Housing Dynamics over the Business Cycle*

Finn E. Kydland[†], Peter Rupert[‡§] and Roman Šustek[¶]

June 15, 2015

Abstract

Housing construction, measured by housing starts, leads GDP in a number of countries. Measured as residential investment, the lead is observed only in the U.S. and Canada; elsewhere, residential investment is coincident. Variants of existing theory, however, predict housing construction lagging GDP. In all countries in the sample, nominal interest rates are low ahead of GDP peaks. Introducing long-term nominal mortgages, and an estimated process for nominal interest rates, into a standard model aligns the theory with observations on starts, as mortgages transmit nominal rates into real housing costs. Longer time to build makes residential investment cyclically coincident.

JEL Classification Codes: E22, E32, R21, R31.

Keywords: Residential investment, housing starts, business cycle, mortgage costs, time to build.

*We thank Tom Cooley, Carlos Garriga, Paul Gomme, Grant Hillier, Haroon Mumtaz, Peter Phillips, Don Schlagenhauf, and Norman Schürhoff for invaluable comments and suggestions. Special thanks go to Martin Gervais, Erik Hembre, and Vincent Sterk for insightful conference discussions. We are also grateful for comments and suggestions to two anonymous referees and the Editor, as well as to seminar participants at Birkbeck, Cal Poly, Cardiff, Cleveland Fed, Concordia, Czech National Bank, Dallas Fed, Edinburgh, Essex, Exeter, Glasgow, Norges Bank, NYU Stern, Sogang, USC, and Yonsei, and to conference participants at the Bank of England, Nottingham, Regensburg, Sciences Po, SED (Cyprus), HULM (St. Louis Fed), and Shanghai.

[†]University of California–Santa Barbara and NBER; kydland@econ.ucsb.edu.

[‡]University of California–Santa Barbara; rupert@econ.ucsb.edu.

[§]Corresponding author: Peter Rupert, address: Department of Economics, 2127 North Hall, Santa Barbara, CA 93106, U.S.A., tel: +1 805 893-2258, fax: +1 805 893-8830, e-mail: rupert@econ.ucsb.edu.

[¶]Queen Mary, University of London and Centre for Macroeconomics; sustek19@gmail.com.

1 Introduction

Over the U.S. business cycle, fluctuations in residential investment (newly constructed homes) are well known to systematically precede fluctuations in real GDP; see, e.g., Leamer (2007). Perhaps due to this leading indicator property, new housing construction attracts considerable attention by professional economists. It has been also repeatedly documented that this observation is at odds with the properties of business cycle models once the aggregate capital stock is disaggregated into two basic components: residential and nonresidential (e.g., Gomme, Kydland and Rupert, 2001; Davis and Heathcote, 2005). The theory predicts that nonresidential investment should lead output while residential investment should lag output.

While the cyclical properties of residential (and nonresidential) investment have been well established for the U.S., little is known about the properties of these data in other countries. Is the U.S. experience unique and data from other countries support the existing theory? Or do the data from other countries make the need for improving the theory even more pressing? This paper has two goals. First, to shed light on the cyclical dynamics of the two types of investment beyond the U.S. and, second, to use these observations to guide the development of the theory.

In a sample of developed economies, only Canada is found to exhibit the lead in residential investment observed in the U.S. Nonetheless, international data do not support existing models either. In other countries, residential investment is, more or less, coincident with GDP; not lagging as the theory predicts. And in all countries nonresidential investment is either lagging or coincident with GDP; not leading as in existing models. The case against the theory is even stronger when international data on housing starts—the number of housing units whose construction commenced in a given period—are taken into account: nearly all countries in the sample exhibit housing starts strongly leading GDP. In other words, residential construction picks up a few quarters before GDP.

Data on housing completions point to longer residential time to build in some countries than in the U.S. During the time to build period, national accounts record in each quarter a construction project's 'value put in place' as a part of residential investment. Time to build thus spreads recorded residential investment over a number of quarters, making it less of a leading indicator in countries where time to build is longer.

An important aspect of housing markets in developed economies is reliance of homeowners on mortgage finance to purchase a property. Mortgage finance takes the form of nominally denominated loans that homeowners gradually repay, with interest, over many years.[1] Furthermore, the cyclical dynamics of nominal mortgage rates—and nominal interest rates, both long and short, more generally—are strikingly similar across countries. Specifically, mortgage rates are negatively correlated with future GDP and positively correlated with past GDP, *suggesting* that mortgage finance is relatively cheap ahead of a peak in GDP.[2]

Motivated by these observations, we investigate (i) if the dynamics of *nominal* interest rates observed in the data transmit into similar cyclical variations in the *real* cost of new mortgage finance and if such variations are sufficient to overturn the standard predictions of the theory; and (ii) if time to build in residential investment can then account for the discrepancies between the timing, in relation to output, of housing starts and residential investment. Various idiosyncracies of individual countries are abstracted from. To this end, long-term fully-amortizing mortgages and residential time to build are introduced into a business cycle model of Gomme et al. (2001). Two main types of mortgages are considered: fixed-rate mortgages (FRM) and adjustable-rate mortgages (ARM).[3] The exogenous input into the model is an estimated VAR process for total factor productivity, the nominal mort-

[1]In contrast, nonresidential fixed investment in advanced economies is predominantly financed by retained earnings and other forms of equity. Rajan and Zingales (1995) document that typically only about 20% of the value of long-term assets in the nonfinancial corporate sector is financed through debt.

[2]In all countries in the sample, nominal mortgage rates have similar dynamics as yields on nominal government bonds of comparable maturities. The 'inverted' lead-lag property of U.S. government bond yields in relation to output has been noted by, for instance, King and Watson (1996) and, more recently, Backus, Routledge and Zin (2010). The same pattern is documented for other countries by Henriksen, Kydland and Šustek (2013). Unfortunately, a theory that would successfully account for this phenomenon is yet to be developed.

[3]Most countries can be broadly classified as having either FRM or ARM as their typical mortgage contract. Research is still inconclusive on the causes of the cross-country heterogeneity in the use of FRM v.s. ARM, but likely reasons seem to be government regulations, historical path dependence, and whether mortgage lenders raise funds through capital markets or bank deposits (e.g., Miles, 2004; Campbell, 2012).

gage interest rate, and the inflation rate. In the absence of an off-the-shelf structural model for the observed lead-lag dynamics of nominal interest rates described above, this guarantees that the cyclical pattern of the nominal mortgage rate (and inflation) in the model is as in the data.

In a baseline case with one-period residential time to build, and multi-period nonresidential time to build, the model exhibits lead-lag patterns of residential and nonresidential investment similar to those in the U.S. and Canada, while also being in line with standard business cycle moments as much as other models in the literature. Introducing into the model a multi-period time to build in residential construction facilitates the distinction between housing starts, completions, and residential investment. While mortgage finance is crucial for producing housing starts leading output, longer time to build pushes residential investment towards being coincident with output. In both versions of the model, mortgage finance has also an indirect effect on nonresidential investment—as households want to keep consumption relatively smooth, when movements in residential investment of the magnitude observed in the data occur ahead of an increase in GDP, nonresidential investment is delayed, making it lag output. The relative price of newly constructed homes responds to housing demand and exhibits cyclical volatility and positive comovement with output similar to those in the data.

A key to understanding the role of mortgages is in the form of an endogenous time-varying wedge in the Euler equation for residential capital. The wedge, working like a tax/subsidy on residential investment, or like a housing taste shock (e.g., Liu, Wang and Zha, 2013), depends on expected future real mortgage payments over the life of the loan, discounted by the household's stochastic discount factor. Thus, unlike observed nominal mortgage rates, the wedge captures the true cost of the mortgage to the household in the model. Due to the long-term and nominal nature of mortgage loans (both FRM and ARM), the movements of the wedge are mainly driven by fluctuations in nominal interest rates. As a result, mortgages are relatively cheap, from households' perspective, when nominal interest rates are low, which occurs ahead of a GDP peak.[4]

The lead in U.S. residential investment has puzzled macroeconomists for several decades. Various models were found to be inconsistent with this observation. In the home production literature (Benhabib, Rogerson and Wright, 1991; Greenwood and Hercowitz, 1991; McGrattan, Rogerson and Wright, 1997) the two types of investment have the opposite lead-lag dynamics to those in the data. Including investment specific shocks (Gomme and Rupert, 2007) or sectoral productivity shocks in a multisector economy with input-output linkages (Davis and Heathcote, 2005) does not help resolve this issue.[5] Gomme et al. (2001) and Fisher (2007) achieve partial success (through nonresidential time to build and production complementarities, respectively), resolving the phase shift of residential investment in relation to nonresidential investment. Nevertheless, both models fail to produce residential investment leading output.

There are three major features that distinguish our model from recent macro models with housing finance, such as Iacoviello (2005) and those that followed. First, we focus only on mortgage costs and how they affect new residential construction. Other models, in contrast, focus on the role of housing in facilitating collateralized borrowing for general consumption purposes. Our model abstracts from that channel. Second, these models usually do not include nonresidential capital (one of the few exceptions is Iacoviello and Pavan, 2013). However, as the home production literature demonstrates, the presence of nonresidential capital has important implications for the cyclical behavior of residential capital. And third, housing finance in this literature involves one-period nominal loans, whereas we consider long-term fully-amortizing nominal loans. Even in the presence of the estimated process for

[4]These findings are consistent with earlier studies of the U.S. housing market (e.g., Kearl, Rosen and Swan, 1975; Kearl, 1979), which find that the nominal interest rate has a negative, statistically significant, coefficient in regression equations for housing investment. We check that the negative effect of nominal interest rates on residential investment is not purely due to the expectations of higher future output (income), following low nominal interest rates.

[5]The reason behind the opposite pattern is that output produced by nonresidential capital has more uses than output produced by residential capital: the former can be either consumed or invested in both nonresidential and residential capital, whereas the latter can only be consumed as housing services. Investment in nonresidential capital thus allows better intertemporal smoothing of consumption. This provides a strong incentive to build up nonresidential capital first, in response to shocks that increase market output.

nominal interest rates and inflation, one-period loans do not generate the lead in residential investment.

A growing literature studies the recent housing boom and bust, focusing on the consequences of the developments in mortgage markets (e.g., the relaxation of collateralized borrowing) and international capital markets (the inflow of foreign capital into U.S. government securities). Both representative agent (e.g. Garriga, Manuelli and Peralta-Alva, 2014) and heterogeneous agent (e.g. Favilukis, Ludvigson and Van Nieuwerburgh, 2015) economies are used. Instead, this paper addresses the typical fluctuations in a number of countries over the past half-century or so. In terms of modeling housing finance, our model differs from the above studies along two dimensions. First, mortgages in our model resemble first lien loans for new house purchases, rather than collateralized borrowing encompassing also second lien loans and home equity loans that can be used for other purposes as well.[6] And second, mortgages in our model are long-term nominal contracts, whereas the above studies consider loans denominated in real terms, either one-period loans (Favilukis et al., 2015) or long-term loans (Garriga et al., 2014). For our result, both the long-term and nominal nature of the loans matters.

The paper proceeds as follows. The next section presents the empirical findings. Section 3 describes the model. Section 4 explains how nominal interest rates affect housing investment. Section 5 reports the main findings. Section 6 demonstrates the quantitative importance of mortgages. Section 7 concludes. The paper is accompanied by a supplemental material containing six appendices. Appendix A provides a description of the international data used in Section 2. Appendix B contains some additional derivations and examples related to Sections 3 and 4 and describes the computation. Appendix C contains estimates of the exogenous stochastic processes used in Sections 5 and 6. Finally, Appendices D, E, and F conduct further sensitivity analysis (stochastic loan-to-value ratio, alternative amortization schedules, and refinancing).

[6] Second lien loans and home equity loans started to play an important role in the U.S. only during the run up to the financial crisis (2002-2007). Furthermore, in some countries in our sample the use of such mortgage products is limited (Calza, Monacelli and Stracca, 2013).

2 Leads and lags in investment data

The empirical analysis is based on quarterly data for the following countries and periods: Australia (1959.Q3-2006.Q4), Belgium (1980.Q1-2006.Q4), Canada (1961.Q1-2006.Q4), France (1971.Q1-2006.Q4), the U.K. (1965.Q1-2006.Q4), and the U.S. (1958.Q1-2006.Q4). These are the only countries for which the breakdown of total investment into residential and nonresidential is available from at least 1980 (we regard a period of about 25 years as the shortest that allows a sensible discussion of business cycles). Appendix A provides a description of the data and lists the availability of the data for other countries.

All investment data are measured as chained-weighted quantity indexes and, subject to slightly different treatment of software expenditures, are conceptually comparable across countries (European Cenral Bank, 2005). As in other business cycle studies, the data are logged and filtered with the Hodrick-Prescott filter and the empirical regularities are summarized in the form of correlations with real (chained-weighted) GDP at various leads and lags; i.e., by $\text{corr}(x_{t+j}, GDP_t)$ for $j = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$, where x_{t+j} and GDP_t are, respectively, the percentage deviations of the variable of interest and real GDP from a HP filter trend. A variable is said to be *leading* the cycle (meaning leading real GDP) if the highest correlation is at $j < 0$, as *lagging* the cycle if the highest correlation is at $j > 0$, and as *coincident* with the cycle if the highest correlation is at $j = 0$.[7]

2.1 Total, residential, and nonresidential investment

To set the stage, Figure 1 plots the cross-correlations for total investment, referred to in national accounts as gross fixed capital formation (GFCF), which accounts on average for a little over 20% of GDP. The figure caption reports the volatility of the investment data, measured by the standard deviation of investment relative to that of real GDP. As the figure

[7] The findings are not particularly sensitive to if, instead, the Christiano and Fitzgerald (2003) band-pass filter is used. Due to the well-known end-point problems of the filters, the ongoing recessions are not included in the sample. Nevertheless, observations of turning points during the 2006-2008 period are consistent with the empirical regularities documented in this section.

shows, in all six countries total investment is coincident with GDP. In addition, the volatility of total investment is between 2.5 times to 4 times the volatility of GDP, which is in the ballpark of the much-cited volatility of U.S. investment (about 3 times the volatility of GDP) and the prediction of a prototypical business cycle model with standard calibration.

Figure 2 displays the cross-correlations for residential and nonresidential structures, which together with equipment & software make up GFCF (nonresidential structures make up on average about 25%, equipment & software 45%, and residential structures 30% of GFCF); volatilities of the data are again reported in the figure caption. Residential structures include new houses, apartment buildings, and other dwellings, whereas nonresidential structures include new office buildings, retail parks, production plants, power plants etc. We will often refer to residential structures as 'residential investment' and to nonresidential structures as 'nonresidential investment'.[8] The empirical regularity discussed in the Introduction that over the U.S. business cycle residential investment leads GDP is clearly evident. The chart for the U.S. also shows that nonresidential investment has the opposite dynamics to those of residential investment, lagging GDP over the business cycle. Such a stark difference in the dynamic properties of residential and nonresidential investment is to a lesser extent observed also in Canada, but in the remaining countries the two types of investment tend to be, more or less, coincident with GDP.

In order to get a sense of the significance of the leads and lags (or their absence) in the charts of Figure 2, the following test is carried out. Using a standard block bootstrap with nonstochastic overlapping blocks (see, e.g., Hardle, Horowitz and Kreiss, 2003), 10,000 pairs of artificial data series for investment and GDP, of the same length as the historical data, are drawn for each country. For each artificial sample, the cross-correlations are computed and the $j \in \{-4, ..., 0, ..., 4\}$ at which the highest correlation occurs is recorded. Figure 3 plots the histograms of these occurrences at different j's.[9] As the histograms show, for

[8] In the case of Belgium and France the cross-correlations are for the sum of nonresidential structures and equipment & software as the two series are not available individually. In the countries for which the breakdown is available, equipment & software behaves, qualitatively, like nonresidential structures.

[9] The length of each block in the bootstrap is set equal to 20 quarters in order to address the serial correlation of around 0.9 in the historical data. While the accuracy of block bootstrap methods can be

residential investment, the U.S. and Canada are the only countries for which the highest correlation is at a lead (i.e., at a $j < 0$) in at least 95% of the draws, while for nonresidential investment only the U.S. has the highest correlation at a lag (i.e., at a $j > 0$) in at least 95% of the draws. Nevertheless, with the exception of Belgium, for which the test is inconclusive even at a 90% confidence level, all countries exhibit residential investment either leading or coincident with GDP; i.e., the highest correlation occurs at a $j \leq 0$ in more than 95% of the draws. And with the exception of the U.K., for which the test is inconclusive, all countries exhibit nonresidential investment either lagging or coincident with GDP; i.e., the highest correlation occurs at a $j \geq 0$ in more than 95% of the draws. The standard predictions of the theory are thus not supported by the available international data.

2.2 Housing starts

While the lead-lag dynamics of residential investment in the U.S. and Canada look different from those in the other countries, there is much more uniformity across the countries in terms of the lead-lag dynamics of housing starts.[10] The 'start' of housing construction is defined consistently across countries as the beginning of excavation for the foundation of a residential building (single family or multifamily). Every month detailed surveys of home builders record the number of such activities. The top half of Figure 4 plots the cross-correlations with GDP for the historical data (volatilities are in the figure caption); the data are again logged and HP-filtered. As is visually apparent, housing starts lead GDP in all countries, possibly with the exception of Belgium. The bottom half of the figure reports the results of a similar robustness check as in the case of investment. In 95% of the draws the lead occurs in the case of Canada, the U.K., and the U.S and in 90% of the draws also in the case of Australia and France.

sensitive to the choice of the block length (Hardle et al., 2003), the main takeaway form Figure 3 is unaffected by changing the length by up to +/- six quarters.

[10]The time periods for housing starts differ slightly from the time periods for residential structures due to differences in data availability. Housing starts are for the following periods: Australia (1965.Q3-2006.Q4), Belgium (1968.Q1-2006.Q4), Canada (1960.Q1-2006.Q4), France (1974.Q1-2006.Q4), the U.K. (1978.Q1-2006.Q4), and the U.S. (1959.Q1-2006.Q4). Details of the data are provided in Appendix A.

