-PERIOD TERM)

Existing loan (119 PERIODS REMAINING)

B. Low persistence (0.5) of the short rate decline

NEW LOAN (120-PERIOD TERM)

Existing loan (119 PERIODS REMAINING)

Figure 2
Illustration of price and income effects
Debt servicing costs under high and low persistence of the mean-reverting short rate decline by three percentage points.
sequence of constraints
\[
c_t + p_{Ht} x_{Ht} + \frac{b_{t+1}}{p_t} = (1 - \tau_N)(u_t n - \tau) + \frac{l_t}{p_t} - \frac{m_t}{p_t} + (1 + i_{t-1} + \Upsilon_{t-1}) \frac{b_t}{p_t} = \psi_t,
\]
(5)
\[
\frac{l_t}{p_t} = \theta p_{Ht} x_{Ht},
\]
(6)
\[
h_{t+1} = (1 - \delta_H) h_t + x_{Ht}.
\]
(7)

Here, \(x_{Ht}\) is newly purchased homes, \(p_{Ht}\) is their relative price, \(b_{t+1}\) is the homeowner’s holdings of a one-period nominal bond between periods \(t\) and \(t+1\), \(u_t\) is a real wage rate, \(n\) is labor, which the household supplies inelastically, \(\tau_N\) is a labor income tax rate, \(\tau\) is a transfer to capital owners, and \(\delta_H \in (0, 1)\) is a depreciation rate of the housing stock. \(\Upsilon_{t-1}\) is a bond market participation cost, governed by a function \(\Upsilon(-b_t)\), where \(b_t = b_t / p_{t-1}\) is the homeowner’s real holdings of the bond. The function \(\Upsilon(.)\) is assumed to be increasing and convex and it controls the extent to which homeowners can smooth out consumption in the presence of income fluctuations. To avoid the participation cost affecting the definition of aggregate output, it is rebated to the homeowner in a lump-sum way as \(\psi_t\), which the homeowner takes as given. The description of mortgage payments follows after describing the capital owner.

3.1.2 Capital owners. A representative capital owner maximizes expected lifetime utility
\[
E_t \sum_{i=0}^{\infty} \beta^i u(c^*_t),
\]
where \(u(.)\) has standard properties, subject to a sequence of constraints
\[
c^*_t + x_K^* + \frac{b^*_t}{p_t} = \frac{l^*_t}{p_t} = [(1 - \tau_K) r_t + \tau_K \delta_K] k_t
\]
\[
+ (1 - \tau_{bt})(1 + i_{t-1}) \frac{b^*_t}{p_t} + \frac{m^*_t}{p_t} + \frac{p_{Lt}}{1 - \Psi}.
\]
(8)

27 As in the three-period model, \(\theta\) is a parameter. Empirical evidence supports this assumption at the aggregate level: over the period 1973–2006, there has been very little variation in the cross-sectional average of the loan-to-value ratio for single family newly built home first mortgages, despite large changes in interest rates and other macroeconomic conditions (Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10).

28 It is further assumed that \(\Upsilon(.) = 0\) when \(b_t = 0\), \(\Upsilon(.) > 0\) when \(b_t < 0\) (the homeowner is borrowing), and \(\Upsilon(.) < 0\) when \(b_t > 0\) (the homeowner is saving). We think of \(\Upsilon(.) > 0\) as capturing a premium for unsecured consumer credit, which is increasing in the amount borrowed. \(\Upsilon(.) < 0\) can be interpreted as higher intermediation costs for homeowners than capital owners, which reduces the homeowners’ returns on savings below those of capital owners. In steady state, \(\dot{b} = 0\). A technical role of the cost function is that, as in two-country business-cycle models with incomplete markets, it prevents the one-period debt from becoming a random walk in a log-linear solution of the model. In other words, it keeps the log-linearized model stationary.
Here, $x_{K_t}$ is investment in capital, $r_t$ is the real rate of return on capital, $\tau_K$ is a capital income tax rate, $\delta_K \in (0, 1)$ is a capital depreciation rate (tax deductible), $k_t$ is capital, $\tau^*_t$ is a lump-sum transfer, $1/(1 - \Psi)$ is new residential land, which the capital owner receives each period as an endowment, and $p_{L_t}$ is its relative price. In addition, $\tau_{bt}$ is a stochastic tax rate on income from the one-period bond market. The tax rate follows a stationary AR(1) process $\tau_{b,t+1} = \rho_b \tau_{b,t} + \epsilon_{b,t+1}$, where $\epsilon_{b,t} \sim i.i.d N(0, \sigma_b)$. The tax proceeds from this tax are rebated back to the capital owner in a lump-sum way as a part of $\tau^*_t$. As discussed below, $\tau_{bt}$ shows up as a wedge in the capital owner’s Euler equation for bonds and affects the ex ante real interest rate. In a reduced form way, this wedge captures various primitive shocks and frictions that affect the ex ante real interest rate (see Šustek 2011). Under no-arbitrage pricing, the capital owner is indifferent across investing in mortgages, bonds, and capital. His composition of period-$t$ investment is therefore pinned down by homeowners’ demand for new mortgages and the one-period bond.

### 3.1.3 Mortgages

In the interest of clarity, we start by describing FRMs and ARMs in their simple forms; that is, no refinancing or choice between the contracts. The modeling of mortgages is based on Kydland, Rupert, and Šustek (2016), who consider infinitely lived loans, which nevertheless contain the key characteristics of standard (finitely lived) mortgages. $d_t$ denotes the period-$t$ stock of outstanding nominal mortgage debt of the homeowner, the nominal mortgage payments the homeowner has to make in period $t$ are

$$m_t = (R_t + \gamma_t) d_t,$$

where $R_t$ and $\gamma_t$ are, respectively, the interest and amortization rates of the outstanding debt. The variables determining $m_t$ are state variables evolving as

$$d_{t+1} = (1 - \gamma_t) d_t + l_t,$$

$$\gamma_{t+1} = (1 - \phi_t)(\gamma_t)^a + \phi_t \kappa,$$

$$R_{t+1} = \begin{cases} (1 - \phi_t) R_t + \phi_t i^F_t, & \text{if FRM}, \\ i_t, & \text{if ARM}, \end{cases}$$

with $\phi_t \equiv l_t / d_{t+1}$ representing the fraction of new loans in the outstanding debt next period. The amortization rate $\gamma_{t+1}$ and the interest rate $R_{t+1}$ in the FRM