2.3 Residential time to build

While housing starts record the number of housing units whose construction commenced, residential investment in national accounts records value put in place on residential projects in a given quarter, as estimated from surveys of home builders (European Commission, 1999; Bureau of Economic Analysis, 2009).[11] Construction projects that take longer to complete therefore have value put in place recorded over more quarters. In the U.S., the Survey of Construction provides details on construction lead times (time to build) for different types of residential structures. The average period from start to completion for a typical single-family structure built for sale is 5.5 months; for an owner-built[12] single-family structure the lead time is 10 months; and for multifamily structures the lead time is 10 months for the aggregate and 13 months for 20+ unit structures. The lead times for the different structure types are approximately constant over time. In national accounts, single-family units make up on average about 80% of new permanent residential structures and owner-built units account on average for only 14% of single-family units. Residential investment in the U.S. thus mainly reflects the relatively short lead time of single-family units built for sale.

In addition to data on housing starts, the U.S. Survey of Construction provides quarterly data on completions for single and multifamily structures (data for the individual structure types within single and multifamily structures are available only from 1999 and thus too short for our purposes). The cross-correlations of starts and completions with GDP are reported in Table 1. They reflect the lead times noted above: for single family units, starts lead GDP by three quarters while completions lead by two quarters; for multifamily units, starts lead GDP by two quarters while completions lag GDP by two quarters (the multifamily data are for 5+ unit structures). The table also reports cross-correlations for single-family and multifamily residential investment. The highest cross-correlations lie in-between the highest cross-correlations for starts and completions for the respective structure types: single-family

[11] Residential investment also includes capital expenses on improvements and brokers' commissions on sales.

[12] Custom-built structures whereby an individual commissions an architect and a builder to build a house for own use.

structures lead GDP by two quarters and multifamily structures are coincident with GDP.

Information on construction lead times in other countries is scarce. However, exploiting the above properties of the U.S. data, we can use available data on housing completions in other countries, published alongside the housing starts data, to obtain estimates of construction lead times. The only countries for which long enough completions data are available are Australia and the U.K. Table 1 shows that in Australia housing starts lead by two quarters while completions are coincident with GDP and in the U.K. housing starts lead by two quarters while completions lag GDP by one quarter. These correlations suggest up to three-quarter time to build in Australia and up to four-quarter time to build in the U.K. As in the case of the U.S., in both Australia and the U.K. the highest cross-correlation of residential investment lies in-between the highest cross-correlations of starts and completions.

Why are there differences in residential time to build across developed economies? Ball (2003) conducts a cross-country comparative study of the structure and practices of home-building industries. He points out substantial variations across countries in the materials used, the extent of pre-fabrication, supply chain efficiency, and regulatory constraints. In addition, the composition of housing investment differs across countries. In Belgium and France, multifamily structures account for almost 40% and owner-built single-family structures for further $45-50\%$ of new construction (Dol and Haffner, 2010). Assuming that multifamily and owner-built structures in Belgium and France take at least as long time to build as in the U.S., the lead times for the residential sectors as a whole in the two countries are likely to be close to four quarters.

2.4 Regulation Q

Regulation Q is sometimes evoked as a reason for the leading behavior of residential investment over the U.S. business cycle (e.g., Bernanke, 2007). This regulation set ceilings on interest rates that savings banks and savings and loans—the main mortgage lenders before mid-1980s—were allowed to pay on deposits. When interest rates increased, these institutions experienced deposit outflows and had to cut mortgage lending, thus causing decline in construction activity and possibly a wider recession. Regulation Q was eventually abolished in 1980 and phased out during the following four years.

In order to assess the effect of Regulation Q, the top panel of Table 2 reports the cross-correlations of single family residential investment with GDP in two subsamples: 1959.Q1-1983.Q4 and 1984.Q1-2006.Q4. The focus is on single family structures as the multifamily market was strongly affected by tax code changes that occurred in the 1980s (Colton and Collignon, 2001). The key observation is that investment in residential structures leads GDP in both periods, even though, admittedly, the correlations at all leads and lags are weaker in the second period than in the first period. Thus, while Regulation Q likely played a role in the cyclical dynamics of residential investment in the first period—possibly accounting for the stronger correlations—it cannot be the only reason for why movements in residential investment precede movements in GDP.

2.5 Mortgage rates

An important feature of housing markets in developed economies is that the acquisition of a residential property relies on debt financing. In the U.S., based on historical data from the Survey of Construction, on average 94% of new single-family house purchases are financed by a mortgage (76% by a 30-year conventional mortgage and 18% by FHA/VA insured mortgages). The remaining 6% are cash purchases. Furthermore, the cross-sectional average of the loan-to-value ratio for newly-built homes conventional mortgages is 76% and this ratio has been remarkably stable over time, fluctuating within a range of a couple of percentage points (Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10; plotted in Appendix D).[13] About 25% of new single-family homes are on average sold at the development stage, 40% are sold during the construction process, and 35% are sold

[13]The Monthly Interest Rate Survey is based on first lien loans. It thus does not capture the rise in the use of second lien and home equity loans in the U.S. during the pre-crisis period 2002-2007 (see, e.g., Favilukis et al., 2015).

after completion (Survey of Construction). Issuance of home mortgage loans therefore, unsurprisingly, exhibits similar lead-lag pattern as single-family residential investment, leading GDP by two quarters, as the middle panel of Table 2 shows.[14] In other countries in the sample, mortgage finance plays an important role as well. The typical loan-to-value ratio varies across-countries from 70 to 90% (Ahearne, Ammer, Doyle, Kole and Martin, 2005; Calza et al., 2013) and mortgage debt outstanding in 2009 was equivalent to $40 - 90\%$ of GDP (International Monetary Fund, 2011); 75% in the U.S.

The next section derives the real cost of mortgage finance to a representative household. Here, Table 3 reports the lead-lag dynamics of two variables that affect that cost, the nominal mortgage interest rate and the inflation rate. According to a number of studies (e.g., Scanlon and Whitehead, 2004; Calza et al., 2013), national mortgage markets can be generally characterized as either FRM dominated or ARM dominated, though the cross-country heterogeneity of mortgage market structures is yet to be understood (Campbell, 2012). For each country in the sample, Table 3 reports the standard deviation (relative to real GDP) and cross-correlations with real GDP of the nominal interest rate on the country's typical mortgage. In addition, the table reports these statistics for government nominal bond yields of maturities comparable to the period for which the mortgage rate in the typical mortgage contract is fixed.[15] The third variable in the table is the inflation rate. For future reference we also include the yield on U.S. 3-month Treasury bills. The table reveals a striking similarity across countries in the cyclical dynamics of these variables: generally, all three variables are negatively correlated with future GDP and positively correlated with past GDP. Thus, on average, nominal interest rates and inflation rates are relatively low before a GDP peak,

[14] The mortgage loan data are for the net change in mortgage debt outstanding obtained from the Flow of Funds Accounts, Table F.217, and deflated with the GDP deflator. Flow of Funds tables report *home* mortgages, defined as mortgages for 1-4 family properties. The fraction of new construction accounted for by 2-4 family properties is, however, negligible (completions data from the Survey of Construction). Home mortgages are thus a good proxy for single family property mortgages. The findings are similar whether or not home equity lines of credit, broadly available from mid-1990s, are included.

[15] Specifically, for FRM countries we take par yields on coupon government bonds of maturities close to the periods for which FRM mortgage rates are fixed; for ARM countries we take 3-month Treasury bill yields, as mortgage rates on ARMs are set, after some initial period, as a margin over a short-term government bond yield.

tend to increase as GDP increases, and reach their peak a few quarters after a peak in GDP. This pattern of nominal interest rates and inflation rates has been previously documented by King and Watson (1996) for the U.S. and by Henriksen et al. (2013) for a number of developed economies. The table also shows that the cross-correlations of mortgage rates are similar to those of government bond yields. Furthermore, Table 2 shows that the general shape of the lead-lag pattern of these variables stayed the same across two U.S. monetary policy regimes—identified, as in Gavin and Kydland (2000), by the appointment of Paul Volcker as the Chairman of the Federal Reserve—even though the correlations became weaker in the second period.

3 A business cycle model with mortgage loans

Motivated by the above observations, this section introduces mortgages into a business cycle model with home and market sectors studied by Gomme et al. (2001), mentioned in the Introduction, henceforth referred to as GKR. It is worth pointing out at the outset that we do not model the underlying reasons giving rise to the demand for mortgages, such as the lumpiness of house purchases, the tax code, or the preference for owning v.s. renting. Modeling demand for mortgages from first principles would make the model unnecessarily complex for the task at hand, which is to investigate the impact of nominal interest rates on the real cost of mortgage finance and, consequently, on residential investment. For this purpose, we simply assume that a fraction of new housing is financed through mortgages and calibrate this fraction from the data. As noted above, in the data this fraction is approximately constant over time.[16]

[16] Gervais (2002), Rios-Rull and Sanchez-Marcos (2008), and Chambers, Garriga and Schlagenhauf (2009) develop models with many of the micro-level details we abstract from. Their focus, however, is on steady-state analysis. Campbell and Cocco (2003) model in detail a single household's mortgage choice in partial equilibrium, while Koijen, Van Hemert and Van Nieuwerburgh (2009) embed a two-period version of such a problem in general equilibrium with aggregate shocks. Iacoviello and Pavan (2013) construct a general equilibrium model with some of the features in Gervais (2002) and aggregate shocks. Housing finance in their model, however, takes the form of one-period loans.

3.1 Preferences and technology

A representative household has preferences over consumption of a market-produced good c_{Mt}, a home-produced good c_{Ht}, and leisure, which is given by $1 - h_{Mt} - h_{Ht}$, where h_{Mt} is time spent in market work and h_{Ht} is time spent in home work. The preferences are summarized by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, 1 - h_{Mt} - h_{Ht}\right), \quad \beta \in (0,1), \tag{1}$$

where $u(.,.)$ has all the standard properties and c_t is a composite good, given by a constant-returns-to-scale aggregator $c(c_{Mt}, c_{Ht})$. Time spent in home work is combined with home capital k_{Ht} to produce the home good according to a production function

$$c_{Ht} = A_H G(k_{Ht}, h_{Ht}), \tag{2}$$

where $G(.,.)$ has all the standard properties. In contrast to the home production literature, we abstract from durable goods and equate home capital with residential structures when mapping the model to data. Home capital will therefore be referred to as 'residential capital'.[17]

Output of the market-produced good y_t is determined by an aggregate production function

$$y_t = A_{Mt} F(k_{Mt}, h_{Mt}), \tag{3}$$

operated by identical perfectly competitive firms. Here, A_{Mt} is total factor productivity (TFP) and k_{Mt} is market capital, which will be referred to as 'nonresidential capital'.[18]

[17] c_{Ht} is thus consumption of housing services and h_{Ht} is interpreted as time devoted to home maintenance and leisure enjoyed at home, as opposed to in a bar. Under enough separability in utility and production functions, which will be imposed under calibration, the period utility function can be rewritten such that it is a function of c_{Mt}, h_{Mt}, and k_{Ht} (Greenwood, Rogerson and Wright, 1995). This makes it comparable to models that put housing directly in the utility function.

[18] Notice that, in contrast to A_{Mt}, which is time varying (due to shocks), A_H is constant. GKR show

Firms rent labor and capital services from households at a wage rate w_t and a capital rental rate r_t, respectively. The market-produced good can be used for consumption, investment in residential capital, x_{Ht}, and investment in nonresidential capital, x_{Mt}.

The production possibilities frontier (PPF) is assumed to be concave in $c_t + x_{Kt}$ and x_{Ht}. Specifically, $c_t + x_{Mt} + q_t x_{Ht} = y_t$, where $q_t = \exp(\sigma(x_{tH} - x_H))$, with $\sigma > 0$ and x_H being steady-state residential investment. Here, q_t measures the rate of transformation between new housing and other uses of output and is increasing and convex in the amount of new housing. q_t is thus the relative price of residential investment. The concavity of the PPF is a stand-in for the costs of changing the composition of an economy's production (Huffman and Wynne, 1999); a concave PPF results also in Davis and Heathcote (2005) due to different factor shares in the economy's production sectors. When the PPF is linear, the two types of investment are too sensitive to the shocks in the model.[19]

We start with one-period residential time to build. Residential capital therefore evolves as

$$k_{H,t+1} = (1 - \delta_H)k_{Ht} + x_{Ht}, \tag{4}$$

where $\delta_H \in (0, 1)$. As in GKR, nonresidential capital has a J-period time to build, where J is an integer greater than one. Specifically, an investment project started in period t becomes a part of the capital stock only in period $t + J$. However, the project requires value to be put in place throughout the construction process from period t to $t + J - 1$. In particular, a fraction $\phi_j \in [0, 1]$ of the project must be invested in period $t + J - j$, $j \in \{1, ..., J\}$, where j denotes the number of periods from completion and $\sum_{j=1}^{J} \phi_j = 1$. Let s_{jt} be the size of

that under enough separability in utility and production functions, which will be imposed under calibration, shocks to A_H do not affect market variables (i.e., time spent in market work, consumption of the market-produced good, and accumulation of the two types of capital). This is convenient as it allows abstracting from home-production TFP shocks, which cannot be measured outside of the model.

[19] An additional source of adjustment costs on residential investment considered by Davis and Heathcote (2005) is a constant endowment of new residential land each period, which is combined in a Cobb-Douglas production function with residential investment to produce new housing. Our model abstracts from residential land. Residential investment and new housing are thus the same thing.

the projects that in period t are j periods from completion. Total nonresidential investment (i.e., investment across all on-going projects) in period t is thus

$$x_{Mt} = \sum_{j=1}^{J} \phi_j s_{jt} \tag{5}$$

and the projects evolve recursively as

$$s_{j-1,t+1} = s_{jt}, \quad j = 2, \ldots, J, \tag{6}$$

$$k_{M,t+1} = (1 - \delta_M) k_{Mt} + s_{1t}, \tag{7}$$

where $\delta_M \in (0, 1)$.

3.2 Mortgage loans

Up until now, with the exception of the concave PPF, the setup is the same as in GKR. What makes the current model different is that residential investment is partially financed by mortgages

$$l_t = \theta p_t q_t x_{Ht}, \tag{8}$$

where l_t is the *nominal* value of a mortgage loan taken out in period t, $\theta \in [0, 1)$ is a loan-to-value ratio, and p_t is the aggregate price level (the price of the market good in 'dollars').

Notice that the constraint (8) is different from that in Iacoviello (2005) and related models. Here, the loan taken out in period t is only used to finance new homes constructed in period t, whereas in Iacoviello (2005), a loan taken out in period t is collateralized by the period-$t+1$ value of the housing stock and can be used for other purposes than acquisition of new housing. In this sense, our loan resembles a first mortgage, whereas that of Iacoviello is closer to a home equity loan as it allows continuous borrowing, for general purposes, against the value of the housing stock.[20]

Mortgage debt is paid off by regular nominal installments. The representative household's budget constraint is therefore

$$c_{Mt} + x_{Mt} + q_t x_{Ht} = (1-\tau_r) r_t k_{Mt} + (1-\tau_w) w_t h_{Mt} + \delta_M \tau_r k_{Mt} + \frac{l_t}{p_t} - \frac{m_t}{p_t} + \tau_t, \tag{9}$$

where τ_r is a tax rate on income from nonresidential capital, τ_w is a tax rate on labor income, τ_t is a lump-sum transfer, and m_t are nominal installments on outstanding mortgage debt.[21] The installments are given as

$$m_t = (R_t + \delta_{Dt}) d_t, \tag{10}$$

where d_t is the nominal mortgage debt outstanding, R_t is an *effective* net interest rate on the outstanding mortgage debt, and $\delta_{Dt} \in (0,1)$ is an *effective* amortization rate of the outstanding mortgage debt. Notice that $\delta_{Dt} \in (0,1)$ implies that $m_t > R_t d_t$; i.e., a part of the outstanding debt is amortized each period. Mortgages are only either FRM or ARM. The variables d_t, R_t, and δ_{Dt} are state variables evolving recursively according to the laws of motion

$$d_{t+1} = (1 - \delta_{Dt}) d_t + l_t, \tag{11}$$

[20] Strictly speaking, the constraint (8) is $l_t \leq \theta p_t q_t x_{Ht}$, but it is assumed to be binding in all states of the world. If it is slack, the choice of x_{Ht} is independent of the choice of l_t and housing finance does not affect equilibrium allocations—the wedge in the Euler equation for housing derived below becomes zero and the properties of the model become the same as in GKR. An empirical justification for our assumption, noted in the previous section, is that the cross-sectional mean of the loan-to-value ratio for conventional single-family newly-built home mortgages has been historically approximately constant (about 0.76, with a standard deviation of one percentage point), despite large changes in nominal interest rates and other economic conditions; Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10, 1963-2007; see the figures in Appendix D. Note that the Survey is based on first-lien loans. It thus does not capture the rise in the use of second lien and home equity loans in the U.S. during the pre-crisis period 2002-2007 (see, e.g., Favilukis et al., 2015).

[21] τ_r and τ_w are constant and, as in the rest of the home production literature, are introduced into the model purely for calibration purposes; τ_t is time-varying and its role is to ensure that the economy's resource constraint holds.

$$\delta_{D,t+1} = (1 - \nu_t) f(\delta_{Dt}) + \nu_t \kappa, \tag{12}$$

$$R_{t+1} = \begin{cases} (1 - \nu_t) R_t + \nu_t i_t, & \text{if FRM,} \\ i_t, & \text{if ARM.} \end{cases} \tag{13}$$

Here, $\nu_t \equiv l_t / d_{t+1}$ is the share of new loans in the new stock of debt and $(1 - \nu_t) \equiv (1 - \delta_{Dt}) d_t / d_{t+1}$ is the share of the outstanding unamortized debt in the new stock of debt. In addition, i_t is the nominal mortgage interest rate on new loans and $\kappa \in (0, 1)$ is the initial amortization rate of new loans. Finally, $f(\delta_{Dt})$, discussed further below, is a smooth function with the following properties: $f(\delta_{Dt}) \in (0, 1)$, $f'(\delta_{Dt}) > 0$, $f''(\delta_{Dt}) > 0$ for δ_{Dt} close to zero, and $f''(\delta_{Dt}) < 0$ for δ_{Dt} close to one. Notice that combining equations (10) and (11) gives the evolution of mortgage debt in a more familiar form: $d_{t+1} = (1 + R_t) d_t - m_t + l_t$.

3.2.1 An example and explanation

It is worth pausing here to explain in more detail the laws of motion (11)-(13) and their implications for the time path of mortgage installments (10). For this purpose, let us suppose that the representative household has no outstanding debt ($d_0 = 0$) and takes out a FRM in period $t = 0$ in the amount $l_0 > 0$. Let us further assume that the household does not take out any new mortgage loans in subsequent periods (i.e., $l_1 = l_2 = ... = 0$). Equations (10)-(13) then yield the following path of mortgage installments: In period $t = 1$, the household's outstanding debt is $d_1 = l_0$, the initial amortization rate at which this debt will be reduced going into the next period is $\delta_{D1} = \kappa$, and the effective interest rate is $R_1 = i_0$. Mortgage payments in $t = 1$ are thus $m_1 = (R_1 + \delta_{D1}) d_1 = (i_0 + \kappa) l_0$. In period $t = 2$ the outstanding debt is $d_2 = (1 - \kappa) l_0$ and is reduced at a rate $\delta_{D2} = f(\kappa) > \kappa$ going into the next period. The interest rate R_2 is again equal to i_0. Mortgage payments in $t = 2$ are thus $m_2 = (R_2 + \delta_{D2}) d_2 = [i_0 + f(\kappa)](1 - \kappa) l_0$ and so on. Notice that whereas the interest part of mortgage payments, $R_t d_t$, declines as debt gets amortized, the amortization part,

$\delta_{Dt} d_t$, increases if δ_{Dt} grows at a fast enough rate. An appropriate choice of $f(.)$ ensures that the amortization part increases at such a rate so as to keep m_t approximately constant for a specified period of time (e.g., 30 years), thus approximating the defining characteristic of a standard FRM. A simple polynomial, which is used in our computational experiments, $f(\delta_{Dt}) = \delta_{Dt}^{\alpha}$, with $\alpha = 0.9946$ (and $\kappa = 0.00162$), is found to work fairly well, but higher-order polynomials can also be used for further precision (see Appendix B for details). An ARM works similarly, except that the interest part varies in line with changes in i_t.

Generally, mortgage payments can be calculated in two equivalent ways: an annuity formula or specifying an increasing sequence of amortization rates, with the final rate equal to one (see, for instance, Fabozzi, Modigliani and Jones, 2010). Here, we use the second method and approximate the finite sequence of amortization rates with an infinite sequence. With the approximation, even though the mortgage never matures, the payments after 30 years are essentially zero and, through out the 30 years, are approximately constant (see Appendix B for further details). This modeling choice is convenient in environments with infinitely-lived households as both the households and their mortgages live forever (even though the payments on these obligations after 30 years are essentially zero). Alternatively, it would be necessary in the household's optimization problem to keep track of the number of periods remaining on a given mortgage and, at the aggregate level, of the different vintages of mortgage debt. Our specification based on amortization rates also allows easy comparison with the alternatives considered in the literature, discussed below, and a simple exposition of the effect of nominal interest rates on the model dynamics, discussed in Section 4.2.

3.2.2 The general case

In the computational experiments below, the representative household starts with the economy's initial (steady-state) outstanding debt and, in response to shocks, chooses x_{Ht}, and thus l_t, every period. In this case, $\delta_{D,t+1}$ evolves as the weighted average of the amortization rate of the outstanding stock, $f(\delta_{Dt})$, and the initial amortization rate of new loans, κ, with the weights being the relative sizes of the current unamortized stock and the current flow in the new stock, respectively. Similarly, in the case of FRM, R_{t+1} evolves as the weighted average of the interest rate paid on the outstanding stock, R_t, and the interest rate charged for new loans, i_t. In the case of ARM, the current interest rate applies to both, the new loan and the outstanding stock.[22]

3.3 Exogenous process and closing the model

The price level p_t evolves as $p_t = (1+\pi_t)p_{t-1}$, where the inflation rate π_t follows an estimated VAR(n) process with the current nominal mortgage rate i_t and market TFP: $z_{t+1}b(L) = \varepsilon_{t+1}$, where $z_t = [\log A_{Mt}, i_t, \pi_t]^\top$, $\varepsilon_{t+1} \sim N(0, \Sigma)$, and $b(L) = I - b_1 L - ... - b_n L^n$ (L being the lag operator). As households in the model have access to only either FRM or ARM, the mortgage rate in the VAR is either an FRM rate or an ARM rate, depending on the experiment. Note that, as we are interested in unconditional moments of the data generated by the model, no identification assumptions on the orthogonality of the shocks in the VAR process are needed.

The model is closed by including a government, ensuring that the economy's resource constraint holds. The government collects revenues from capital and labor income taxes and operates the mortgage market by collecting mortgage instalments and providing new mortgage loans. Each period the government balances its budget by lump-sum transfers to the household, $\tau_t = \tau_r r_t k_{Mt} + \tau_w w_t h_{Mt} - \tau_r \delta_M k_{Mt} + m_t/p_t - l_t/p_t$, which can be negative.

The exogenous VAR process is a reduced form capturing the aspects of financial markets behind the observed lead-lag dynamics of nominal interest rates, both at the long end (FRM) and the short end (ARM) of the yield curve. As mentioned above, in the absence of an

[22] Most existing business cycle models with housing assume one-period loans. The interest rate applied to the loan is either the current short-term interest rate (e.g., Iacoviello, 2005, and many others), a weighted average of the current and past interest rates (Rubio, 2011), or evolving in a sticky Calvo-style fashion (Graham and Wright, 2007). The loan in Iacoviello (2005) is equivalent to $\delta_{Dt} = 1$ for all t, whereas the loans in Rubio (2011) and Graham and Wright (2007) are equivalent to $\delta_{Dt} = 1$ for all t in equation (11), but not in equation (13). Calza et al. (2013) model FRM as a two-period loan and ARM as a one-period loan. The housing debt of Campbell and Hercowitz (2006) and Garriga et al. (2014) is equivalent to equations (11)-(13) when the loan is ARM and the amortization rate $\delta_{Dt} \in (0, 1)$ is held constant (and the loans are denominated in real terms).

off-the-shelf structural model, the VAR process ensures that the lead-lag pattern of the mortgage rate (and the inflation rate) is as in the data. Koijen et al. (2009) take a similar approach, appending their model economy with a reduced-form model for interest rates in order to generate their realistic dynamics. As mortgages are priced exogenously, the stochastic discount factor of the household in the model is implicitly different from the pricing kernel reflected in the exogenous process for mortgage rates. If the two were the same, mortgage finance would not play any role. Inequality between the stochastic discount factor of the household and the pricing kernel in financial markets (due to, e.g., market incompleteness, segmentation, or regulation) is a necessary condition for mortgages to affect housing decisions in any model. This is not to say that otherwise there would be no borrowing and lending, but rather that the form of the loan contract, short-term v.s. long-term or FRM v.s. ARM, would be irrelevant.

4 The effect of mortgages on housing investment

This section characterizes the effect of mortgages on housing investment. Due to space constraints, equilibrium conditions that are not essential for the current discussion are relegated to Appendix B (this appendix also describes computation).

4.1 Equilibrium

The equilibrium is defined as follows: (i) the representative household solves its utility maximization problem, described below, taking all prices and transfers as given; (ii) r_t and w_t are equal to their marginal products and q_t is the marginal rate of transformation; (iii) the government budget constraint is satisfied; and (iv) the exogenous variables follow the VAR(n) process. The aggregate resource constraint, $c_{Mt} + x_{Mt} + q_t x_{Ht} = y_t$, then holds by Walras' Law. To characterize the equilibrium, it is convenient to work with a recursive formulation of the household's problem

$$V\left(s_{1t},...,s_{J-1,t},k_{Mt},k_{Ht},d_t,\delta_{Dt},R_t\right)=\max\{u\left(c_t,1-h_{Mt}-h_{Ht}\right)$$

$$+\beta E_t V\left(s_{1,t+1},...,s_{J-1,t+1},k_{M,t+1},k_{H,t+1},d_{t+1},\delta_{D,t+1},R_{t+1}\right)\},$$

subject to (2) and (4)-(13). After substituting the constraints into the Bellman equation, the maximization is only with respect to h_{Mt}, h_{Ht}, s_{Jt}, and x_{Ht}. Here, x_{Ht} affects the period utility function u through its effect on the budget constraint and the value function V through its effect on the laws of motion for $k_{H,t+1}$, d_{t+1}, $\delta_{D,t+1}$, and R_{t+1}.

There is enough separability in this problem that the variables related to mortgage finance (d_t, δ_{Dt}, R_t; i_t, π_t) show up only in the first-order condition for x_{Ht}. In this section we simply state the optimality condition. Its interpretation and how it impacts on the results is delayed until the next section. The optimality condition may appear somewhat cumbersome, but as the next section shows, its interpretation is fairly straightforward.

The first-order condition for x_{Ht} is:

(14)

$$u_{1t}c_{1t}(1-\theta)q_t-\theta q_t\beta E_t\left[\widetilde{V}_{d,t+1}+\zeta_{Dt}(\kappa-\delta_{Dt}^{\alpha})V_{\delta_D,t+1}+\zeta_{Dt}(i_t-R_t)V_{R,t+1}\right]=\beta E_t V_{kH,t+1}.$$

Here, $\widetilde{V}_{d,t+1}\equiv p_t V_{d,t+1}$, $\widetilde{d}_t\equiv d_t/p_{t-1}$ and $V_{kH,t}$, V_{dt}, $V_{\delta_D,t}$, and V_{Rt} are the derivatives of the value function with respect to the state variables specified in the subscript.[23] The above redefinitions of $V_{d,t+1}$ and d_t are required to ensure that the optimization problem is well-defined in the presence of nonzero steady-state inflation. Further, ζ_{Dt} measures the marginal contribution of unamortized debt to the stock of new debt, $\zeta_{Dt}\equiv\left(\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t\right)/\left(\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t+\theta q_t x_{Ht}\right)^2$, and the terms $\zeta_{Dt}(\kappa-\delta_{Dt}^{\alpha})V_{\delta_D,t+1}$ and $\zeta_{Dt}(i_t-R_t)V_{R,t+1}$ capture the marginal effects of changes in the effective amortization and interest rates, respectively, on the household's lifetime utility, occurring due to changes in the stock of unamortized debt. Notice that these

[23]We also adopt the convention of denoting by u_{2t}, for example, the first derivative of the u function with respect to its second argument.

two terms are equal to zero when old debt and new loans carry the same amortization and interest rates.

It is instructive to rearrange the first-order condition (14) as

$$u_{1t}c_{1t}q_t(1+\tau_{Ht}) = \beta E_t V_{kH,t+1}, \tag{15}$$

where $V_{kH,t+1}$ is decreasing in $k_{H,t+1}$ (see Appendix B) and

$$\tau_{Ht} = -\theta\left\{1 + \frac{\beta E_t \widetilde{V}_{d,t+1}}{u_{1t}c_{1t}} + \frac{\zeta_{Dt}(\kappa - \delta_{Dt}^{\alpha})\beta E_t V_{\delta_D,t+1}}{u_{1t}c_{1t}} + \frac{\zeta_{Dt}(i_t - R_t)\beta E_t V_{R,t+1}}{u_{1t}c_{1t}}\right\} \tag{16}$$

is an endogenous time-varying wedge, further discussed below. For $\tau_{Ht} = 0$, equation (15) has a straightforward interpretation: it equates this period's marginal utility of market consumption with expected marginal life-time utility of housing from next period on. The wedge acts like an ad-valorem tax, making an additional unit of housing more or less expensive in terms of current market consumption. Alternatively, it resembles a housing 'taste shock' (e.g., Liu et al., 2013), affecting the marginal rate of substitution between market consumption and housing. If $\theta = 0$ (i.e., no mortgage finance), the wedge is equal to zero and the equilibrium is the same as in GKR; the same results if the finance constraint is specified with inequality and is slack.

For future reference, note that $\widetilde{V}_{dt}$ is obtained by the Benveniste-Scheinkman theorem as

$$\widetilde{V}_{dt} = -u_{1t}c_{1t}\left(\frac{R_t + \delta_{Dt}}{1+\pi_t}\right) + \beta\left(\frac{1-\delta_{Dt}}{1+\pi_t}\right)E_t\left[\widetilde{V}_{d,t+1} + \zeta_{xt}(\delta_{Dt}^{\alpha} - \kappa)V_{\delta_D,t+1} + \zeta_{xt}(R_t - i_t)V_{R,t+1}\right], \tag{17}$$

where ζ_{xt} measures the marginal contribution of new loans to the stock of new debt, $\zeta_{xt} \equiv \theta q_t x_{Ht} / \left(\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t + \theta q_t x_{Ht}\right)^2$. The associated terms have analogous interpretation as in the case of old unamortized debt in equation (14).

4.2 Nominal interest rates and the wedge

An insight into the interpretation of the wedge is gained by again considering a once-and-for-all house purchase with no outstanding initial debt (i.e., $\widetilde{d}_t = 0$ and $x_{H,t+j} = 0$, for $j = 1, 2, ...$). In this case, $\zeta_{Dt} = 0$ in equation (16) and $\zeta_{x,t+j} = 0$, for $j = 1, 2, ...$, in equation (17), shifted one period forward. Further, the laws of motion (11)-(13) simplify as in the example in Section 3.2.1. The wedge (16) becomes

$$\tau_{Ht} = -\theta \left[1 + \beta E_t \left(\frac{\widetilde{V}_{d,t+1}}{u_{1t} c_{1t}} \right) \right] \tag{18}$$

and equation (17), shifted one period forward, is

$$\widetilde{V}_{dt+1} = -u_{1,t+1} c_{1,t+1} \left(\frac{i_t + \delta_{D,t+1}}{1 + \pi_{t+1}} \right) + \beta \left(\frac{1 - \delta_{D,t+1}}{1 + \pi_{t+1}} \right) E_{t+1} \widetilde{V}_{d,t+2}, \tag{19}$$

where i_t is either a FRM rate, an thus constant throughout the life of the loan, or an ARM rate, and thus time-varying. By forward substitution of equation (19)

$$\tau_{Ht} = -\theta \left\{ 1 - E_t \left[Q_{t+1} \frac{i_t + \delta_{D,t+1}}{1 + \pi_{t+1}} + Q_{t+1} Q_{t+2} \frac{(i_{t+1} + \delta_{D,t+2})(1 - \delta_{t+1})}{(1 + \pi_{t+1})(1 + \pi_{t+2})} + ... \right] \right\}, \tag{20}$$

where $Q_{t+j} \equiv \beta(u_{1,t+j} c_{1,t+j})/(u_{1,t+j-1} c_{1,t+j-1})$ is the stochastic discount factor of the representative household. The expression inside the square brackets states the present value of new mortgage debt, given as the expected discounted sum of marginal per-period real mortgage installments, weighted by the marginal utility of market consumption, over the life-time of the loan. The wedge is thus equal to $-\theta$ times the difference between the 'out-of-pocket' cost of financing an additional unit of housing, which is one unit of foregone market consumption today, and the mortgage cost of doing so, which is the present value of expected foregone market consumption in the future. A decline in the cost of mortgage finance (i.e., a decline of the expression in the square brackets) leads to a decline in the wedge. Through equation (15), this then, ceteris paribus, increases x_{Ht}.

Continuing the exposition with the simplified wedge, it is apparent from equation (20) that the behavior of the wedge depends on the exogenous stochastic process for the mortgage and inflation rates, guiding the expectations of these variables through out the life of the loan, and the endogenous behavior of consumption. A ceteris paribus decline in the mortgage rate reduces the wedge. In the case of FRM, the decline applies to interest payments in all periods of the loan's life; in the case of ARM, the expected persistence of the decline matters: a more persistent decline in i_t reduces the wedge by more. In contrast, a ceteris paribus decline in expected inflation increases the wedge; a more persistent decline increases the wedge by more. Recall that, over the business cycle, the inflation rate has similar cyclical dynamics as the nominal interest rate (Table 3): inflation declines when nominal interest rates decline and inflation increases when nominal interest rates increase. Which of the two variables is going to affect the wedge more? Suppose, for the sake of the argument, that both rates decline by one percentage point. Because $\delta_{Dt} < 1$ and at the front end of the loan's life very small—for instance, κ, the initial amortization rate, is 0.00162 for a 30-year mortgage—the real value of mortgage installments at the front end declines. This is because an equal decline in i_t and π_{t+1} reduces the numerator in the first expression on the right-hand side of equation (20) by more than the denominator. The effect of lower inflation gains strength only in later periods of the loan's life (if the decline in inflation is persistent) as its cumulative effect starts to sufficiently increase the real value of future installments. If the 'front-end effect' dominates this 'back-end effect', for instance due to discounting, the wedge declines.

One may wonder if a mortgage set in real, instead of nominal, terms would work equally well. By setting in equation (20) $\pi_t = 0$ for all t and letting i_t be the real interest rate, one obtains a wedge for a real mortgage. Thus, if the real interest rate leads output negatively, the wedge would decline prior to an increase in output. The issue with real interest rates, however, is that they are not directly observable and have to be constructed as a difference between nominal interest rates and estimates of inflation expectations.[24] This has consequences for the cyclical dynamics of real rates and Hornstein and Uhlig (2000) demonstrate

[24] Time series for yields on inflation protected government bonds are too short for business cycle analysis.

that their lead-lag pattern in relation to output is inconclusive. Real rates can lead negatively or positively, depending on the way inflation expectations are estimated. In this sense things are easier with nominal mortgages. With sufficient discounting, as explained above, the first-order effect on the wedge comes from the nominal interest rate. The computational experiments below indeed confirm that the wedge inherits the lead-lag dynamics of nominal mortgage rates. A nominal long-term mortgage thus transmits nominal interest rates into real mortgage costs.