---

29 By subjecting only the capital owner to this wedge, it is assumed that the wedge captures frictions related only to the “wholesale” financial market, in which mortgage investors participate, as opposed to the “retail” financial market, in which homeowners participate. The main results are not sensitive as to which agent is subjected to the wedge.
Mortgages and Monetary Policy

case thus evolve as weighted averages of the amortization and interest rates, respectively, on the existing stock and new loans. \( \kappa, \alpha \in (0, 1) \) are parameters. Specifically, \( \kappa \) is the initial amortization rate of a new loan and \( \alpha \) controls the evolution of the amortization rate over time. Notice that setting \( \alpha = 0 \) and \( \kappa = 1 \) implies \( \gamma_t = 1 \) \( \forall \, t \). That is, \( l_t \) becomes a one-period loan. Setting \( \alpha = 1 \) results in a constant amortization rate \( \gamma_t = \kappa \) and thus declining nominal mortgage payments over the life of the loan. Recall from Section 2 that to keep mortgage payments constant over the life of a loan, the amortization rate has to be increasing. When \( \kappa, \alpha \in (0, 1) \), the amortization rate increases, converging to one. Kydland, Rupert, and Šustek (2016) show that \( \kappa \) and \( \alpha \) can be chosen so as to approximate the payments of a standard 30-year mortgage.30

Mortgage payments received by the capital owner are specified analogously

\[
m^*_t = (R^*_t + \gamma^*_t) d^*_t, \tag{14}
\]

\[
d^*_t+1 = (1 - \gamma^*_t) d^*_t + l^*_t, \tag{15}
\]

\[
\gamma^*_{t+1} = (1 - \phi^*_t) (\gamma^*_t)^\alpha + \phi^*_t \kappa, \tag{16}
\]

\[
R^*_{t+1} = \begin{cases} (1 - \phi^*_t) R^*_t + \phi^*_t i^*_F & \text{if FRM}, \\ i_t & \text{if ARM}, \end{cases} \tag{17}
\]

where \( \phi^*_t \equiv l^*_t / d^*_t \). Under FRM, the first-order condition for \( l^*_t \) pins down \( i^*_F \) such that the capital owner is indifferent between new mortgages and rolling over the one-period bond from period \( t \) on. The first-order condition is an infinite-horizon counterpart to Equation (1) (see Online Appendix A).

3.1.4 Refi loans. Chen, Michaux, and Roussanov (2013) document that 60% of refi loans in the United States are unrelated to interest rate changes (instead serving the purpose of liquidity withdrawals). Given our interest, we abstract from this type of refinancing. Chen, Michaux, and Roussanov (2013) further demonstrate that the remaining fraction is negatively related to a difference between the current FRM interest rate and an average of its past values. This is the relationship we intend to capture.

With refinancing, newly originated loans consist of mortgages used for new house purchases (first mortgages) and refi loans

\[
l_t = \theta(p_t p^H_t x^H_t) + q_t (1 - \gamma_t) d_t. \tag{18}
\]

30 Under appropriate choice of \( \kappa \) and \( \alpha \), even though the loan has an infinite life, it gets essentially repayed within 30 years and the nominal payments are approximately constant for most of these 30 years, conditional on a constant mortgage rate. Such modeling of mortgages is convenient, as both the agents and loans have an infinite life, thus allowing a simple recursive representation of the model.
Here, $q_t$ is the fraction of the outstanding debt that is being refinanced in period $t$ (refinancing takes place after the regular period-$t$ amortization payment is made). Mortgage payments are still given by $m_t = (R_t + \gamma_t) d_t$, but now with laws of motion

$$d_{t+1} = (1 - q_t)(1 - \gamma_t) d_t + l_t,$$

$$\gamma_{t+1} = (1 - \phi_t)(\gamma_t)^{\alpha} + \phi_t \kappa,$$

$$R_{t+1} = (1 + \Theta_t) R_t + \Theta_t i_t^F,$$

where $\phi_t = \theta(p_t, p_{Ht}, x_{Ht})/d_{t+1}$ is the fraction of first mortgages in the stock of debt next period and $\Theta_t = l_t/d_{t+1}$ is the fraction of all newly originated loans in the stock of debt next period. Observe that combining Equations (18) and (19) gives back the original law of motion for debt (11); that is, prepayments and refi loans cancel out and only first mortgages are added to the stock of debt. An implicit assumption in the law of motion (20) is that a new refi loan starts with an amortization rate of the loan that it replaces. This leads to a sharp characterization of refinancing. Namely, in the homeowner’s constraints, refinancing shows up only in the law of motion (21), as a change in the weights on the old effective interest rate $R_t$ and the current market interest rate $i_t^F$, without tying this change to new house purchases. When $q_t = 1$, that is, the whole existing stock is refinanced, $\Theta_t = 1$ and the evolution of $R_{t+1}$ becomes the same as under ARM. When $q_t = 0$, $\Theta_t = \phi_t$ and the evolution of $R_{t+1}$ becomes the same as in the case of the basic FRM loan, Equation (13).

The capital owner’s laws of motion are analogous: $d^*_{t+1} = (1 - q_t)(1 - \gamma_t^*) d^*_t + l_t^*$, $\gamma^*_{t+1} = (1 - \phi_t^*)(\gamma_t^*)^{\alpha} + \phi_t^* \kappa^*$, and $R^*_{t+1} = (1 + \Theta_t^*) R^*_t + \Theta_t^* i_t^F$, where $\phi_t^* = [l_t - q_t(1 - \gamma_t^*) d_t^*/d_{t+1}^*]$ and $\Theta_t^* = l_t^*/d_{t+1}^*$, with $\phi_t^*$ taken as given. The capital owner’s first-order condition for $i_t^F$ (gross lending) again determines $i_t^F$. Observe that, as the evolution of $d^*_{t+1}$ depends on $q_t$, the FRM rate $i_t^F$ depends on the refinancing behavior of homeowners (i.e., $\rho_t$ shows up in a first-order condition for $l_t^*$). The current FRM rate $i_t^F$ thus prices in the expected refinancing behavior of homeowners.$^{31}$

The homeowner’s decision to refinance is modeled in a simple way

$$q_t = \left\{ \begin{array}{ll} \Delta & \text{and } \chi_t = 0 \quad \text{if } i_t^F > R_t, \\
\frac{\Delta}{\theta_t} & \text{and } \chi_t = \Gamma(\widehat{q}) \quad \text{if } i_t^F < R_t. \end{array} \right.$$  

Here, $\Delta$ is a parameter, representing a refi fraction occurring due to exogenous reasons (e.g., moving house), $\widehat{q}$ is chosen optimally, and $\Gamma(\widehat{q})$ is a function with $\Gamma(\widehat{q}) > 0$ and a minimum $\Gamma(\widehat{q}) = 0$ at $\widehat{q} = \Delta$. The utility cost $\chi_t$ can be interpreted, for instance, as a time loss. When we abstract from refinancing, we set $q_t \equiv 0$ and $\chi_t \equiv 0.$

$^{31}$ With refinancing, the left-hand side of the capital owner’s budget constraint needs to be modified: $i_t^F$ is replaced with $i_t^F - \rho_t (1 - \gamma_t^*) d_t^*$ (i.e., it is first mortgages in both cases).
3.1.5 Mortgage choice. Allowing for endogenous choice between new FRM and ARM loans requires keeping track of the two types of debt separately. Therefore, mortgage payments of the homeowner are $m_t = m_{1t} + m_{2t}$, where $m_{1t}$ are payments on outstanding FRM debt and $m_{2t}$ are payments on outstanding ARM debt. These two variables are determined by separate laws of motion (11)–(13), as applicable, each with its own new loans, $l_{1t}$ for FRM and $l_{2t}$ for ARM. The mortgage payments received by the capital owner are specified analogously. The financing constraint of the homeowner becomes

$$l_{1t} + l_{2t} = l_t,$$

with

$$l_t = \theta p_t p_{Ht} x_{Ht}.$$  \hspace{1cm} (22)