Notice that if the mortgage was modeled as a one-period loan (i.e., $\delta_{D,t+1} = 1$), equation (20) would reduce to

$$\tau_{Ht} = -\theta \left[1 - E_t \left(\mu_{t+1} \frac{1 + i_t}{1 + \pi_{t+1}} \right) \right]$$

and a one-for-one decline in the nominal and inflation rates would cancel each other out, leaving the wedge unaffected; holding inflation constant, a decline in i_t reduces the wedge even in this case, but less than in the case of the mortgage where the decline, if persistent, affects mortgage installments over many periods. In this sense, a long-term mortgage provides, ceteris paribus, a stronger propagation mechanism for persistent shocks than a one-period loan.

5 Computational experiments

This section calibrates the model and reports findings from the main experiments. As the lead-lag patterns of mortgage and inflation rates are roughly similar across countries, the computational experiments are for a generic parameterization based on U.S. data.

5.1 Calibration

The parameter values are summarized in Table 4. One period in the model corresponds to one quarter and the functional forms are as in GKR: $u(.,.) = \omega \log c + (1-\omega) \log(1 - h_M - h_H)$; $c(.,.) = c_M^{\psi} c_H^{1-\psi}$; $G(.,.) = k_H^{\eta} h_H^{1-\eta}$; and $F(.,.) = k_M^{\varrho} h_M^{1-\varrho}$. The parameter A_H is normalized to be equal to one and the value of A_{Mt} in a nonstochastic steady state is chosen so that y_t in the nonstochastic steady state is equal to one.

As mentioned above, we abstract from consumer durable goods. In addition, housing services are modeled explicitly in the home sector. The data equivalent to y_t is thus GDP less expenditures on consumer durable goods and the gross value added of housing. Nonresidential capital in the model is mapped into the sum of nonresidential structures and equipment & software. If only nonresidential structures were used, the share of capital income in output, ϱ, would be too low, making the model dynamics difficult to compare with the literature. As in GKR, J is set equal to 4 and ϕ_j is set equal to 0.25 for all j. The parameter ϱ is set equal to 0.283, based on measurement from the National Income and Product Accounts (NIPA) obtained by Gomme, Ravikumar and Rupert (2011). Their NIPA-based estimate of $\tau_w = 0.243$ is also used. The depreciation rates are given as the average ratios of investment to the corresponding capital stocks. This yields $\delta_H = 0.0115$ and $\delta_M = 0.0248$. These are a little higher than the average depreciation rates from BEA Fixed Assets Accounts because the model abstracts from long-run population and TFP growth.

The parameter θ is set equal to 0.76, the average loan-to-value ratio for conventional single family newly-built home mortgages (Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10, 1963-2006). The steady-state mortgage rate i is set equal to 9.28% per annum, the average interest rate for the conventional 30-year FRM, 1971-2007, the dominant mortgage contract in the U.S. The initial amortization rate κ equals 0.00162 while α, the parameter governing the evolution of the amortization rate, is set equal to 0.9946. These choices are guided by an approximation of installments of a 30-year mortgage (see Appendix B). The steady-state inflation rate is set equal to 4.54% per annum, the average inflation rate for 1971-2007, which is the same period as that used to parameterize i. Given these values, the law of motion (12) implies a (quarterly) steady-state amortization rate of 0.0144, which—as in the data—is higher than the depreciation rate of residential structures. These values imply that in steady state the difference between the receipts of mortgage payments on outstanding debt and new loans is equal to 2.1% of output.

The discount factor β, the share of consumption in utility ω, the share of market good in consumption ψ, the share of capital in home production η, and the tax rate on income from nonresidential capital τ_r are calibrated jointly. Namely, by matching the average values of h_M, h_H, k_M/y, k_H/y, and the after-tax real rate of return on nonresidential capital, using the steady-state versions of the first-order conditions for h_M, h_H, s_J, and x_H (see Appendix B), and the model's after-tax real rate of return on nonresidential capital, $(1-\tau_r)(A_M F_1 - \delta_M)$, evaluated in steady state. According to the American Time-Use Survey (2003), individuals aged 16+ spent on average 25.5% of their available time working in the market and 24% in home production. We assume that half of home hours correspond to our notion of h_H. The average capital-to-output ratios are 4.88 for nonresidential capital and 4.79 for residential capital (in both cases expenditures on consumer durable goods and gross value added of housing are subtracted from GDP). The average (annual) after-tax real rate of return on nonresidential capital is measured by Gomme et al. (2011) to be 5.16%. These five targets yield $\beta = 0.988$, $\omega = 0.47$, $\psi = 0.69$, $\eta = 0.30$, and $\tau_r = 0.61$. As is common in models with disaggregated capital, the tax rate on market capital is higher than the statutory tax rate or an implicit tax rate calculated from NIPA. The calibration implies that in steady state the wedge τ_H is small, equal to -1.17%.

The parameterization of the exogenous stochastic process is based on point estimates of a VAR(3) process for TFP, the mortgage rate for a 30-year conventional FRM, and the inflation rate (see Appendix C for details). By construction, this process generates dynamic correlations of the mortgage and inflation rates with output similar to those in Table 3.

The parameter σ, which controls the curvature of the PPF, is chosen by matching the ratio of the standard deviations (for HP-filtered data) of residential investment (single family structures) and GDP. This yields $\sigma = 6.4$. The percentage deviation of q_t from its steady-state value $q = 1$ is related to the percentage deviation of x_{Ht} from its steady-state value $x_H = 0.055$ as $\widehat{q}_t = (x_H \sigma)\widehat{x}_{Ht}$. If we interpret q_t as the relative price of newly constructed homes, its volatility in the model is comparable to that in the data. In both cases the standard deviation, for HP-filtered logged data, is about 3 (the data counterpart used is the average sales prices of new homes sold, 1975-2007, from the Department of Commerce). In addition, in both the model and the data, the contemporaneous correlation with output is around 0.5. However, due to the absence of housing supply shocks, the model overstates the correlation between q_t and x_{Ht}.

5.2 Findings for one-period residential time to build

Table 5 reports the cyclical behavior of the model economy. Specifically, it contains the standard deviations (relative to that of y_t) of the key endogenous variables and their cross-correlations with y_t at various leads and lags. The upper panel shows the results for the baseline case of one-period residential time to build. The first thing to notice is that, despite the introduction of mortgages, the basic variables, y_t, h_{Mt}, c_{Mt}, and x_t behave like in other business cycle models and the behavior of total investment is broadly in line with the international evidence on GFCF in Figure 1.

Second, residential investment leads output. It is also more volatile than nonresidential investment. In addition, although not strictly lagging, nonresidential investment is substantially more positively correlated with past output than future output. The reason behind the lead in residential investment can be understood from the cyclical behavior of the wedge, which leads negatively. Referring back to our discussion in Section 4.2 and Table 3, notice that the lead-lag pattern of the wedge is similar to that of the mortgage rates.[25]

5.3 Multi-period residential time to build

When residential construction takes more than one period, a distinction needs to be made between finished houses and ongoing residential projects. With some small modifications, residential time to build is modeled in the same way as nonresidential time to build. The

[25] As new mortgage lending in the model (in real terms) is a constant fraction θ of residential investment, it leads output exactly as residential investment. This is consistent with the empirical findings in Table 2.

household makes an out-of-pocket investment in residential projects and, upon completion, sells the finished homes at a price q_t^*. The household also buys finished homes for its own use (think of the household as a homebuilder who likes houses of other makes than its own). Let n_t^* denote the number of newly constructed homes, occupiable next period, that the household wants to purchase for its own use and let n_{1t} denote the number of homes, occupiable next period, built by the household. With these modifications, the household's budget constraint becomes

$$c_{Mt}+x_{Mt}+q_t x_{Ht}+q_t^* n_t^* = (1-\tau_r)r_t k_{Mt}+\tau_r \delta_M k_{Mt}+(1-\tau_w)w_t h_{Mt}+q_t^* n_{1t}+l_t/p_t-m_t/p_t+\tau_t,$$

where $l_t = \theta p_t q_t^* n_t^*$ and $x_{Ht} = \sum_{\iota=1}^{N} \mu_\iota n_{\iota t}$, with $n_{\iota t}$ denoting residential projects ι periods from completion ($\sum_{\iota=1}^{N} \mu_\iota = 1$). The stock of houses for the household's own use evolves as

$$k_{H,t+1} = (1-\delta_H)k_{Ht} + n_t^*$$

and the on-going residential projects evolve as

$$n_{\iota-1,t+1} = n_{\iota t}, \quad \text{for} \quad \iota = 2, ..., N.$$

In equilibrium, $n_t^* = n_{1t}$. The economy's resource constraint is the same as before, $c_{Mt} + x_{Mt} + q_t x_{Ht} = y_t$, except that

$$x_{Ht} = \sum_{\iota=1}^{N} \mu_\iota n_{\iota t},$$

with $n_{1t}, ..., n_{N-1,t}$ being a part of the vector of state variables. Section 2.3 suggests that residential time to build in the countries in the sample other than the U.S. and Canada may be as long as one year. N is therefore set equal to 4. In the absence of information on the distribution of value put in place over the construction period, the μ's are assumed to be the same as the ϕ's in nonresidential time to build, $\mu_\iota = 0.25\ \forall \iota$. This parameterization has the additional advantage of treating the two types of time to build symmetrically. Shifting the weights towards the first period makes residential investment behave more like starts while shifting the weights towards the last period makes residential investment behave more like completions. The findings in Table 1 suggest that evenly distributed μ's are plausible.

The results are reported in the lower panel of Table 5. In addition to the variables reported in the case of one-period time to build, the table also reports results for 'housing starts' n_{4t} (i.e., structures four periods from completion) and 'completions' n_{0t} (i.e., structures that in period t have become a part of the usable housing stock h_t). As the table shows, x_{Ht} now reaches the highest correlation at $j = 0$, while starts lead by two quarters and completions lag by two quarters. The lead in housing starts occurs despite the fact that housing construction is financed by out-of-pocket expenses. The cyclical properties of the basic aggregates y_t, h_{Mt}, c_{Mt} and x_t are left, more or less, unaffected.

6 The quantitative importance of mortgages

In order to further investigate the quantitative role of mortgages, Table 6 reports the dynamic properties of the investment variables and the wedge for various specifications of the model (with one-period residential time to build). Recall that the model contains two forces pulling in opposite directions: the standard consumption smoothing effect, pushing residential investment to lag, and the wedge, inducing residential investment to lead. As σ is a free parameter, in each experiment it is recalibrated so as to match the relative volatility of residential investment, like in the baseline experiment. For easy comparison, panel (a) repeats the results for the baseline experiment.

We start, in panel (b), by removing mortgages ($\theta = 0$). The exogenous VAR process, however, stays the same. This guarantees that the underlying probability space of the economy remains unchanged.[26] When $\theta = 0$, the mortgage and inflation rates matter only to the extent that they help forecast future TFP. Specifically, referring back to the dynamics of these variables in Table 3, a low mortgage or inflation rate forecasts high TFP. Thus the

[26]The VAR is kept the same across experiments (a)-(d).

two nominal variables work as ‘news shocks’, signalling higher output and income in the future. As we see in panel (b), with $\theta = 0$, the lead-lag patterns observed in the baseline case disappear: both x_{Ht} and x_{Mt} are now coincident with output, with x_{Ht} being more strongly correlated with output at lags and x_{Mt} being more strongly correlated with output at leads. As GKR show, this inverted lead-lag pattern would be even more pronounced if there was no time to build in nonresidential capital.

Notice that even though the behavior of its components changes, the behavior of total investment, x_t, remains broadly unaffected by removing mortgage finance. In fact, the dynamics of x_t stay, more or less, unchanged across all model specifications in the table. This is because consumption smoothing constrains the response of total investment to shocks. A corollary of this result is that x_{Mt} has to lag output when x_{Ht} leads with sufficiently high volatility and vice versa.[27] The results of the current experiment also mean that, by themselves, expectations of higher future TFP (positive ‘news shocks’) are insufficient to produce residential investment leading output.

As noted in Section 2.5, typical loan-to-value ratios for new mortgage loans are similar across the countries in the sample. However, Belgium and France have only about half as high mortgage debt-to-GDP ratios than Australia, the U.K., and the U.S., with Canada being somewhere in-between (International Monetary Fund, 2011). This partly reflects historically smaller fraction of new homes financed through mortgages in these countries. Setting θ equal to 0.36 yields steady-state debt-to-output ratio about half as high as in the baseline. Panel (c) shows that in this case x_{Ht} still exhibits a lead, though less pronounced than in the baseline. This is because with a lower θ the wedge responds less to shocks than in the baseline.

Panel (d) considers the case of a one-period loan ($\delta_{Dt} = 1\ \forall t$), which results when $\alpha = 0$ and $\kappa = 1$. In line with our discussion in Section 4.2, the wedge is now little volatile and essentially uncorrelated with output, resulting in an absence of any lead in x_{Ht}.[28]

[27] Arguably, this consumption smoothing constraint would be weaker if the model economy was an open economy.

[28] The same result is obtained if the VAR process in this experiment includes a 3-month T-Bill rate, instead

Panel (e) investigates the role of the interest and inflation rate dynamics. Specifically, it considers the extreme case in which i_t and π_t are held constant at their steady-state values. The estimated VAR process is replaced with an AR(1) process for TFP, with persistence 0.94 (the highest eigenvalue of the original process) and the standard deviation of the innovation equal to 0.008. The household understands that i_t and π_t are now constant. We consider this to be a 'policy' experiment and therefore, unlike in the other cases, do not recalibrate σ. Under this specification, the lead in x_{Ht} again disappears. A corollary of this result is that the time series properties of residential investment observed in the data are likely to change when the dynamics of the two nominal variables (and especially of the nominal interest rate) change.

Finally, FRM is compared with ARM. Under ARM, the mortgage rate in the model is reset every period (a quarter). A natural choice for an ARM rate is therefore the yield on 3-month T-bills (the VAR process is re-estimated using this interest rate and is reported in Appendix C). Panel (f) shows that in this case a positive correlation of x_{Ht} with output occurs only at leads of two or more quarters. The highest positive correlation (0.39) occurs at $j = -5$, which falls outside of the table, and the contemporaneous correlation is negative. This long lead and the negative contemporaneous correlation are due to the wedge starting to increase well ahead of a peak in output; compare the behavior of the wedge with its behavior in the baseline case. The early rise in the wedge reflects the anticipated future increases in the short-term nominal interest rate, occurring alongside increases in output (refer back to Table 3).[29]

Bucks and Pence (2008) compare survey evidence on the perceived adjustability of ARM rates by households to administrative data on ARM terms and show that households systematically underestimate the extent to which their ARM rates can rise as short-term interest rates increase. To the extent that this is the case, the model—in which households under-

of the FRM interest rate.

[29]Koijen et al. (2009) argue that the changes in the relative cost of FRM v.s. ARM are mainly driven by cyclical variations in term premia. Such variations are here implicitly reflected in the VAR processes for FRM and ARM.

stand the stochastic process for the short rate—overstates, relative to the actual economy, the responses to expected future rises in interest rates. Panel (g) carries out the same exercise as panel (f), but using the initial interest rate charged on ARMs, instead of the 3-month T-bill rate. This is the interest rate that most ARMs carry for a specified initial period before interest payments become tied to an index, such as a T-bill rate. In the data, the initial ARM rate tends to stay low for longer than the 3-month T-bill rate and increases less sharply over the business cycle. Panel (g) of Table 6 shows that in this case x_{Ht} leads by two quarters, instead of five, with a positive contemporaneous correlation.

Notice that in cases (b)-(g), σ needs to be smaller than in the baseline in order to achieve the observed volatility of x_{Ht}. As a result, house prices in these cases are less volatile than in the baseline.

7 Concluding remarks

In a sample of developed economies, residential construction, measured by housing starts, leads real GDP. When measured by residential investment in national accounts, the lead is observed in the U.S. and Canada; in other countries in the sample, residential investment is more or less coincident with GDP. Such cyclical properties are at odds with the predictions of existing business cycle models that disaggregate capital into residential and nonresidential.

Motivated by a striking similarity, across countries, of the cyclical properties of nominal mortgage interest rates, and the dependence of house purchases on mortgage finance, we introduce mortgages into an otherwise standard business cycle model with home and market sectors. The mortgage in the model resembles first-lien loans used for house purchases only. Feeding into the model the observed cyclical dynamics of nominal mortgage interest rates and inflation rates produces a lead-lag pattern of residential and nonresidential investment similar to those in the U.S. and Canada. Increasing time to build in residential construction then makes residential investment coincident with GDP as in most other countries in the sample. Housing starts, however, still lead output as in the data. The results come at no cost in terms of deteriorating the model's ability to account for standard business cycle moments as much as other models in the literature.

Due to the absence of an off-the-shelf theory for the cyclical lead-lag pattern of mortgage rates, and nominal interest rates more generally, the stochastic process for mortgage rates is taken as exogenous. However, by itself, the process is not sufficient to reproduce the lead in housing starts and residential investment observed in the data. The other necessary element is the long-term and nominal nature of mortgage loans, which allows the transmission of nominal interest rates into real housing costs. The model also predicts that the cyclical lead in residential construction is not structural in nature: once the cyclical dynamics of nominal interest rates and inflation change, the empirical regularities of residential investment change as well.

It is beyond the scope of this paper to answer the question what drives the observed movements of mortgage rates. We have documented that their cyclical behavior is similar to that of government nominal bond yields of comparable maturities but leave open for future research the issue of the lead-lag pattern and causality between government bond yields and output.

While the main aim of the paper was to enhance our understanding of the lead-lag dynamics of residential investment, a broader lesson of the analysis is that nominal interest rates, in conjunction with long-term nominal mortgage loans, may have a quantitatively significant effect on the economy. In our framework this shows up only in the composition of total investment, not in other aggregate variables. It is, however, worth exploring channels through which such effects could transmit into the broader economy. This, of course, requires a richer framework than the one used here. Extensions of existing models along these lines may provide insights into channels of the transmission of monetary policy above and beyond the standard channels. This is where explicit modeling of long-term nominal loans is likely to be most fruitful.