The homeowner faces a cost function $\Phi(l_{2t}/l_t)$, assumed to satisfy $\Phi(l_{2t}/l_t)^{\prime\prime} > 0$ with a minimum $\Phi(l_{2t}/l_t) = 0$ at $l_{2t}/l_t = \Lambda$, where $\Lambda > 0$ is a parameter. As in the case of refinancing, it is assumed that the cost takes the form of the utility loss $\chi_t$. A way to think about the parameter $\Lambda$ is as representing a “normal” share of ARM loans in the economy, determined by various institutional and historical factors, noted in footnote 1 and not modeled here, and the cost function represents the economy’s costs of deviating from this norm. This simple modeling strategy is in the spirit of the ideas suggested by Badarinza, Campbell, and Ramadorai (2014). The capital owner takes the mortgage choice by the homeowner as given but the first-order condition for $l_{1t}^*$ again determines the mortgage rate $i_F$, so as to leave the capital owner indifferent between the two types of new loans (and other assets).

3.1.6 Technology. An aggregate production function, operated by perfectly competitive producers, is given by $Y_t = A_t f(K_t, N_t)$, where $K_t$ is the aggregate capital stock, $N_t$ is aggregate labor, and $f(.,.)$ has the standard neoclassical properties. Total factor productivity (TFP) follows a stationary AR(1) process

$$\log A_{t+1} = (1 - \rho_A) \log A_t + \rho_A \log A_t + \epsilon_{A,t+1},$$

where $A$ is the unconditional mean and $\epsilon_{A,t} \sim iid N(0,\sigma_A)$. The real rate of return on capital, $r_t$, and the real wage rate, $w_t$, are determined by the marginal products of capital and labor, respectively. The resource constraint of the economy is $C_t + C_t^* + X_{Kt} + q_t X_{St} + G = Y_t$, where $X_{St}$ is new housing structures and $G$ is constant government expenditures (capital letters denote aggregate counterparts to lower case letters introduced earlier). Here, $q_t$ is the marginal rate of transformation between new housing structures and the other uses of output, and hence the relative price of new housing structures. It is given as $q_t = \eta_t q(X_{St})$, where $q(X_{St})$ is a convex function that makes the economy’s production possibilities frontier (PPF) concave in the space of $(C_t + X_{Kt} + G)$ and $(X_{St})$—a specification akin to that of Huffman and Wynnd (1999), a stand-in for the costs of moving factors of production across different sectors of the economy. Further, $\eta_t$ is a shock following a stationary AR(1) process $\eta_{t+1} = (1 - \rho_q) \eta_t + \rho_q \eta_t + \epsilon_{\eta,t+1}$, with unconditional mean equal to one and $\epsilon_{\eta,t} \sim iid N(0,\sigma_\eta)$. This shock can be thought of as a housing construction shock. The shocks $A_t$ and $\eta_t$ are relevant only for cross-validating the model against business-cycle moments of the data.
As in [Davis and Heathcote (2005)], new homes consist of new housing structures and land and are produced by perfectly competitive homebuilders according to an aggregate production function \( X_{Ht} = g(X_{St}, X_{Lt}) \). Here, \( X_{Ht} \) is the aggregate number of new homes constructed in period \( t \), \( X_{Lt} \) is the aggregate new residential land, and \( g \) has the standard neoclassical properties.

### 3.1.7 Monetary policy.

Monetary policy is characterized by a Taylor-type rule with a stochastic inflation target (e.g., Ireland 2007):

\[
i_t = (i + \pi_t - \pi) + \nu (\pi_t - \pi), \quad \nu > 1,
\]

where \( i \) is the steady-state short-term nominal interest rate, and \( \pi_t \) is the inflation target. The inflation target follows a stationary AR(1) process:

\[
\pi_{t+1} = (1 - \rho) \pi_t + \rho \pi + \epsilon_{\pi, t+1}, \quad \epsilon_{\pi, t+1} \sim iidN(0, \sigma_\pi).
\]

Notice that the interest rate rule can be rewritten in a more typical form as:

\[
i_t = i + \nu (\pi_t - \pi) + \xi_t, \quad \xi_t = -(\nu - 1)(\pi_t - \pi).
\]

### 3.2 Equilibrium

To economize on space and notation, the equilibrium is defined only for the case of separate FRM and ARM loans in their basic form. Let \( z_t = [\log A_t, \pi_t, r, n_t, p_{t-1}] \) be the vector of exogenous state variables and a lagged value of the price level, \( s^*_{t} = [k_t, b_t^*, d_t^*, \gamma_t^*, R_t^*] \) the vector of the capital owner’s state variables, \( s_t = [h_t, h_t, d_t, \gamma_t, R_t] \) the vector of the homeowner’s state variables, and \( S_t = [K_t, H_t, B_t, D_t, \Gamma_t, B_t] \) the vector of aggregate endogenous state variables, where the elements are, respectively, aggregate capital, housing stock, bonds, outstanding mortgage debt, and its amortization and interest rates.

Next, write the capital owner’s optimization problem as:

\[
U(z, S, S^*) = \max_{[x, (b^*)]} \left\{ u(c^*) + \beta E[U(z', S', (s^*)']|z] \right\},
\]

where \( u(c^*) = \max_{[x, (b^*)]} \left\{ u(c^*) + \beta E[U(z', S', (s^*)']|z] \right\}, \)

where a prime denotes a value next period and the constraints (5)–(7) and (10)–(13) are thought to have been substituted in the utility and value functions. Similarly, write the homeowner’s problem as:

\[
V(z, S, S) = \max_{[x, h]} \left\{ v(c, h) - \chi + \beta E[V(z', S', (s^*)']|z] \right\},
\]

where the constraints (8)–(10) are thought to have been substituted in the utility and value functions. Let \( W_t = [X_{Kt}, p_t, i_t^M, X_{Ht}, B_{t+1}] \) be the
vector of aggregate decision variables and prices, where \( i^M = i^F \) under FRM and \( i^H = i \) under ARM. Define a function \( W = W(z_t, S_t) \).

A recursive competitive equilibrium consists of the functions \( U, V, \) and \( W \) such that: (1) \( U \) and \( V \) solve (24) and (25), respectively; (2) \( r_t \) and \( w_t \) are given by the respective marginal products of capital and labor, \( p_H \) and \( \rho_L \) are given by the respective marginal products of structures and land, and \( q_t = \eta \varphi(X_S) \); (3) \( i \) is given by the monetary policy rule (23); (4) the bond, mortgage, housing, and land markets clear: (1) \( 1 / \rho_L t = \Psi \rho L H_t, X_{Ht} = g(X_S, X_L), \) and \( X_{Lt} = 1 \); (5) aggregate consistency is ensured: \( K_t = (1 - \Psi) k_t, x_{Kt} = (1 - \Psi) x_{Kt}, X_{Ht} = \Psi x_{Ht}, B_t = \Psi h_t, H_t = \Psi h_t, (1 - \Psi) m_t^2 = \Psi m_t, (1 - \Psi) \gamma_t^2 = \Psi \gamma_t, R_t = \Psi \gamma_t, \) and \( (1 - \Psi) \tau_t = x_K (r_t - \delta_K) K_t + \tau (w_t N - \Psi t) + \Psi t + \tau_t (1 - i_t) B_t / p_t - G_t; \) (6) the exogenous state variables follow their respective stochastic processes and the endogenous aggregate state variables evolve according to aggregate counterparts to the laws of motion for the respective individual state variables; and (7) the individual optimal decision rules of the capital owner (for \( x_K, (b^*) \) and \( f^* \)) and the homeowner (for \( x_H \) and \( b^* \)) are consistent with \( W(z, S) \).

Online Appendix B describes the computational procedure. Before computing the equilibrium, the model is made stationary by dividing all nominal variables by either \( p_t \) or \( p_{t - 1} \). Given a relatively large number of state variables, the model is solved by log-linear approximation, adopting a version of the method of Hansen and Prescott (1995). Thus, in the quantitative results presented, the linear equilibrium decision rules and pricing functions are characterized by certainty equivalence, whereby only current realizations and the conditional first moments of future shocks (not higher moments) affect decisions and market clearing prices. In the case of refinancing, the computational method is modified by utilizing the ideas developed by Guerrieri and Iacoviello (2015).

33 With refinancing, the capital owner’s laws of motion (24) and (25) are replaced with their refi counterparts. Similarly, the homeowner’s laws of motion (24) and (25) are replaced with their refi counterparts. In addition, the homeowner’s maximization is also with respect to \( \varphi_t \) and an aggregate counterpart to \( \varphi_t \) is an element of \( W_t \). Further, the mortgage market clearing condition noted below is replaced with \( (1 - \Psi) [b^* \gamma_t^2 / p_t = \Psi [ith_H, x_{Ht}] + \varphi_t (1 - \gamma_t) h_t / p_t] \). With mortgage choice, the homeowner’s maximization is also with respect to \( l_{1t} \) and \( l_{2t} \), subject to \( l_{1t} = (p_t p_{1t} x_{1t}) - l_{2t} \), and the aggregate counterpart to \( l_{2t} \) is a part of \( W_t \). In the capital owner’s problem, maximization with respect to \( l^* \) is replaced with \( l^*_t \). Mortgage market clearing requires \( (1 - \Psi) [l^*_{1t} + l^*_{2t}] = \Psi [ith_H, x_{Ht}] + \varphi_t [ith_H, x_{Ht}] \). The choice of \( x_{Ht} \) and \( l_{2t} \) by the homeowner thus pins down \( l^*_{1t} \) and \( l^*_{2t} \).

34 In the case of ARM, \( i^H = i \) makes the capital owner indifferent between new mortgages and bonds and the first-order condition for \( i^F \) can be dropped from the description of the equilibrium. In the case of FRM, the first-order condition is needed to determine \( i^F \).

35 We have explored the effects of (constant) second moments of the shocks on the equilibrium decision rules and pricing functions, using a special case of Epstein-Zin preferences (i.e., with unitary intertemporal elasticity of substitution) that is easy to handle by the computational method. But like Tallarini (2015) and Backus, Ferriere, and Zin (2015), we have found little effect of the second moments on the model dynamics for a wide range of the risk-aversion parameter.
3.3 Equilibrium nominal interest rates and inflation

The capital owner’s first-order conditions for $b^{*}_{t+1}$ and $x_{Kt}$ yield the Fisher equation. In a linearized form

$$i_t - E_t\pi_{t+1} \approx E_t r_{t+1} + E_t \tau_{t+1},$$  \hspace{1cm} (26)

where (abusing notation) the variables are in percentage point deviations from steady state. Notice that, for a given expected rate of return on capital $E_t r_{t+1}$, determined by the expected marginal product of capital, an increase in $E_t \tau_{t+1}$ increases the ex ante real interest rate, $i_t - E_t\pi_{t+1}$. In this sense, $\tau_{t+1}$ is a real rate shock.

Given stochastic processes for $r_t$ and $\tau_{t+1}$, the Fisher equation and the monetary policy rule (23) determine, by forward substitutions, $i_t$ and $\pi_t$. For $\rho_\pi$ close to one, excluding explosive paths for inflation (a common assumption), the resultant expression for $i_t$ is

$$i_t \approx \sum_{j=0}^{\infty} \left( \frac{1}{\nu_\pi} \right)^j E_t r_{t+1+j} + \frac{\nu_\pi \rho_\phi}{\nu_\pi - \rho_\phi} \tau_{t+1} + \bar{\pi}_t,$$  \hspace{1cm} (27)

Substituting $i_t$ from Equation (27) back into the policy rule (23) gives the equilibrium inflation rate

$$\pi_t \approx \frac{1}{\nu_\pi} \sum_{j=0}^{\infty} \left( \frac{1}{\nu_\pi} \right)^j E_t r_{t+1+j} + \frac{\rho_\phi}{\nu_\pi - \rho_\phi} \tau_{t+1} + \bar{\pi}_t.$$ \hspace{1cm} (28)

Thus, under the assumption that $\rho_\pi$ is close to one, the equilibrium short-term nominal interest rate and inflation move, subject to general equilibrium adjustments in the expected path of $r_t$, one for one with the highly persistent $\pi_t$ shock. In this sense, in contrast to the $\tau_{t+1}$ shock, the $\pi_t$ shock is a purely nominal shock.

Because movements in $i_t$ occurring due to the $\pi_t$ shock are highly persistent, the long rate $i_F$ moves almost as much as the short rate in response to this shock. In this sense, the $\pi_t$ shock works like a level factor shock. In contrast, the $\tau_{t+1}$ shock, if less persistent, has only a temporary effect on the short rate. It thus has only a small effect on the long rate and moves predominantly the long-short spread. We exploit these properties in calibration.\footnote{In the present case, where $\tau_{t+1}$ affects only the capital owner’s Euler equation, the ex ante real rate is given by Equation (26) and, in a log-linearized form, the Euler equations of the two agents give $E_t c^{*}_{t+1} - c^*_t = E_t \tau_{t+1} + \gamma_t$ (again abusing notation). If $\tau_{t+1}$ affects both the capital owner’s and homeowner’s Euler equations, by entering both agents’ budget constraints as a tax on bonds, then Equation (26) holds again, but $E_t c^{*}_{t+1} - c^*_t = E_t \tau_{t+1} + \gamma_t$. If $\tau_{t+1}$ affects only homeowners, then we can write $i_t - E_t\pi_{t+1} = E_t r_{t+1} - i_t - \gamma_t + E_t \tau_{t+1}$ and $E_t c^{*}_{t+1} - c^*_t = E_t \tau_{t+1} - i_t - \gamma_t$. Notice that in all three cases, $\tau_{t+1}$ (in so far as it is autocorrelated) affects the ex ante real rate, $i_t - E_t\pi_{t+1}$. The only difference across the three cases is how the real rate relates to the consumption paths of the two agents. Experimenting with the three cases, the quantitative differences on our findings are small.}
4. Model Cross-Validation

Before proceeding to quantitatively assess the transmission mechanism under investigation, we use three different sets of available empirical observations to cross-validate the model: (1) business-cycle moments, (2) responses to a real interest rate shock, and (3) the marginal propensity to consume (MPC) of ARM homeowners. As the empirical evidence is conditioned on FRM versus ARM, or based on ARM alone, the model in this section is also solved under only either FRM or ARM, without refinancing.