References

Ahearne, A. G., J. Ammer, B. M. Doyle, L. S. Kole and R. F. Martin, "House Prices and Monetary Ploicy: A Cross-country Study," International Finance Discussion Papers No. 841, Board of Governors of the Federal Reserve System, 2005.

Backus, D. K., B. R. Routledge and S. Zin, "The Cyclical Component of U.S. Asset Returns," Mimeo, New York University, 2010.

Ball, M., "Markets and the Structure of the Housebuilding Industry: An International Perspective," *Urban Studies* 40 (2003), 897–916.

Benhabib, J., R. Rogerson and R. Wright, "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," *Journal of Political Economy* 99 (1991), 1166–87.

Benigno, P. and M. Woodford, "Optimal Taxation in an RBC Model: A Linear-Quadratic Approach," *Journal of Economic Dynamics and Control* 30 (2006), 1445–89.

Bernanke, B. S., "Housing, Housing Finance, and Monetary Policy," *Spech at the Federal Reserve Bank of Kansas City's Economic Symposium, Jackson Hole, Wyoming* (2007).

Bucks, B. K. and K. M. Pence, "Do Borrowers Know Their Mortgage Terms?," *Journal of Urban Economics* 64 (2008), 218–33.

Bureau of Economic Analysis, "Updated Summary of NIPA Methodologies," *Survey of Current Business* November (2009), 11–29.

Calza, A., T. Monacelli and L. Stracca, "Housing Finance and Monetary Policy," *Journal of the European Economic Association* 11 (2013), 101–122.

Campbell, J. R. and Z. Hercowitz, "The Role of Collateralized Household Debt in Macroeconomic Stabilization," Working Paper, Federal Reserve Bank of Chicago 2004-24, 2006.

Campbell, J. Y., "Mortgage Market Design," NBER Working Paper 18339, 2012.

Campbell, J. Y. and J. F. Cocco, "Household Risk Management and Optimal Mortgage Choice," *Quarterly Journal of Economics* 118 (2003), 1449–94.

Chambers, M., C. Garriga and D. Schlagenhauf, "Accounting for Changes in the Homeownership Rate," *International Economic Review* 50 (2009), 677–726.

Christiano, L. J. and T. J. Fitzgerald, "The Band Pass Filter," *International Economic Review* 44 (2003), 435–65.

Colton, K. W. and K. Collignon, "Multifamily Rental Housing in the 21st Century," Working Paper W01-1, Joint Center for Housing Studies, Harvard University, 2001.

Davis, M. A. and J. Heathcote, "Housing and the Business Cycle," *International Economic Review* 46 (2005), 751–84.

Dol, K. and M. Haffner, *Housing Statistics in the European Union* (Ministry of the Interior and Kingdom Relations, The Hague, Netherlands, 2010).

European Cenral Bank, "Comparability of Statistics for the Euro Area, the United States and Japan," *Monthly Buletin* April (2005), 61–72.

European Commission, "Handbook on Quarterly National Accounts," in *Methods and Nomenclatures* (European Commission, Brussels, Belgium, 1999).

Fabozzi, F. J., F. Modigliani and F. J. Jones, *Foundations of Financial Markets and Institutions*, fourth edition (Pearson, 2010).

Favilukis, J., S. C. Ludvigson and S. Van Nieuwerburgh, "The Macroeconomic Effects of Housing Wealth, Housing Finance, and Limited Risk-Sharing in General Equilibrium," Mimeo, 2015.

Fisher, J., "Why Does Household Investment Lead Business Investment over the Business Cycle?," *Journal of Political Economy* 115 (2007), 141–68.

Garriga, C., R. Manuelli and A. Peralta-Alva, "A Macroeconomic Model of Price Swings in the Housing Market," Mimeo, 2014.

Gavin, W. T. and F. E. Kydland, "The Nominal Facts and the October 1979 Policy Change," *Reserve Bank of St. Louis Quarterly Review* November/December (2000), 39–61.

Gervais, M., "Housing Taxation and Capital Accumulation," *Journal of Monetary Economics* 49 (2002), 1461–89.

Gomme, P., F. Kydland and P. Rupert, "Home Production Meets Time to Build," *Journal of Political Economy* 109 (2001), 1115–1131.

Gomme, P., B. Ravikumar and P. Rupert, "The Return to Capital and the Business Cycle," *Review of Economic Dynamics* 14 (2011), 262–78.

Gomme, P. and P. Rupert, "Theory, Measurement and Calibration of Macroeconomic Models," *Journal of Monetary Economics* 54 (2007), 460–97.

Graham, L. and S. Wright, "Nominal Debt Dynamics, Credit Constraints and Monetary Policy," *The B.E. Journal of Macroeconomics* 7 (2007), Article 9.

Greenwood, J. and Z. Hercowitz, "The Allocation of Capital and Time over the Business Cycle," *Journal of Political Economy* 99 (1991), 1188–1214.

Greenwood, J., R. Rogerson and R. Wright, "Household Production in Real Business Cycle Theory," in T. F. Cooley, ed., *Frontiers of Business Cycle Research* (Princeton University Press, 1995).

Hansen, G. D. and E. C. Prescott, "Recursive Methods for Computing Equilibria of Business Cycle Models," in T. F. Cooley, ed., *Frontiers of Business Cycle Research* (Princeton University Press, 1995).

Hardle, W., J. Horowitz and J.-P. Kreiss, "Bootstrap methods for time series," *International Statistical Review* 71 (2003), 435–59.

Henriksen, E., F. E. Kydland and R. Šustek, "Globally Correlated Nominal Fluctuations," *Journal of Monetary Economics* 60 (2013), 613–31.

Hornstein, A. and H. Uhlig, "What is the Real Story for Interest Rate Volatility?," *German Economic Review* 1 (2000), 43–67.

Huffman, G. W. and M. A. Wynne, "The Role of Intratemporal Adjustment Costs in a Multisector Economy," *Journal of Monetary Economics* 43 (1999), 317–50.

Iacoviello, M., "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle," *American Economic Review* 95 (2005), 739–64.

Iacoviello, M. and M. Pavan, "Housing and Debt over the Life Cycle and over the Business Cycle," *Journal of Monetary Economics* 60 (2013), 221–38.

International Monetary Fund, "Housing Finance and Financial Stability—Back to Basics?," in *Global Financial Stability Report, April 2011: Durable Financial Stability—Getting There from Here* (International Monetary Fund, Washington, DC, 2011).

Kearl, J. R., "Inflation, Mortgage, and Housing," *Journal of Political Economy* 87 (1979), 1115–38.

Kearl, J. R., K. Rosen and C. Swan, "Relationships between the Mortgage Instruments, the Demand for Housing, and Mortgage Credit," in *New Mortgage Designes for Stable Housing in an Inflationary Environment* (Federal Reserve Bank of Boston, Boston, 1975).

King, R. G. and M. W. Watson, "Money, Prices, Interest Rates and the Business Cycle," *Review of Economics and Statistics* 78 (1996), 35–53.

Koijen, R. S. J., O. Van Hemert and S. Van Nieuwerburgh, "Mortgage Timing," *Journal of Financial Economics* 93 (2009), 292–324.

Leamer, E. E., "Housing Is the Business Cycle," Working Paper 13428, NBER, 2007.

Liu, Z., P. Wang and T. Zha, "Land-Price Dynamics and Macroeconomic Fluctuations," *Econometrica* 81 (2013), 1147–84.

McGrattan, E., R. Rogerson and R. Wright, "An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy," *International Economic Review* 38 (1997), 267–90.

Miles, D., "The UK Mortgage Market: Taking a Longer-Term View," Special report, HM Treasury, 2004.

Rajan, R. G. and L. Zingales, "What Do We Know about Capital Structure? Some Evidence from International Data," *Journal of Finance* 50 (1995), 1421–60.

Rios-Rull, J. V. and V. Sanchez-Marcos, "An Aggregate Economy with Different Size Houses," *Journal of the European Economic Association* 6 (2008), 705–14.

Rubio, M., "Fixed- And Variable-Rate Mortgages, Business Cycles, and Monetary Policy," *Journal of Money, Credit, and Banking* 43 (2011), 657–88.

Scanlon, K. and C. Whitehead, "International Trends in Housing Tenure and Mortgage Finance," Special report for the Council of Mortgage Lenders, London School of Economics, 2004.

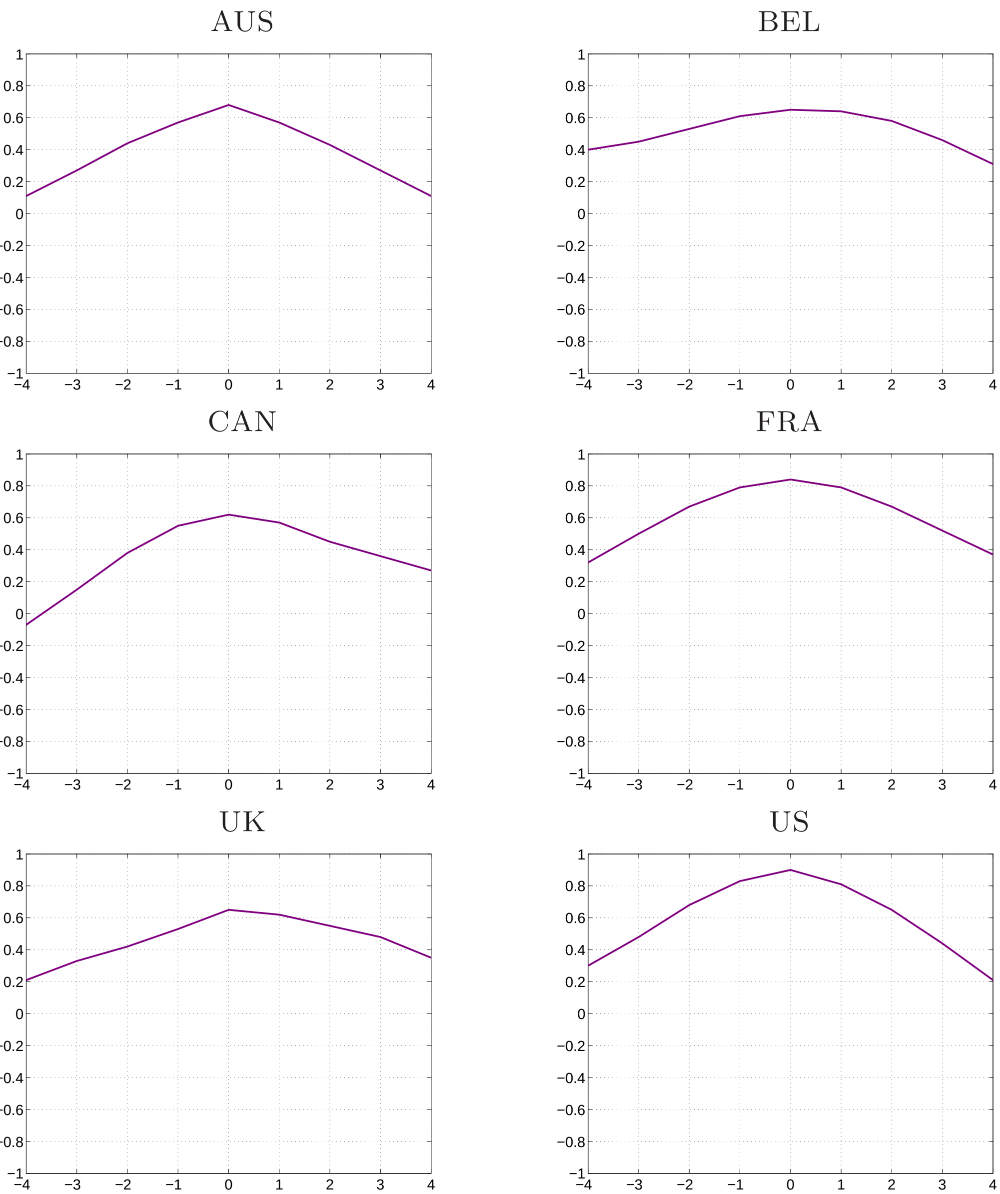

Figure 1: Cyclical dynamics of total fixed investment (gross fixed capital formation). The plots are correlations of real investment in $t+j$ with real GDP in t; the data are logged and filtered with Hodrick-Prescott filter. The volatility of total fixed investment (measured by its standard deviation relative to that of real GDP) is: AUS = 3.98, BEL = 3.93, CAN = 3.32, FRA = 2.65, UK = 2.55, US = 3.23.

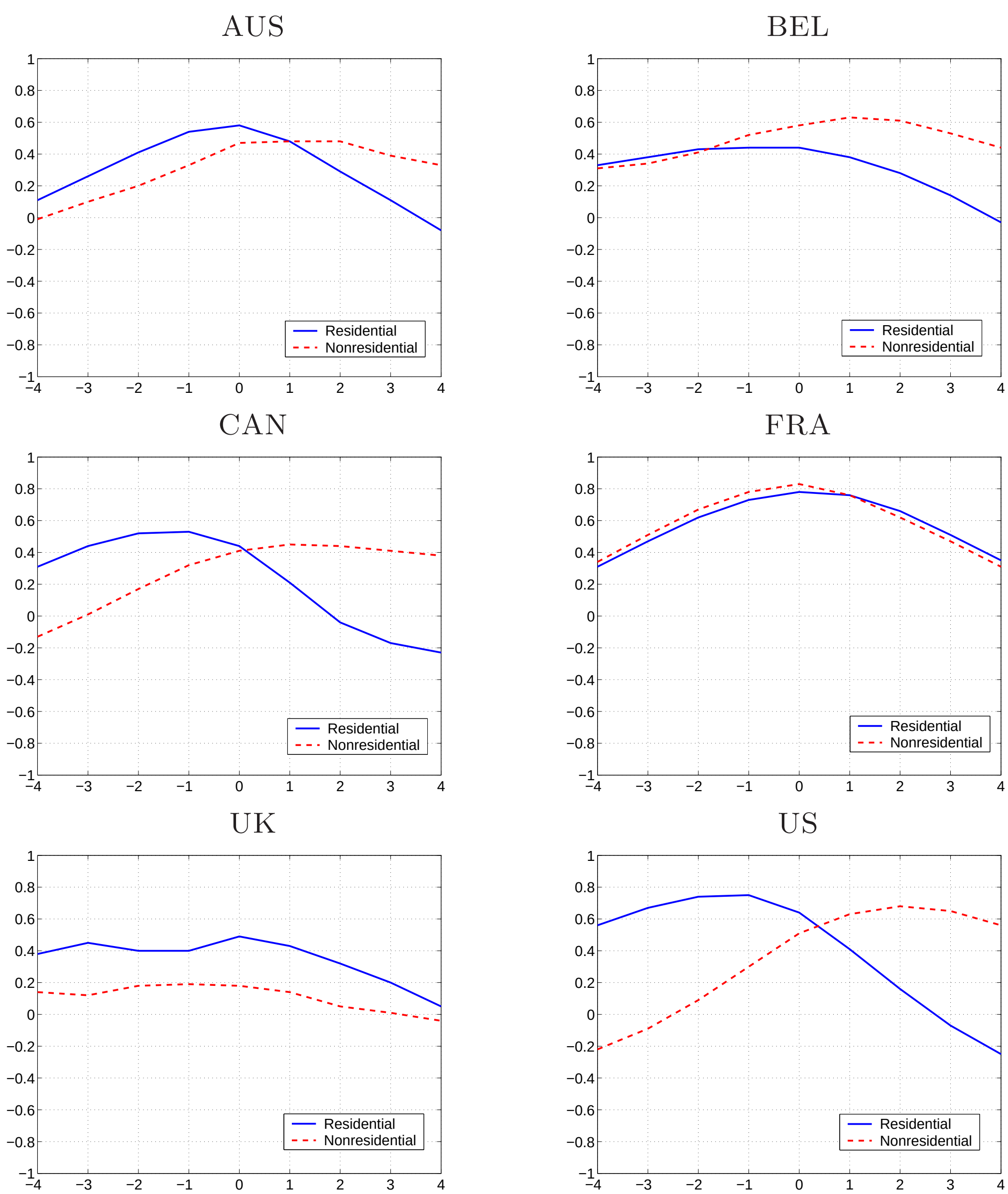

Figure 2: Cyclical dynamics of residential and nonresidential investment. The plots are correlations of real investment in $t + j$ with real GDP in t; the data are logged and filtered with Hodrick-Prescott filter. The volatility of residential (nonresidential) investment, relative to that of real GDP, is: AUS = 5.95 (6.96), BEL = 7.97 (4.36), CAN = 4.39 (3.97), FRA = 3.05 (3.24), UK = 5.02 (3.24), US = 6.42 (3.40).