4.1 Parameterization

For the quantitative analysis in this section and in Section 5, the following standard functional forms are used: 

\[ u(c^*) = \log c^* \],

\[ v(c,h) = \xi \log c + (1 - \xi) \log h \],

\[ f(K,N) = K^\varsigma N^{1-\varsigma} \],

\[ g(X_S,X_L) = X_S^{1-\phi} X_L^\phi \].

Further, 

\[ q(X_{St}) = \exp(\zeta(X_{St} - X_S)) \],

where \( \zeta > 0 \) and \( X_S \) is the steady-state ratio of new housing structures to output (\( Y \) is normalized to be equal to one in steady state). A similar functional form is used also for the bond market participation cost:

\[ \Upsilon(-\tilde{b}_t) = \exp(-\vartheta \tilde{b}_t) - 1 \],

where \( \vartheta > 0 \) and \( \tilde{b}_t = 0 \) in steady state. It is straightforward to check that these functions satisfy the properties set out in Section 3.

The model is then calibrated to be consistent with long-run aggregate and cross-sectional moments of U.S. data. For space constraints, Online Appendix E describes the calibration in detail. Here, we just note that the persistence of the \( \pi_t \) and \( \tau_{bt} \) shocks is calibrated by matching the autocorrelation of a long-term nominal interest rate (ten-year government bond yield, serving as a proxy for the 30-year mortgage rate due to longer availability) and the long-short spread. Table 1 summarizes all parameter values.

4.2 Business-cycle moments

First, the business-cycle moments—standard deviations and correlations with output—are reported in panel A of Table 2. Six standard deviations in the table are a part of the set of moments used to calibrate the model. Nonetheless, the remaining standard deviations—of consumption, the short rate, and inflation—line up well with the data. The model also correctly predicts positive correlations with output of consumption, the short rate, and inflation—line up well with the data. The model also correctly predicts positive correlations with the two types of investment, with the correlation of consumption being the highest and the correlation of housing investment being the lowest of the three. However, it overpredicts the strength of these relations. This also applies to house prices. Such finding should be expected, as the number of shocks in the model is limited. The short and long interest rates, the long-short spread, and the inflation rate, nonetheless, have correlations with output similar to those in the data.

The findings in Table 2 show that the cyclical properties of the model are not particularly sensitive to whether FRM or ARM contract is used to simulate

---

37 The data are quarterly. Both the actual and simulated data are expressed as percentage (or percentage point, for interest rates and the inflation rate) deviations from HP-filter trend.
Table 1
Calibration

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ψ</td>
<td>2/3</td>
<td>Share of homeowners</td>
</tr>
<tr>
<td>Preferences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ</td>
<td>0.9883</td>
<td>Discount factor</td>
</tr>
<tr>
<td>ξ</td>
<td>0.5003</td>
<td>Weight on nonhousing consumption</td>
</tr>
<tr>
<td>Technology</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ζ</td>
<td>0.283</td>
<td>Capital share of output</td>
</tr>
<tr>
<td>η</td>
<td>0.02225</td>
<td>Depreciation rate of capital</td>
</tr>
<tr>
<td>ζ_1</td>
<td>0.01021</td>
<td>Depreciation rate of housing</td>
</tr>
<tr>
<td>ω_1</td>
<td>3.0</td>
<td>Curvature of PPF</td>
</tr>
<tr>
<td>ζ</td>
<td>0.1</td>
<td>Land share of new housing</td>
</tr>
<tr>
<td>Fiscal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>0.138</td>
<td>Government expenditures</td>
</tr>
<tr>
<td>τ_1</td>
<td>0.235</td>
<td>Labor income tax rate</td>
</tr>
<tr>
<td>τ_2</td>
<td>0.3362</td>
<td>Capital income tax rate</td>
</tr>
<tr>
<td>τ</td>
<td>0.4693</td>
<td>Transfer</td>
</tr>
<tr>
<td>Mortgage contract</td>
<td></td>
<td></td>
</tr>
<tr>
<td>θ</td>
<td>0.6</td>
<td>Loan-to-value ratio</td>
</tr>
<tr>
<td>κ</td>
<td>0.00162</td>
<td>Initial amortization rate</td>
</tr>
<tr>
<td>α</td>
<td>0.9946</td>
<td>Amortization adjustment factor</td>
</tr>
<tr>
<td>Bond market</td>
<td></td>
<td></td>
</tr>
<tr>
<td>σ</td>
<td>0.0345</td>
<td>Participation cost function</td>
</tr>
<tr>
<td>Monetary policy rule</td>
<td></td>
<td></td>
</tr>
<tr>
<td>νπ</td>
<td>1.5</td>
<td>Weight on inflation</td>
</tr>
<tr>
<td>τ_3</td>
<td>0.0113</td>
<td>Steady-state inflation rate</td>
</tr>
<tr>
<td>Exogenous processes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ_A</td>
<td>0.9641</td>
<td>Persistence of TFP shock</td>
</tr>
<tr>
<td>σ_A</td>
<td>0.0148</td>
<td>SD of TFP innovations</td>
</tr>
<tr>
<td>ρ₀</td>
<td>0.994</td>
<td>Persistence of nominal shock</td>
</tr>
<tr>
<td>σ₀</td>
<td>0.0012</td>
<td>SD of nominal innovations</td>
</tr>
<tr>
<td>ρ₀</td>
<td>0.903</td>
<td>Persistence of real rate shock</td>
</tr>
<tr>
<td>σ₀</td>
<td>0.00075</td>
<td>SD of real rate innovations</td>
</tr>
<tr>
<td>ρ_q</td>
<td>0.798</td>
<td>Persistence of PPF shock</td>
</tr>
<tr>
<td>σ_q</td>
<td>0.012</td>
<td>SD of PPF innovations</td>
</tr>
<tr>
<td>Refinancing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δ</td>
<td>0.02</td>
<td>Long-run fraction of refi in outst. debt</td>
</tr>
<tr>
<td>ω</td>
<td>12.0</td>
<td>Cost function parameter</td>
</tr>
<tr>
<td>Mortgage choice</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Λ</td>
<td>0.3</td>
<td>Long-run share of ARMs</td>
</tr>
<tr>
<td>ω</td>
<td>0.0055</td>
<td>Cost function parameter</td>
</tr>
</tbody>
</table>

the model. This may seem surprising as one would expect that, especially, the moments of housing investment should be sensitive to the type of the loan. Housing investment is indeed less positively correlated with output under ARM than under FRM, and its volatility is lower under ARM than under FRM, but the differences are small. Is this true in the data? Panel B of the table shows that the differences in the model are similar to those between a set of FRM countries and a set of ARM countries. The reason for the similarity is that the TFP shock is the dominating shock in the model and housing investment responds similarly to such a shock under both contracts. However, as in the data, it responds a

---

38 The countries used are the only countries for which quarterly housing investment data are available going back to at least 1980. FRM counties include Belgium, France, and the United States: ARM countries include Australia, Canada, and the United Kingdom. The moments are taken from Kydland, E., R. Rupert, and Šustek (2016).