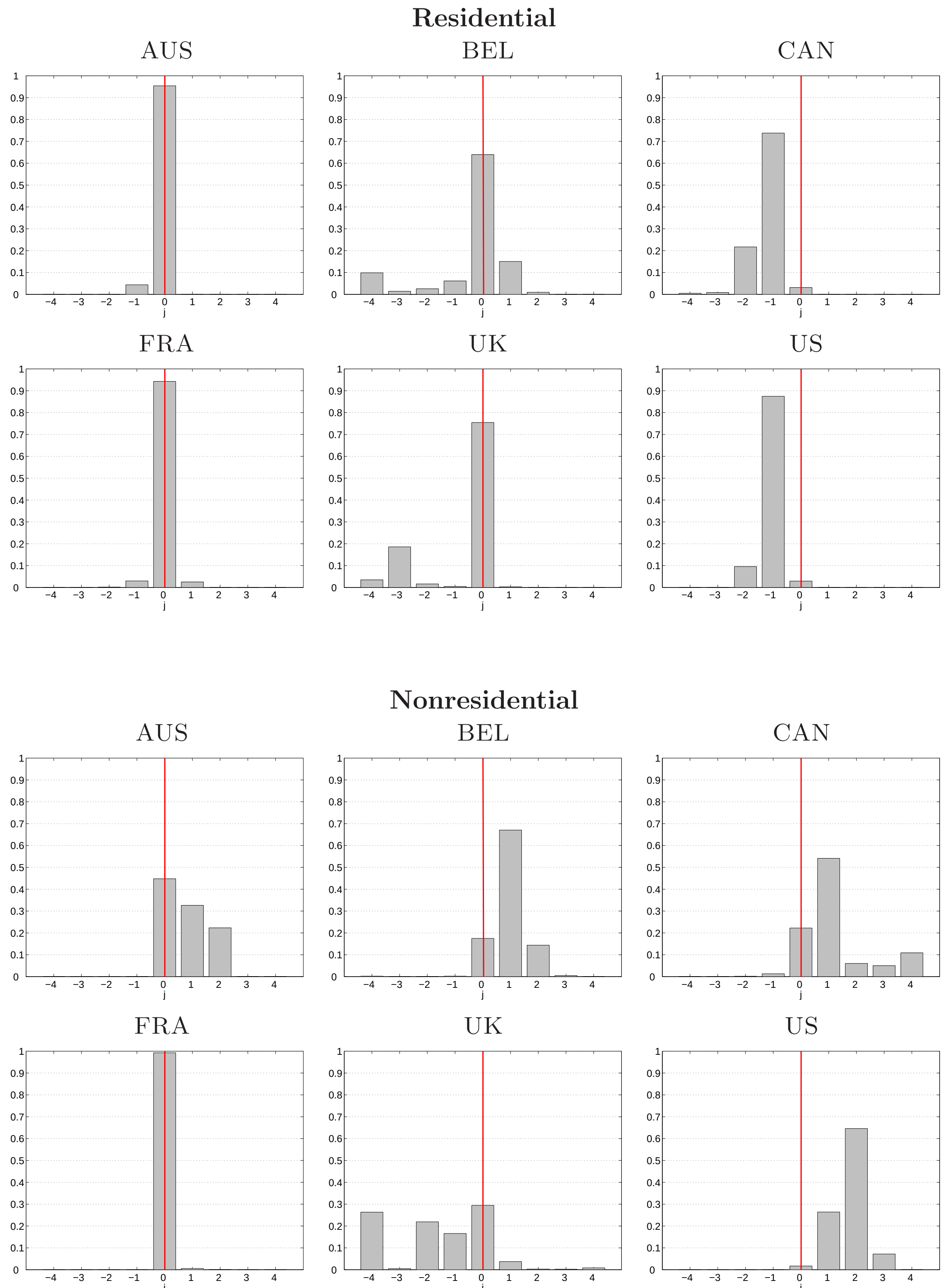

Figure 3: Statistical significance of leads and lags in investment dynamics. Histograms show the frequency with which a given j has the highest correlation coefficient in a sample of 10,000 cross-correlograms based on bootstrapped data.

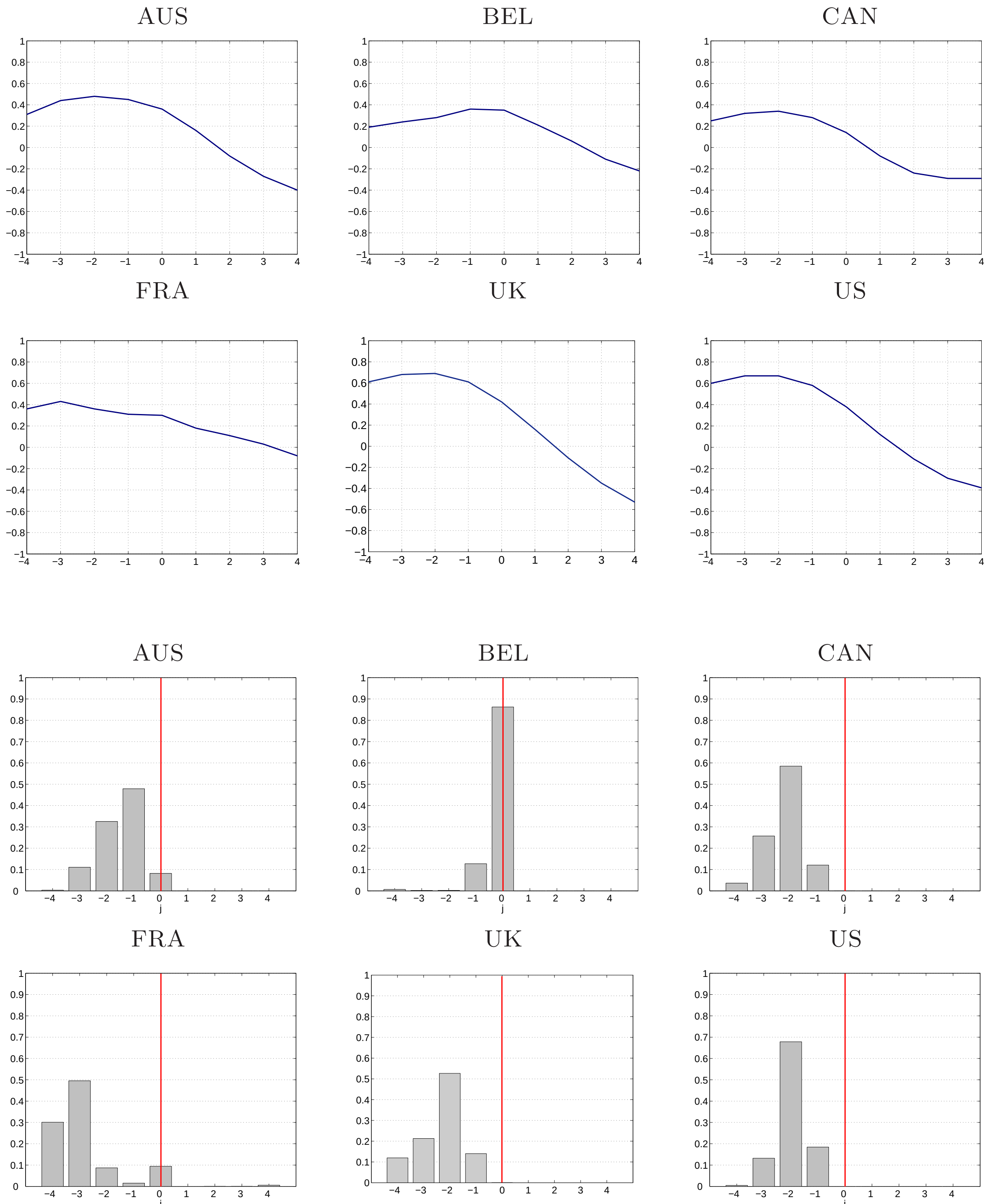

Figure 4: Housing starts. The top six charts plot cross-correlations in historical data (logged and HP-filtered); the bottom six charts show the statistical significance of leads and lags in the data; i.e., the frequency with which a given j has the highest correlation coefficient in a sample of 10,000 cross-correlograms based on bootstrapped data. The volatility of housing starts in the historical data, relative to that of real GDP, is: AUS = 8.80, BEL = 11.67, CAN = 9.95, FRA = 6.24, UK = 9.81, US = 9.72.

Table 1: Starts, completions, and residential investment[a]

	Relative std. dev.[b]	Correlations of real GDP in t with a variable in $t+j$: j = -4	-3	-2	-1	0	1	2	3	4
United States										
Starts										
1 unit	8.85	0.65	0.70	0.68	0.56	**0.33**	0.05	-0.18	-0.35	-0.42
5+ units	14.54	0.40	0.50	0.55	0.53	**0.43**	0.27	0.09	-0.06	-0.17
Completions										
1 unit	7.17	0.64	0.74	0.79	0.76	**0.72**	0.53	0.29	0.07	-0.12
5+ units	10.56	0.09	0.19	0.31	0.43	**0.53**	0.62	0.64	0.57	0.48
Resid. invest.										
Single-family	8.77	0.62	0.71	0.76	0.73	**0.60**	0.35	0.08	-0.17	-0.33
Multifamily	11.22	0.16	0.28	0.39	0.47	**0.49**	0.43	0.32	0.19	0.07
Australia										
Starts	8.80	0.31	0.44	0.48	0.45	**0.36**	0.16	-0.08	-0.27	-0.40
Completions	6.87	0.06	0.18	0.30	0.42	**0.50**	0.47	0.33	0.14	-0.05
Resid. invest.	5.95	0.11	0.26	0.41	0.55	**0.57**	0.48	0.29	0.11	-0.08
United Kingdom										
Starts	9.81	0.61	0.68	0.69	0.61	**0.42**	0.16	-0.11	-0.35	-0.53
Completions	4.62	-0.07	0.10	0.29	0.46	**0.55**	0.59	0.56	0.49	0.37
Resid. invest.	5.02	0.38	0.45	0.40	0.40	**0.49**	0.43	0.32	0.20	0.05

[a] The series are logged and filtered with Hodrick-Prescott filter.
[b] Standard deviations are expressed relative to that of a country's real GDP.

Table 2: U.S. data—further details[a]

	Relative std. dev.[b]	Correlations of real GDP in t with a variable in $t+j$: j = -4	-3	-2	-1	0	1	2	3	4
The effect of Regulation Q										
Resid. invest.										
1959.Q1–1983.Q4	8.84	0.58	0.65	0.73	0.72	**0.62**	0.39	0.14	-0.11	-0.30
1984.Q1–2006.Q4	8.40	0.51	0.57	0.60	0.57	**0.48**	0.28	0.05	-0.13	-0.25
Mortgage lending and resid. investment[c]										
Home mortgages[d]	15.01	0.47	0.56	0.62	0.57	**0.42**	0.23	0.10	-0.08	-0.19
Resid. invest.	8.77	0.62	0.71	0.76	0.73	**0.60**	0.35	0.08	-0.17	-0.33
Monetary policy regimes										
3-m nom. int. rate										
1959.Q1–1979.Q3	0.56	-0.59	-0.39	-0.17	0.07	**0.32**	0.52	0.61	0.63	0.62
1979.Q4–2006.Q4	0.96	-0.40	-0.21	-0.03	0.25	**0.43**	0.53	0.59	0.55	0.43
10-yr nom. int. rate										
1959.Q1–1979.Q3	0.28	-0.60	-0.51	-0.43	-0.30	**-0.17**	-0.02	0.11	0.25	0.39
1979.Q4–2006.Q4	0.60	-0.44	-0.33	-0.16	-0.02	**0.19**	0.24	0.23	0.18	0.07
Inflation rate (CPI)										
1959.Q1–1979.Q3	1.00	-0.53	-0.39	-0.16	0.03	**0.25**	0.46	0.56	0.67	0.68
1979.Q4–2006.Q4	1.32	-0.10	-0.02	0.23	0.31	**0.44**	0.45	0.28	0.23	0.20

[a] The series are logged and filtered with Hodrick-Prescott filter.
[b] Standard deviations are expressed relative to that of a country's real GDP.
[c] Both for 1959.Q1–2006.Q4
[d] Net change in home mortgages, deflated with GDP deflator (home mortgages = 1-4 family properties). The fraction of new construction accounted for by 2-4 family structures is small making home mortgages a good proxy for single-family housing mortgages, for which data are not available.

Table 3: Cyclical dynamics of nominal mortgage interest rates[a]

		Relative std. dev.[b]	Correlations of real GDP in t with a variable in $t+j$: j = -4	-3	-2	-1	0	1	2	3	4
Mortgage rates[c]											
AUS	ARM	0.59	-0.29	-0.22	-0.16	-0.03	**0.12**	0.25	0.39	0.48	0.50
BEL	FRM 10 yrs	0.89	-0.17	0.01	0.19	0.38	**0.56**	0.63	0.60	0.53	0.41
CAN	FRM 5 yrs	0.77	-0.52	-0.41	-0.24	-0.04	**0.19**	0.38	0.45	0.45	0.43
FRA	FRM 15 yrs	0.87	-0.10	-0.02	0.10	0.20	**0.30**	0.36	0.35	0.31	0.27
UK	ARM[d]	1.29	-0.68	-0.52	-0.31	-0.06	**0.17**	0.36	0.49	0.55	0.56
US	FRM 30 yrs	0.55	-0.59	-0.55	-0.46	-0.29	**-0.07**	0.09	0.16	0.21	0.23
Government bond yields[e]											
AUS	3-m	1.07	-0.19	-0.06	0.10	0.24	**0.34**	0.44	0.52	0.45	0.34
BEL	10-yr	0.75	-0.01	0.20	0.33	0.49	**0.53**	0.50	0.43	0.31	0.19
CAN	3-5-yr	0.73	-0.42	-0.25	-0.06	0.17	**0.39**	0.52	0.54	0.50	0.41
FRA	10-yr	0.86	-0.12	-0.02	0.10	0.21	**0.29**	0.31	0.28	0.25	0.24
UK	3-m	1.29	-0.68	-0.52	-0.31	-0.06	**0.17**	0.36	0.49	0.55	0.56
US	10-yr	0.53	-0.45	-0.39	-0.29	-0.11	**0.04**	0.09	0.10	0.12	0.09
	3-m	0.88	-0.45	-0.30	-0.10	0.17	**0.39**	0.48	0.51	0.49	0.46
Inflation rates[f]											
AUS		1.60	-0.31	-0.19	0.01	0.24	**0.43**	0.54	0.56	0.51	0.42
BEL		0.76	-0.03	-0.13	-0.23	-0.25	**-0.17**	0.02	0.22	0.39	0.44
CAN		1.10	-0.29	-0.12	0.06	0.23	**0.37**	0.46	0.52	0.54	0.51
FRA		1.08	-0.34	-0.25	-0.15	-0.08	**-0.06**	-0.05	0.01	0.11	0.24
UK		2.16	-0.68	-0.61	-0.45	-0.24	**0.01**	0.20	0.36	0.45	0.51
US		1.24	-0.27	-0.12	0.02	0.21	**0.42**	0.49	0.51	0.52	0.51

[a] GDP is in logs; all series are filtered with Hodrick-Prescott filter; time periods differ across countries due to different availability of mortgage rate data: AUS (59.Q3-06.Q4), BEL (80.Q1-06.Q4), CAN (61.Q1-06.Q4), FRA (78.Q1-06.Q4), UK (65.Q1-06.Q4), US (71.Q2-06.Q4).
[b] Standard deviations are expressed relative to that of a country's real GDP.
[c] Based on a typical mortgage for each country, as reported by Calza et al. (2013) and Scanlon and Whitehead (2004). Mortgages rates are APR. ARM = adjustable rate mortgage (interest rate can be reset within one year), FRM = fixed rate mortgage (interest rate can be at the earliest reset only after 5 years). The numbers accompanying FRMs refer to the number of years for which the mortgage rate is typically fixed.
[d] U.K. mortgage rate data are available only from 1995.Q1. 3-m T-bill rate is used as a proxy for the adjustable mortgage rate for the period 1965.Q1-1994.Q4; the correlation between the two interest rates for the period 1995.Q1-2006.Q4 is 0.97.
[e] Constant maturity rates; APR; periods correspond to those of mortgage rates.
[f] Consumer price indexes; q-on-q percentage change at annual rate; periods correspond to those of mortgage rates.

Table 4: Calibration

Symbol	Value	Definition
Preferences		
β	0.988	Discount factor
ω	0.472	Consumption share in utility
ψ	0.692	Share of market good in consumption
Home technology		
δ_H	0.0115	Depreciation rate
η	0.305	Capital share in production
Nonresidential time to build		
J	4	Number of periods
ϕ_j	0.25	Fraction completed at stage j
Market technology		
δ_M	0.0248	Depreciation rate
ϱ	0.283	Capital share in production
σ	6.4	PPF curvature parameter
Tax rates		
τ_w	0.243	Tax rate on labor income
τ_r	0.612	Tax rate on capital income
Mortgages		
θ	0.76	Loan-to-value ratio
κ	0.00162	Initial amortization rate
α	0.9946	Adjustment factor
i	0.0232	Steady-state mortgage rate
π	0.0113	Steady-state inflation rate

Note: The parameters of the exogenous stochastic process are contained in Appendix C.

Table 5: Cyclical behavior of the model economy[a]

	Rel.	Correlations of y in period t with variable υ in period $t+j$:								
υ_{t+j}	st.dev.[b]	j = -4	-3	-2	-1	0	1	2	3	4
1-period residential time to build										
Main aggregates										
y	1.01	-0.03	0.19	0.48	0.75	**1.00**	0.75	0.48	0.19	-0.03
h_M	0.56	0.10	0.31	0.57	0.76	**0.89**	0.68	0.41	0.07	-0.21
c_M	0.48	-0.21	-0.09	0.13	0.38	**0.70**	0.52	0.38	0.29	0.28
x	4.42	0.07	0.29	0.56	0.78	**0.93**	0.71	0.43	0.10	-0.18
Investment components and the wedge										
x_H	8.45	0.19	0.34	0.50	0.55	**0.51**	0.31	0.11	-0.13	-0.32
x_M	4.33	-0.12	0.03	0.25	0.50	**0.78**	0.70	0.52	0.31	0.12
τ_H	3.26	-0.21	-0.33	-0.43	-0.43	**-0.32**	-0.17	-0.02	0.18	0.34
4-period residential time to build										
Main aggregates										
y	1.01	-0.03	0.17	0.45	0.73	**1.00**	0.73	0.45	0.17	-0.03
h_M	0.54	0.11	0.30	0.55	0.76	**0.92**	0.66	0.37	0.05	-0.21
c_M	0.44	-0.23	-0.10	0.14	0.41	**0.76**	0.58	0.43	0.31	0.29
x	4.32	0.08	0.28	0.54	0.77	**0.95**	0.69	0.40	0.08	-0.17
Investment components, starts, completions, and the wedge[c]										
x_H	6.51	0.18	0.32	0.47	0.57	**0.60**	0.42	0.14	-0.16	-0.40
n_4	8.89	0.33	0.40	0.50	0.48	**0.38**	-0.10	-0.33	-0.40	-0.34
n_0	8.88	-0.05	-0.02	0.06	0.18	**0.33**	0.40	0.50	0.48	0.38
x_M	4.11	-0.13	0.05	0.31	0.60	**0.90**	0.80	0.62	0.38	0.14
τ_H	3.17	-0.22	-0.34	-0.43	-0.42	**-0.29**	-0.16	-0.02	0.18	0.34

[a] Calibration is as in Table 4. The statistics are averages for 200 artificial data samples. All variables are in percentage deviations from steady state, except the wedge, which is in percentage point deviations from steady state. Before computing the statistics for each sample, the artificial data were filtered with the HP filter.
[b] Standard deviations are measured relative to that of y; the standard deviation of y is in absolute terms.
[c] n_4 = housing starts (houses that in period t are four periods from completion), n_0 = housing completions (houses that in period $t-1$ were one period away from completion and in period t are a part of the housing stock).

Table 6: Impact of mortgage finance on investment dynamics

υ_{t+j}	Rel. st.dev.	Correlations of y in period t with variable υ in period $t+j$: j = -4	-3	-2	-1	0	1	2	3	4
(a) Baseline; $\sigma = 6.4$										
x	4.42	0.07	0.29	0.56	0.78	**0.93**	0.71	0.43	0.10	-0.18
x_H	8.45	0.19	0.34	0.50	0.55	**0.51**	0.31	0.11	-0.13	-0.32
x_M	4.33	-0.12	0.03	0.25	0.50	**0.78**	0.70	0.52	0.31	0.12
τ_H	3.26	-0.21	-0.33	-0.43	-0.43	**-0.32**	-0.17	-0.02	0.18	0.34
(b) No mortgage finance; i_t and π_t are only news shocks; $\sigma = 0.03$										
x	4.71	0.15	0.27	0.49	0.72	**0.99**	0.68	0.41	0.17	0.04
x_H	8.45	-0.05	-0.01	0.12	0.33	**0.67**	0.54	0.49	0.44	0.56
x_M	5.06	0.24	0.37	0.57	0.73	**0.83**	0.51	0.19	-0.11	-0.37
τ_H	–	–	–	–	–	–	–	–	–	–
(c) Low mortgage finance ($\theta = 0.36$); $\sigma = 2.87$										
x	4.50	0.05	0.27	0.54	0.77	**0.96**	0.72	0.44	0.11	-0.16
x_H	8.45	0.16	0.32	0.50	0.60	**0.59**	0.37	0.15	-0.11	-0.30
x_M	4.36	-0.08	0.07	0.30	0.56	**0.84**	0.71	0.51	0.28	0.08
τ_H	1.53	-0.19	-0.31	-0.41	-0.42	**-0.31**	-0.18	-0.03	0.18	0.35
(d) 1-period loan ($\delta_{Dt} = 1\ \forall t$); $\sigma = 0.41$										
x	4.97	0.11	0.19	0.49	0.63	**0.95**	0.63	0.42	0.15	-0.02
x_H	8.45	0.08	-0.03	0.28	0.24	**0.63**	0.39	0.29	0.22	0.16
x_M	4.78	0.10	0.32	0.51	0.76	**0.93**	0.64	0.40	0.05	-0.16
τ_H	0.41	-0.12	0.14	-0.15	0.12	**-0.18**	-0.01	-0.12	-0.04	-0.01
(e) Constant i_t and π_t (held at steady-state values); $\sigma = 6.4$										
x	3.49	0.16	0.30	0.49	0.71	**0.98**	0.69	0.44	0.20	-0.02
x_H	0.72	0.09	0.24	0.44	0.68	**0.99**	0.69	0.45	0.28	0.17
x_M	4.65	0.16	0.31	0.49	0.71	**0.97**	0.69	0.43	0.19	-0.04
τ_H	0.19	-0.05	0.07	0.24	0.47	**0.79**	0.49	0.33	0.31	0.45
(f) ARM (3m T-Bill rate); $\sigma = 1.97$										
x	4.02	0.11	0.15	0.39	0.66	**0.94**	0.68	0.35	0.05	-0.05
x_H	8.45	0.37	0.33	0.17	-0.05	**-0.31**	-0.45	-0.54	-0.56	-0.52
x_M	8.26	-0.11	-0.05	0.20	0.49	**0.82**	0.71	0.52	0.32	0.23
τ_H	1.22	-0.25	-0.22	0.02	0.29	**0.64**	0.61	0.60	0.55	0.56
(g) ARM (ARM rate); $\sigma = 0.96$										
x	4.20	0.17	0.36	0.59	0.79	**0.96**	0.75	0.50	0.21	-0.04
x_H	8.45	0.33	0.42	0.45	0.38	**0.24**	0.03	-0.17	-0.29	-0.36
x_M	5.85	-0.05	0.09	0.30	0.56	**0.83**	0.77	0.64	0.42	0.21
τ_H	0.78	-0.27	-0.26	-0.15	0.06	**0.35**	0.34	0.36	0.38	0.42

Notes: Except case (e), σ is recalibrated so as to keep std(x_{Ht})/std(y_t) constant across experiments. Cases (a)-(d): the underlying probability space (i.e., the VAR process) is kept constant. Case (e): the process is changed to an AR(1) for TFP, with a persistence parameter 0.94 and the standard deviation of the innovation 0.008. Cases (f) and (g): a VAR process with the short-term interest rate noted in the brackets, instead of the FRM rate.

Supplemental material (not part of the manuscript)

A. International data used in Section 2

Australia. Real volumes: GDP, private GFCF, private GFCF nondwelling construction total, private GFCF dwellings total (all in chained dollars, SA, 1959.Q3-2006.Q4, *Australian Bureau of Statistics, National Accounts*); Mortgage rate: standard variable housing loans lending rate, banks (1959.Q3-2006.Q4, *Reserve Bank of Australia*); Interest rate: 3-month T-bill yield (1960.Q1-2006.Q4, *Global Financial Data*); Inflation rate: CPI q-on-q percentage change at annual rate (SA adjusted by authors, 1959.Q3-2006.Q4, *OECD-MEI*); Housing starts: units, SA, 1965.Q3-2006.Q4,*OECD-MEI*; Housing completions: units, SA, 1965.Q3-2006.Q4,*Australian Bureau of Statistics*.

Belgium. Real volumes: GDP at market prices, GFCF total, GFCF in dwellings, GFCF by enterprises, self-employed workers and non-profit institutions (all in chained 2006 euros, SA, 1980.Q1-2006.Q4, *BelgoStat Online, National Accounts*); Mortgage rate: fixed rate on loans for house purchasing (1980.Q1-2006.Q4, *Global Financial Data*); Interest rate: 3-month T-bill yield (1980.Q1-2006.Q4, *Global Financial Data*); Inflation rate: CPI-ex q-on-q percentage change at annual rate (SA adjusted by authors, 1980.Q1-2006.Q4, *OECD-MEI*); Housing starts: units, SA, 1968.Q1-2006.Q4, *OECD-MEI*.

Canada. Real volumes: GDP, residential structures, nonresidential structures, single dwellings, multiple dwellings (all in chained 2002 dollars, SA, *Statistics Canada, National Accounts*, 1961.Q1-2006.Q4, except for single and multiple dwellings, which are for 1981.Q1-2006.Q4); Mortgage rate: conventional mortgage lending rate, 5-year term (1961.Q1-2006.Q4, *Statistics Canada*); Interest rate: 3-month T-bill yield (1961.Q1-2006.Q4, *Global Financial Data*); Inflation rate: CPI q-on-q percentage change at annual rate (SA adjusted by authors, 1961.Q1-2006.Q4, *OECD-MEI*); Housing starts: units, SA, 1960.Q1-2006.Q4, *OECD-MEI*.

France. Real volumes: GDP, total GFCF, GFCF of non financial enterprises—including uninc. entrep., GFCF of households—excluding uninc. entrep. (all in chained euros, SA, 1971.Q1-2006.Q4, *INSEE, National Accounts*); Mortgage rate: mortgage lending rate (1978.Q1-2006.Q4, *Global Financial Data*); Interest rate: money market rate (1971.Q1-2006.Q4, *International Financial Statistics* and *Datastream*); Inflation rate: CPI-ex q-on-q percentage change at annual rate (SA adjusted by authors, 1978.Q1-2006.Q4, *OECD-MEI*); Housing starts: units, SA, 1974.Q1-2006.Q4, *OECD-MEI*.

United Kingdom. Real volumes: GDP at market prices, GFCF total, GFCF dwellings, GFCF other new buildings and structures (all in chained 2002 pounds, SA, 1965.Q1-2006.Q4, *Office for National Statistics, United Kingdom Economic Accounts*); Mortgage rate: sterling standard variable mortgage rate to households (1995.Q1-2006.Q4, *Bank of England*); Interest rate: 3-month T-bill yield (1965.Q1-2006.Q4, *Office for National Statistics*); Inflation rate: CPI q-on-q percentage change at annual rate (SA adjusted by authors, 1965.Q1-2006.Q4, *OECD-MEI*); Housing starts and completions: units, SA adjusted by authors, 1978.Q1-2006.Q4, *Department of Communities and Local Government*.

United States. REAL VOLUMES: GDP, private fixed investment, private residential fixed investment, private fixed investment single family, private fixed investment multifamily, private fixed investment structures (all in chained 2000 dollars, SA, 1958.Q1-2006.Q4, *FRED* and *Bureau of Economic Analysis, National Income and Product Accounts*); MORTGAGE RATE: 30-year conventional mortgage rate (1971.Q1-2006.Q4, *FRED*); INTEREST RATE: 3-month T-bill yield (1958.Q1-2006.Q4, *FRED*); INFLATION RATE: CPI q-on-q percentage change at annual rate (SA, 1971.Q2-2006.Q4, *OECD-MEI*); HOUSING STARTS: units, SA, 1959.Q1-2006.Q4, *FRED*; HOUSING STARTS AND COMPLETIONS FOR STRUCTURE TYPES: units, SA, 1968.Q1-2006.Q4, *Survey of Construction, Census Bureau*; HOME MORTGAGES: Millions of current dollars, SAAR, *Flow of Funds, Table F217.*

Availability of residential and nonresidential investment data for other countries: Austria from 1988.Q1, Denmark from 1990.Q1, Finland from 1990.Q1, Germany from 1991.Q1 (annually from 1970), Ireland from 1997.Q1 (annually from 1970), Italy from 1990.Q1, the Netherlands from 1987.Q1, New Zealand from 1987.Q2, (annually from 1972), Portugal from 1995.Q1, and Spain from 1995.Q1. The data sources are the OECD Main Economic Indicators database, the OECD National Accounts database, and national statistical agencies. The data are also available for Japan from 1980.Q1, Norway from 1978.Q1, and Sweden from 1980.Q1. However, for these time periods residential investment in these countries does not exhibit 'cyclical' fluctuations in the sense of recurrent random ups and downs. Instead, in each of these countries the data are dominated by one episode: the financial and housing market crises in Norway (1987-1992) and Sweden (1990s) and the late 1980s/early 1990s housing boom and bust in Japan.

B. Model: further details and computation

B.1 Full set of the household's optimality conditions

The household's optimal decisions are characterized by four first-order conditions for h_{Mt}, h_{Ht}, s_{Jt}, and x_{Ht}. These are, respectively,

$$u_{1t}c_{1t}(1-\tau_w)w_t = u_{2t},$$

$$u_{1t}c_{2t}A_H G_{2t} = u_{2t},$$

$$u_{1t}c_{1t}\phi_J = \beta E_t V_{s_{J-1},t+1},$$

$$u_{1t}c_{1t}(1-\theta)q_t - \theta q_t \beta E_t \left[\widetilde{V}_{d,t+1} + \zeta_{Dt}(\kappa - \delta_{Dt}^{\alpha})V_{\delta_D,t+1} + \zeta_{Dt}(i_t - R_t)V_{R,t+1}\right] = \beta E_t V_{kH,t+1}.$$

Here $\widetilde{V}_{d,t+1}$ and ζ_{Dt} are defined as in the main text; that is, $\widetilde{V}_{d,t+1} \equiv p_t V_{d,t+1}$ and $\zeta_{Dt} \equiv \left(\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t\right) / \left(\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t + \theta q_t x_{Ht}\right)^2$, where $\widetilde{d}_t \equiv d_t/p_{t-1}$.

The first-order condition for s_{Jt} is accompanied by Benveniste-Scheinkman conditions for s_{jt} $(j = J-1, ..., 2)$, s_{1t}, and k_{Mt}, respectively,

$$V_{s_j t} = -u_{1t}c_{1t}\phi_j + \beta E_t V_{s_{j-1},t+1}, \quad j = J-1, ..., 2,$$

$$V_{s_1 t} = -u_{1t}c_{1t}\phi_1 + \beta E_t V_{k_M,t+1},$$

$$V_{k_M,t} = u_{1t}c_{1t}\left[(1-\tau_r)r_t + \tau_r\delta_M\right] + \beta(1-\delta_M)E_tV_{k_M,t+1}.$$

The first-order condition for x_{Ht} is accompanied by Benveniste-Scheinkman conditions for d_t, δ_{Dt}, R_t, and k_{Ht}. These are, respectively,

$$\widetilde{V}_{dt} = -u_{1t}c_{1t}\frac{R_t+\delta_{Dt}}{1+\pi_t} + \beta\frac{1-\delta_{Dt}}{1+\pi_t}E_t\left[\widetilde{V}_{d,t+1} + \zeta_{xt}(\delta_{Dt}^{\alpha}-\kappa)V_{\delta_D,t+1} + \zeta_{xt}(R_t-i_t)V_{R,t+1}\right],$$

$$V_{\delta_D,t} = -u_{1t}c_{1t}\left(\frac{\widetilde{d}_t}{1+\pi_t}\right) + \left[\zeta_{xt}(\kappa-\delta_{Dt}^{\alpha}) + \frac{(1-\delta_{Dt})\alpha\delta_{Dt}^{\alpha-1}}{\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t + \theta q_t x_{Ht}}\right]\left(\frac{\widetilde{d}_t}{1+\pi_t}\right)\beta E_tV_{\delta_D,t+1}$$
$$-\left(\frac{\widetilde{d}_t}{1+\pi_t}\right)\beta E_t\widetilde{V}_{d,t+1} + \zeta_{xt}(i_t-R_t)\left(\frac{\widetilde{d}_t}{1+\pi_t}\right)\beta E_tV_{R,t+1},$$

$$V_{Rt} = -u_{1t}c_{1t}\left(\frac{\widetilde{d}_t}{1+\pi_t}\right) + \frac{\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t}{\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t + \theta q_t x_{Ht}}\beta E_tV_{R,t+1},$$

$$V_{k_Ht} = u_{1t}c_{2t}A_{Ht}G_{1t} + \beta E_tV_{k_H,t+1}(1-\delta_H),$$

where ζ_{xt} is defined as in the main text, $\zeta_{xt} \equiv \theta q_t x_{Ht}/\left(\frac{1-\delta_{Dt}}{1+\pi_t}\widetilde{d}_t + \theta q_t x_{Ht}\right)^2$. Notice that the terms involving $\widetilde{V}_{d,t+1}$, $V_{\delta_D,t+1}$, and $V_{R,t+1}$ appear only in the first-order condition for x_{Ht}, as claimed in the main text. These terms drop out if $\theta = 0$. In this case the optimal decisions are characterized by the same conditions as in GKR, implying the same allocations as in their model (subject to the presence of a different exogenous stochastic process).

B.2 A numerical example for the mortgage loan

Here we elaborate further on the discussion in Section 3.2.1 by providing a numerical example for the evolution of mortgage installments implied by the mortgage in the model in the case of the once-and-for-all housing investment considered in that section. In particular, Figure A.1 tracks the main characteristics of the mortgage over its life and compares them with the characteristics of a standard fully-amortizing mortgage in the real world. Here, one period corresponds to one quarter, $l_0 = \$250,000$ and $i_0 = 9.28\%/4$ (the long-run average interest rate for the U.S. 30-year conventional FRM, which was used to calibrate the model). The polynomial governing the evolution of the amortization rate of the mortgage is, as in the model, δ_{Dt}^{α} with $\alpha = 0.9946$ and $\kappa = 0.00162$. Panels A and B plot mortgage installments, m_t, and outstanding debt, d_t, respectively, for 120 quarters. Panel C then plots the shares of interest payments, R_td_t, and amortization payments, $\delta_{Dt}d_t$, in mortgage installments, m_t. For the real-world mortgage, the variables are obtained from the Yahoo Mortgage Calculator. We see that the mortgage loan in the model captures two key features of the real-world mortgage. First, mortgage payments are approximately constant in the model for the first 70 or so periods (17.5 years). Second, interest payments are front-loaded: they make up most of mortgage installments at the beginning of the life of the mortgage and their share gradually declines; the opposite is true for amortization payments. If α

was equal to one, the share of interest payments in m_t would be constant and m_t would be declining over time. How close are the mortgage installments in the model to those of the real-world mortgage? By comparing the time paths in panel A one may conclude that the model approximates the real-world installments poorly, as after the 70th period the installments in the model significantly deviate from the installments in the real-world contract. This deviation, however, matters only little for the housing investment decision in period 0. This is because mortgage installments far out in the life of the mortgage are heavily discounted (by inflation and the real discount factor) and thus affect the present value cost of the mortgage—and hence the wedge—only little. A more suitable metric for comparing the two mortgages is therefore the difference between the two installment paths in present value terms (here we use $1/i$ as the discount factor), normalized by the size of the loan (i.e., \$250,000). This metric is plotted in panel D of the figure, which shows that throughout the 120 periods the approximation error is of the order of magnitude of $1e^{-4}$. The sum of the absolute values of these present-value errors is equal to about 3% of the size of the loan. For comparison, this is about the same as the typical transaction costs of obtaining a mortgage in the United States.

Figure A.2 shows the same plots as Figure A.1, but for a more complex polynomial governing the evolution of the amortization rate: $(1-\delta_{Dt})\delta_{Dt}^{\alpha_1} + \delta_{Dt}\delta_{Dt}^{\alpha_2}$, with $\alpha_1 = 0.9974$ and $\alpha_2 = 0.7463$ (κ is the same as before). This specification implies that as δ_{Dt} increases, its evolution gets relatively more governed by α_2 than by α_1. As $\alpha_2 < \alpha_1$, this means that the amortization rate increases at an increasingly faster rate as it gets closer to one. This improves the approximation. The approximation errors are plotted in panel D and (in absolute values) add up to less than one percent. For the results in the paper, however, this improvement in precision makes almost no difference.