### Table 2

**Model cross-validation I: Business-cycle moments**

<table>
<thead>
<tr>
<th></th>
<th>A. U.S. data</th>
<th>Model</th>
<th>U.S. data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FRM</td>
<td>ARM</td>
<td>FRM</td>
<td>ARM</td>
</tr>
<tr>
<td><strong>SD Corr with Y</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Y )</td>
<td>1.92</td>
<td>1.90</td>
<td>1.91</td>
<td>1.00</td>
</tr>
<tr>
<td>( C )</td>
<td>0.85</td>
<td>0.78</td>
<td>0.78</td>
<td>0.79</td>
</tr>
<tr>
<td>( X_S )</td>
<td>13.32</td>
<td>12.98</td>
<td>12.77</td>
<td>0.60</td>
</tr>
<tr>
<td>( X_K )</td>
<td>4.70</td>
<td>5.09</td>
<td>5.38</td>
<td>0.73</td>
</tr>
<tr>
<td>( i )</td>
<td>0.28</td>
<td>0.29</td>
<td>0.29</td>
<td>0.36</td>
</tr>
<tr>
<td>( i^F )</td>
<td>0.17</td>
<td>0.18</td>
<td>N/A</td>
<td>0.01</td>
</tr>
<tr>
<td>( i^F - i )</td>
<td>0.20</td>
<td>0.20</td>
<td>N/A</td>
<td>-0.49</td>
</tr>
<tr>
<td>( \pi )</td>
<td>0.34</td>
<td>0.23</td>
<td>0.22</td>
<td>0.36</td>
</tr>
<tr>
<td>( pH )</td>
<td>3.01</td>
<td>2.90</td>
<td>2.87</td>
<td>0.55</td>
</tr>
</tbody>
</table>

|                  | B. Other countries | Model | FRM | ARM |
|                  | FRM              | ARM   |     |     |
| \( S(D(X_S)/SD(Y)) \) | 5.80             | 5.50  |     |     |
| \( C(X_S,Y) \)     | 0.65             | 0.55  |     |     |

**Notes A**: All moments are for HP-filtered series, quarterly data. U.S. data: 1958–2006. Ten-year government bond yield is used as a proxy for \( i_F \) due to longer time availability; CPI ex-energy inflation rate is used for \( \pi \); three-month Treasury bill yield is used for \( i \); the ratio of the average price of new homes sold (Census Bureau) and the GDP deflator is used for \( pH \) (1975–2006). The model moments are averages of moments for 150 runs; the artificial data of each run are of the same length as the U.S. data and are also HP filtered. [Online Appendix C](#) provides the details of U.S. data.

**Notes B**: FRM countries include Belgium, France, and the United States; ARM countries include Australia, Canada, and the United Kingdom. The moments, based on HP-filtered data, are taken from Kydland, Rupert, and Šustek (2016). The classification is based on footnote 1.

little less under ARM than under FRM because the responses are dampened by procyclical movements of interest rates, to which housing investment is more sensitive under ARM than FRM, as shown below.

#### 4.3 Responses to a real rate shock

Second, we scrutinize the model responses to the \( \tau_{bt} \) shock, which directly affects the ex-ante real interest rate. Recall that, in a reduced-form way, \( \tau_{bt} \) captures various channels through which primitive shocks affect the ex ante real interest rate. The identification of such shocks in the data, often associated with monetary policy surprises, is well understood. [Calza, Monacelli, and Stracca (2013)](#) employ a typical VAR identification strategy to study the responses of housing investment in a sample of FRM and ARM countries. They find that, in response to a one-percentage-point (annualized) increase in the short-term nominal interest rate, leading to an increase in the real rate, housing investment declines more in ARM than in FRM countries. Figure[8] shows that the model is consistent with this finding. Panel A shows responses under the calibrated persistence of the \( \tau_{bt} \) shock, and panel B shows responses under calibration that reproduces the persistence of the nominal interest rate following the VAR shock in [Calza, Monacelli, and Stracca (2013)](#). In the latter case, the quantitative response under FRM falls within the error bands reported in their paper. Under ARM, the initial decline somewhat exceeds their error bands (−1.95% in the model vs. −1.4% in their paper). This is likely due to the fact that the interest
4.4 MPC of homeowners

Finally, in Table 3 we compare the MPC of ARM homeowners in the model with the empirical MPC from the study by Di Maggio, Kermani, and Ramcharan (2014) briefly described in Section 1 (the study does not contain MPC of FRM homeowners). In both the data and their study, the MPC is calculated as an increase in consumption of ARM homeowners divided by the extra income brought about by a decline in real mortgage payments occurring due to a drop in the nominal interest rate. In the model, the nominal interest rate can decline due to both the \( \tau_b \) and \( \pi_t \) shocks (i.e., it can decline due to a decline in the real rate or in line with inflation), whereas the empirical study is silent on the sources of the decline. We therefore consider both scenarios. As the empirical study is based on reduced form analysis, it does not address the question of the agents’ expectations of the persistence of the interest rate decline. When we use the calibrated persistence of the shocks, the model implies MPC of around 0.28. However, a modest reduction in the persistence of the \( \pi_t \) shock, from 0.994 to 0.95, replicates the empirical MPC of 0.17 (the MPC value of 0.28,
Table 3
Model cross-validation III: MPC of ARM homeowners

<table>
<thead>
<tr>
<th>Shock persistence</th>
<th>Calibrated</th>
<th>Matched</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nom. shock</td>
<td>Real rate shock</td>
<td>Nom. shock</td>
</tr>
<tr>
<td>N/A</td>
<td>0.994</td>
<td>0.90</td>
</tr>
<tr>
<td>MPC</td>
<td>0.17*</td>
<td>0.29</td>
</tr>
</tbody>
</table>

Note: Real rate shock = direct effect on the ex ante real interest rate (τₜₘ shock); nominal shock = no direct effect on the ex ante real interest rate (τₙ shock). Calibrated = persistence as in Table 1; matched = persistence so as to reproduce MPC in Di Maggio, Kermani, and Ramcharan (2014). MPC, marginal propensity to consume.

* Based on their reported $150 increase in consumption in response to $900 increase in disposable income.

however, is in the ballpark of estimates from the fiscal stimulus literature, e.g., Johnson, Parker, and Souleles 2006; Parker et al. 2013. As demonstrated below, reducing the persistence of the shock to 0.95 does not change the main message of the paper. In the case of the τₜₘ shock, the persistence needs to be brought down to 0.7 to match the MPC of 0.17 (interestingly, a value in-between our calibrated value of 0.9 and the value of 0.5 implied by Calza, Monacelli, and Stracca 2013).

5. Findings: Responses to the Nominal Shock

We start by reporting the responses of the model economy to the nominal shock under the two contracts separately, abstracting from the choice between them and refinancing. This exercise, while not strictly applicable to the United States, is relevant for other countries, in addition to its purpose of isolating the effects of each contract. In many countries, ARMs (of various types) are the only contract available; see footnote 1. And in some FRM countries (Germany, France) refinancing is less common than in the United States; see Villar Burke (2015). We then check the sensitivity of the basic findings to refinancing and mortgage choice. While our model does not provide a full treatment of refinancing and mortgage choice, it should give us a ballpark estimates of the effects of these decisions on the basic findings.

5.1 FRM versus ARM

The quantitative assessment of the transmission mechanism under the two separate contracts is contained in Figure 4. The figure plots the responses of selected variables to a positive (one percentage point, annualized) πₜₙ shock in period 1. The first two charts demonstrate the nominal and level factor nature of the shock: the short-term nominal interest rate and the inflation rate, and in the case of the FRM economy also the FRM rate, all increase more or less in parallel by approximately one percentage point (annualized). The model is stationary. Thus, even though convergence back to steady state may not be apparent from the plots, eventually all variables converge back to steady state. This, however, takes longer than the 40 periods displayed in the charts.
Figure 4
Findings I: The transmission mechanism under separate contracts
Responses to the nominal ($\pi_t$) shock under FRM and ARM; no refinancing, no mortgage choice. Interest rates and the inflation rate are measured as percentage point (annualized) deviations from steady state; quantities are in percentage deviations. One period = one quarter.

plots the responses of real mortgage payments ($m_t/p_t$). The chart confirms the effects discussed in Section 2. Due to higher inflation, real mortgage payments decline on impact (period 1) under both contracts by the same amount. But this decline is dwarfed by the magnitudes in subsequent periods. Under FRM, real mortgage payments display a persistent gradual decline, while under ARM the payments increase sharply (and persistently) one period after the shock. Under FRM, both homeowners' consumption ($C_t$) and housing investment ($X_{St}$) increase in response to growing real disposable income (the figure plots responses of $X_{St}$ but the responses of $X_{Ht}$ are similar). In contrast, under ARM, both variables decline in response to the drop in real disposable income. Further, the responses under ARM are stronger than under FRM. For instance, the decline in housing investment is 1.8% on impact under ARM, compared with a 0.7% increase under FRM [14]. House prices ($p_{Ht}$), not plotted due to space constraints, increase on impact by 0.2% under FRM, and decline by 0.5% under ARM. The last chart shows that capital owners' consumption ($C^*_t$) responses are opposite to those of homeowners, but are smaller and smoother. This reflects that mortgage payments are a quantitatively smaller fraction of capital owners' than homeowners' income (6.4% vs. 24.2% of after-tax income) and that capital owners can better smooth out fluctuations in income.

40 The responses of consumption and housing investment do not exactly track the responses of real mortgage payments as homeowners have (costly) access to the one-period bond market to smooth out the impact of the changes in disposable income.
5.1.1 Comparison with a real rate channel. It was argued in Section 2 that, under ARM, the effect of a one-for-one increase in the nominal interest rate and inflation works like an increase in the real interest rate. How does, then, the transmission mechanism under ARM quantitatively compare with a real interest rate channel? To answer this question, we rely again on the $\tau_{bt}$ shock that affects the ex ante real interest rate. Using the calibration based on the VAR study of Calza, Monacelli, and Stracca (2013), $p_b$ is set equal to 0.5. Figure 5 contains the results of this comparison. It plots the responses of selected variables ($i_t - E_t \pi_{t+1}$, $m_t / p_t$, $X_s$, and $C_t$) to a one-percentage-point (annualized) increase in the short term nominal interest rate, occurring due to either the $\tau_{bt}$ or the $\pi_t$ shock. Observe that under the $\tau_{bt}$ shock the ex ante real interest rate ($i_t - E_t \pi_{t+1}$) increases, whereas under the $\pi_t$ shock it stays almost unchanged (it gradually declines due to small general equilibrium effects working through capital accumulation). The immediate increase in real mortgage payments, however, is the same under the two shocks and the...
$\rho_{\pi} = 0.994$

$\rho_{\tau_b} = 0.95$

$\rho_{\tau_b} = 0.5$

Figure 6
Findings III: The effect of the persistence of the nominal ($\pi_t$) shock under FRM and ARM; no refinancing, no mortgage choice

Interest rates and the inflation rate are measured as percentage point (annualized) deviations from steady state; quantities are in percentage deviations. One period = one quarter.

Subsequent effect is, in fact, more persistent under the $\pi_t$ shock than under the $\tau_b$ shock. Housing investment under both shocks declines by roughly 2% on impact, but the decline is more persistent under the $\pi_t$ shock. The same applies to consumption. The transmission mechanism studied in this paper is thus at least as potent, under ARM, as the traditional real rate channel.

5.1.2 The role of persistence. The calibration of $\rho_{\pi}$ so far was based on replicating the persistence of the ten-year nominal government bond yield, used as a proxy for the 30-year mortgage rate. How do the quantitative findings change when the persistence of the shock is reduced? Figure 6 provides the answer. Focusing on the ARM case, in which the real effects are generally larger, the initial response of housing investment is reduced from 1.8% to 1.1% as the persistence is reduced from the calibrated value of 0.994 to 0.95 (recall that the latter value is required to match the MPC of homeowners in Di Maggio, Kermani, and Ramcharan 2014). When the persistence is further reduced to 0.5, the real effects become negligible. At this degree of persistence, the real effects
working through the traditional real rate channel are much stronger (refer to the bottom panel of Figure 3). Notice that as the persistence is reduced, the $\pi_t$ shock starts to affect the long-short spread, rather than the level of the yield curve. The transmission mechanism proposed in this paper is thus quantitatively more relevant for monetary policy shocks that, under expectations hypothesis, primarily affect the level of the yield curve, rather than its slope.

5.2 Optimal refinancing
Does optimal refinancing make the responses of the economy under FRM look more like under ARM when interest rates decline? Recall that the refi cost takes the form of a utility loss $\chi_t$ in the homeowner’s utility function (4).

To make the cost function operative, we assume a quadratic parametric form $\nu(\tilde{\varrho}_t - \Delta)^2$, with $\nu > 0$. Under this assumption, the optimal refi problem yields a simple solution with an intuitive interpretation, summarized by a first-order condition for $\tilde{\varrho}_t$ (see Online Appendix F for details)

$$\tilde{\varrho}_t = \frac{1}{1 + \gamma_t} \left( -\beta E_t \frac{V_{R_{t+1}}}{2\nu} \right) \left( R_t - i^F_t \right). \tag{29}$$

In Equation (29), $\tilde{d}_t = d_t / p_{t-1}$ and $V_{R_t} < 0$ is the derivative of the homeowner’s value function with respect to $R_t$, representing the marginal lifetime gain of reducing the effective interest rate on outstanding debt by one unit. By refinancing, the homeowner thus trades off the current marginal refi cost against the expected marginal lifetime benefit.

The parameter $\Delta$ is set equal to 0.02, which is the long-run average fraction of outstanding debt that is refinanced per quarter. The parameter $\nu$ is set equal to 12, so as to match, in equilibrium, the local elasticity of the share of refi loans in new loans with respect to the FRM rate (see Online Appendix F for details). Figure 7 shows the responses of selected variables to a 25 basis point quarterly (one percentage point, annualized) decline in the FRM rate, occurring due to the $\pi_t$ shock, with the calibrated persistence of $\rho_\pi$ = 0.994. For comparison, the figure also plots responses under FRM without refinancing and under ARM. As expected, the responses under refinancing lie in-between the other two cases. Quantitatively, they are still closer to the FRM responses. This is because the increase in the quarterly refi share of outstanding debt, $\tilde{\varrho}_t$, is fairly small: an increase from 0.02 to 0.026 on impact. Notice, however, that (given the size of the stock) such a modest increase in $\varrho_t$ shows up as a large increase in the refi share of new loans: an increase from 0.4 to 0.48 on impact. Due to the attenuated response of real mortgage payments under refi, the negative responses of consumption and housing investment observed under the basic FRM contract are now weaker, close to zero, in fact, for several periods.

5.3 Optimal mortgage choice
Various studies document that the ARM share in the United States moves positively with the long-short spread, or its various proxies (Koijen, Van
Responses to a one-percentage-point (annualized) decline in the FRM rate due to the nominal ($\pi_t$) shock, with the calibrated persistence $\rho_{\pi} = 0.994$. Refi shares are measured as percentage point deviations from steady state; quantities are in percentage deviations. One period = one quarter.

Hemert, and Van Nieuwerburgh 2007; Moench, Vickery, and Aragon 2010; Badarinza, Campbell, and Ramadorai 2014). Is our model consistent with this behavior once we allow for mortgage choice? And does mortgage choice affect the responses of the model economy to the $\pi_t$ shock? As in the case of refinancing, the mortgage choice cost function is made operative by assuming a quadratic form, 

$$\Phi(l) = \omega (l_{2t}/lt - \Lambda)^2,$$

where $l_{2t}/lt$ is the fraction of ARM loans in newly originated loans and $\Lambda$ is a parameter capturing the institutional norm of the ARM share in the economy. Optimal mortgage choice is characterized by the first-order condition for $l_{2t}$, which for the above cost function takes a simple form

$$l_{2t}/lt - \Lambda = \frac{v_{ct}P_H \tau_{H1}t}{2\omega} (\tau_{H1t} - \tau_{H2t}), \quad (30)$$

where $\tau_{H1}$ and $\tau_{H2}$ are the wedges capturing the price effect under FRM and ARM respectively (see Online Appendix G for details). The optimality condition (30) thus states that the ARM share in new loans $l_{2t}/lt$ increases when the homeowner perceives ARM loans as cheaper than FRM loans; that is, $\tau_{H1t} > \tau_{H2t}$. 

3370
A Real rate shock, $\rho_b = 0.9$

Figure 8
Findings V: The transmission mechanism under mortgage choice
Both shocks are scaled so that, on impact, the short rate increases by one percentage point (annualized). ARM share is measured as percentage point deviations from steady state; housing investment is in percentage deviations. One period = one quarter.

The parameter $\Lambda$ is set equal to 0.3, the long-run average of ARM share in the U.S. economy, and the cost parameter $\omega$ is set so as to replicate the empirical elasticity of the ARM share to the long-short spread, equal to 7.0 (i.e., a seven-percentage-point increase in the ARM share in response to a one-percentage-point increase in the long-short spread). This elasticity implies $\omega = 0.0055$. Panel A of Figure 8 shows that the model is consistent with the basic feature of the data: the ARM share of new loans declines when the long-short spread declines. Given that the model is consistent with the basic observations on mortgage choice in the data, we ask if mortgage choice plays an important role in the transmission mechanism with respect to the $\pi_t$ shock. Panel B of Figure 8 provides a negative answer. As the $\pi_t$ shock is highly persistent, it has almost no effect on the long-short spread and thus no effect on mortgage choice. The response of housing investment is only affected by the parameter $\Lambda$ and lies in-between the responses under FRM and ARM, reported in Figure 4.

6. Concluding Remarks

Mortgage payments constitute a substantial part of homeowners’ mandatory expenses. In combination with the fact that mortgages are long-term loans set in nominal terms, it is natural to ask: What role do mortgage contracts play in the transmission of monetary policy? This paper attempts to establish these
connections. Like goods market imperfections provide a breeding ground for nominal price rigidities to play a role in the transmission of monetary policy in New-Keynesian models, financial market imperfections (incomplete asset markets) facilitate transmission of monetary policy through mortgage contracts in our framework.

Three key properties of the mortgage transmission mechanism emerge. First, the transmission mechanism is found to be stronger under adjustable-rate mortgages compared with fixed-rate mortgages. Second, monetary policy shocks affecting, under the expectations hypothesis, the level of the nominal yield curve have larger real effects than transitory shocks affecting its slope. And third, persistently higher inflation gradually benefits homeowners under FRMs, but hurts them immediately under ARMs. In terms of quantities, on impact, housing investment increases by 1.8% under ARM and declines by 0.7% (0.15% with refinancing) under FRM, in response to a one-percentage-point (annualized) downward shift of the nominal yield curve (and inflation). Under ARM, the strength of the transmission mechanism is comparable to a traditional real rate channel. This is despite the fact that, in equilibrium, the real interest rate in our model hardly moves in response to monetary policy shocks.

In the interest of transparency, we have abstracted from the usual nominal frictions and other channels through which housing finance affects the macroeconomy. The number of shocks was also limited. A natural extension is therefore to incorporate these additional features and study their interaction with the mechanism proposed here.

Another interesting avenue is to study the price and income effects in an overlapping generations setup with realistic life-cycle dynamics. The agents who face the price effect are likely different from those who face the income effect. In addition, the importance of the income effect is likely to vary with the homeownership life cycle.

Finally, an important normative question considers optimal monetary policy. Such policy is likely to depend on the prevalent mortgage type, FRM or ARM, in the economy. This question is particularly complex for the Eurozone, which includes countries with very different mortgage markets. Extending the model to allow for banks may also generate nontrivial interactions between monetary and macroprudential policies.

The framework developed in this paper provides a groundwork that can be extended to address these additional questions.

References


Mortgages and Monetary Policy


Mortgages and Monetary Policy