Finally, Figure A.3 plots the same variables as Figure A.2 (with the addition of the amortization rate) but tracks them for 40 years, instead of 30 years. The figure shows that the amortization rate indeed converges to one and mortgage installments become essentially zero by the 140th period (the 35th year). Higher order polynomials, such as $(1-\delta_{Dt}-\delta_{Dt}^2)\delta_{Dt}^{\alpha_1} + (\delta_{Dt})\delta_{Dt}^{\alpha_2} + (\delta_{Dt}^2)\delta_{Dt}^{\alpha_3}$, can improve the precision even further, but again, the gains in our case are minuscule.

B.3 Computation

The equilibrium is computed by combining the linear-quadratic approximation methods of Hansen and Prescott (1995) and Benigno and Woodford (2006). Specifically, after transforming the model so that it is specified in terms of stationary variables π_t and $\widetilde{d}_t \equiv d_t/p_{t-1}$ (instead of nonstationary variables p_t and d_t), the home production function (2) and the budget constraint (9), with l_t and m_t substituted out from equations (8) and (10), are substituted in the period utility function $u(.,.)$. The utility function is then used to form a Lagrangian that has the nonlinear laws of motion (11)-(13) as constraints. This Lagrangian forms the return function in the Bellman equation to be approximated with a linear-quadratic form around a nonstochastic steady state, with the variables expressed as percentage deviations from steady state. The steps for computing equilibria of distorted linear-quadratic economies, described by Hansen and Prescott (1995), then follow; with a vector of exogenous state variables $\Omega_t = [z_t, ..., z_{t-n}]$ (where $z_t = [\log A_{Mt}, i_t, \pi_t]^\top$), a vector of endogenous state variables $\Phi_t = [s_{1t}, ..., s_{J-1,t}, k_{Mt}, k_{Ht}, \widetilde{d}_t, \delta_{Dt}, R_t]$, and a vector of decision variables $\Upsilon_t = [h_{Mt}, h_{Ht}, x_{Ht}, s_{Jt}, \widetilde{d}_{t+1}, \delta_{D,t+1}, R_{t+1}, \lambda_{1t}, \lambda_{2t}, \lambda_{3t}]$, where $\lambda_{1t}, \lambda_{2t}, \lambda_{3t}$ are Lagrange multipliers for the non-linear constraints (11)-(13).[30] The use of the Lagrangian ensures that second-order cross-derivatives of the nonlinear laws of motion (11)-(13), evaluated at steady state, appear in equilibrium decision rules (Benigno and Woodford, 2006). The usual procedure of substituting out $\widetilde{d}_{t+1}$, $\delta_{D,t+1}$, and R_{t+1} from these laws of motion into the period utility function is not feasible here as these three variables are interconnected in a way that does not allow such substitution. The Lagrangian is

$$L_t = u\left(c(c_{Mt}, c_{Ht}), 1 - h_{Mt} - h_{Ht}\right) + \lambda_{1t}\left[d_{t+1} - (1-\delta_{Dt})d_t - l_t\right]$$

$$+\lambda_{2t}\left[\delta_{D,t+1} - (1-\nu_t)\delta_{Dt}^{\alpha} - \nu_t\kappa\right] + \lambda_{3t}\left[R_{t+1} - (1-\nu_t)R_t - \nu_t i_t\right],$$

with the remaining constraints of the household's problem substituted in the consumption aggregator $c(.,.)$, as mentioned above. For our calibration, the steady-state values of the Lagrange multipliers (λ_{1t}, λ_{2t}, λ_{3t}) are positive, implying that the above specification of the Lagrangian is correct in the neighborhood of the steady state.

The Lagrange multipliers are convenient for computing the wedge, τ_{Ht}. Notice from equation (16) that the wedge depends on conditional expectations of the derivatives of the value function. The multipliers, which are obtained as an outcome of the solution method, provide a straightforward way of computing these expectations. The mapping between the multipliers and the expectations is obtained from the first-order conditions for d_{t+1}, $\delta_{D,t+1}$, and R_{t+1} in the household's problem. Forming the Bellman equation

$$V\left(z_t, ..., z_{t-n}, s_{1t}, ..., s_{J-1,t}, k_{Mt}, k_{Ht}, d_t, \delta_{Dt}, R_t\right)$$

$$= \max\left\{L_t + \beta E_t V\left(z_{t+1}, ..., z_{t-n+1}, s_{1,t+1}, ..., s_{J-1,t+1}, k_{M,t+1}, k_{H,t+1}, d_{t+1}, \delta_{D,t+1}, R_{t+1}\right)\right\},$$

the respective first-order conditions are

$$\lambda_{1t} + \lambda_{2t}\left[\frac{(1-\delta_{Dt})\delta_{Dt}^{\alpha}d_t + p_t\theta\kappa q_t x_{Ht}}{d_{t+1}^2}\right] + \lambda_{3t}\left[\frac{(1-\delta_{Dt})d_t R_t + p_t\theta i_t q_t x_{Ht}}{d_{t+1}^2}\right] + \beta E_t V_{d,t+1} = 0,$$

$$\lambda_{2t} + \beta E_t V_{\delta_D,t+1} = 0,$$

$$\lambda_{3t} + \beta E_t V_{R,t+1} = 0.$$

When the model is transformed so that it is specified in terms of π_t and $\widetilde{d}_t$, rather than p_t and d_t, the first of these conditions changes to

$$\widetilde{\lambda}_{1t} + \lambda_{2t}\left[\frac{(1-\delta_{Dt})\delta_{Dt}^{\alpha}\widetilde{d}_t}{(1+\pi_t)\widetilde{d}_{t+1}^2} + \frac{\theta\kappa q_t x_{Ht}}{\widetilde{d}_{t+1}^2}\right] + \lambda_{3t}\left[\frac{(1-\delta_{Dt})\widetilde{d}_t R_t}{(1+\pi_t)\widetilde{d}_{t+1}^2} + \frac{\theta i_t q_t x_{Ht}}{\widetilde{d}_{t+1}^2}\right] + \beta E_t \widetilde{V}_{d,t+1} = 0,$$

where $\widetilde{\lambda}_{1t} \equiv p_t\lambda_{1t}$. As we can see, the three optimality conditions give a simple mapping from the multipliers to the conditional expectations.

An alternative computational procedure would be to use log-linearization of the equi-

[30] In the version with residential time to build, the $n_{\iota t}$'s become a part of Φ_t and n_t^* becomes a part of Υ_t, but with q_t^* being its counterpart in the aggregate counterpart to Υ_t.

librium conditions around the nonstochastic steady state. This procedure yields the same decision rules as the one employed here (Benigno and Woodford, 2006). An advantage of our procedure is the convenience for computing the conditional expectations and thus the wedge.

C. VAR processes

The exogenous VAR process used in Section 5 is estimated on U.S. data for logged and linearly detrended Solow residual, the interest rate on the conventional 30-year FRM, and the CPI inflation rate. The series for the Solow residual is taken from the data accompanying Gomme and Rupert (2007). The capital stock used in their construction of the residual is the sum of structures and equipment & software (current costs deflated with the consumption deflator), which is consistent with our mapping of k_{Mt} into the data. The number of lags in the VAR is determined by the likelihood ratio test. The point estimates (ignoring the constant term) are

$$z_{t+1} = \begin{pmatrix} 0.933 & -0.543 & -0.283 \\ 0.023 & 0.953 & 0.020 \\ 0.021 & 0.431 & 0.246 \end{pmatrix} z_t + \begin{pmatrix} 0.118 & -0.070 & 0.183 \\ -0.016 & -0.134 & 0.036 \\ 0.111 & -0.249 & 0.164 \end{pmatrix} z_{t-1}$$

$$+ \begin{pmatrix} -0.147 & 0.633 & 0.117 \\ 0.036 & -0.011 & 0.043 \\ -0.084 & -0.197 & 0.187 \end{pmatrix} z_{t-2} + \begin{pmatrix} 0.0049 & 0 & 0 \\ 0.0002 & 0.0009 & 0 \\ -0.0011 & 0.0009 & 0.0026 \end{pmatrix} \epsilon_{t+1},$$

where $z_t = [\log A_{Mt}, i_t, \pi_t]^\top$ and $\epsilon_{t+1} \sim N(0, I)$. These point estimates are used to solve the model and run the computational experiments in Sections 5 and 6. Note that as in our computational experiments we are interested only in unconditional moments, the ordering of the variables in the VAR is irrelevant.

In some experiments in Section 6, the FRM interest rate is replaced with the 3-month T-bill yield and the ARM rate. The point estimates are, respectively,

$$z_{t+1} = \begin{pmatrix} 0.912 & -1.491 & -0.164 \\ 0.049 & 1.449 & 0.030 \\ 0.076 & 0.719 & 0.255 \end{pmatrix} z_t + \begin{pmatrix} 0.063 & 2.124 & 0.217 \\ -0.046 & -0.412 & -0.014 \\ 0.101 & -0.777 & 0.158 \end{pmatrix} z_{t-1}$$

$$+ \begin{pmatrix} -0.295 & -2.329 & 0.055 \\ -0.003 & -0.039 & -0.029 \\ -0.143 & 0.431 & 0.204 \end{pmatrix} z_{t-2} + \begin{pmatrix} 0.311 & 1.294 & 0.130 \\ 0.020 & -0.048 & -0.019 \\ -0.016 & -0.179 & -0.175 \end{pmatrix} z_{t-3}$$

$$+ \begin{pmatrix} 0.0044 & 0 & 0 \\ 0.0001 & 0.0008 & 0 \\ -0.0010 & 0.0007 & 0.0026 \end{pmatrix} \epsilon_{t+1}$$

and

$$z_{t+1} = \begin{pmatrix} 0.885 & -1.173 & -0.284 \\ 0.025 & 1.214 & 0.026 \\ 0.048 & 0.402 & 0.267 \end{pmatrix} z_t + \begin{pmatrix} 0.170 & 0.033 & 0.200 \\ -0.039 & -0.219 & 0.024 \\ 0.084 & -0.074 & 0.151 \end{pmatrix} z_{t-1}$$

$$
+\begin{pmatrix} -0.121 & 1.006 & 0.198 \\ 0.033 & -0.109 & -0.011 \\ -0.099 & -0.275 & 0.153 \end{pmatrix} z_{t-2} + \begin{pmatrix} 0.0047 & 0 & 0 \\ 0.0001 & 0.0007 & 0 \\ -0.0011 & 0.0006 & 0.0027 \end{pmatrix} \epsilon_{t+1}.
$$

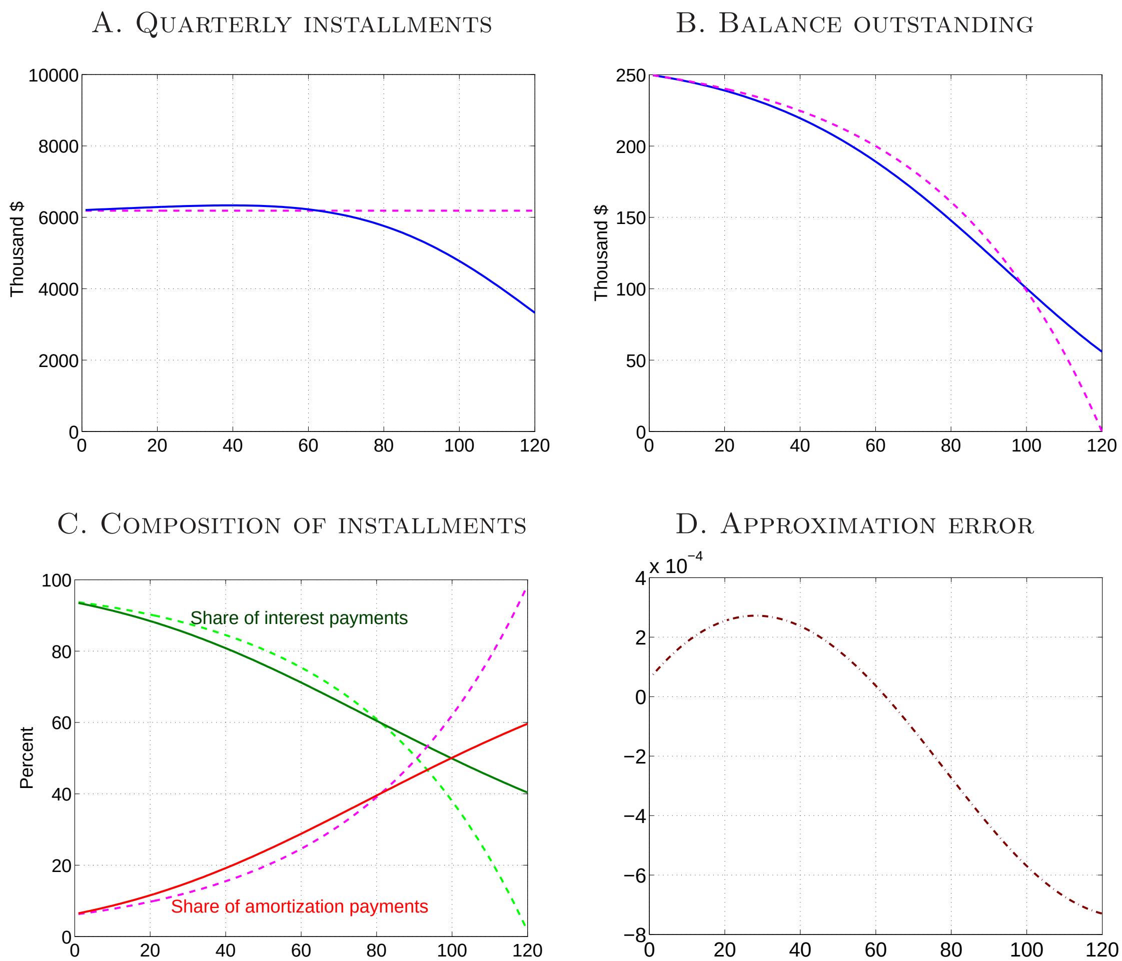

Figure A.1: Mortgage loan: model ($\kappa = 0.00162$ and $\alpha = 0.9946$) v.s. real-world mortgage (Yahoo mortgage calculator). Solid line=model, dashed line=real-world mortgage. Here, $l_0 = \$250,000$ and $i = 9.28\%/4$. The approximation error is expressed as the present value (using $1/i$) of the difference between the installments in the model and in the mortgage calculator, divided by the size of the loan.

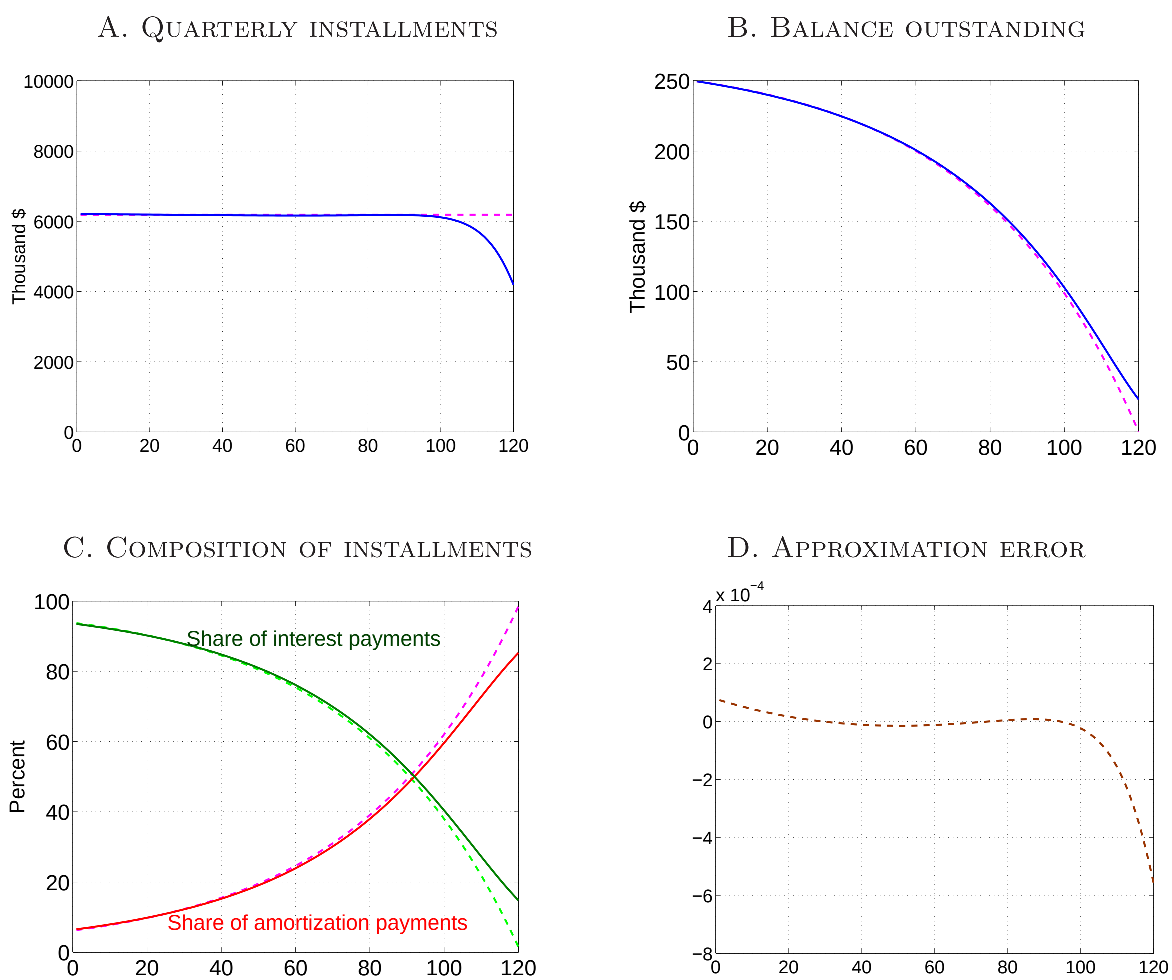

Figure A.2: Mortgage loan: model ($\kappa = 0.00162$, $\alpha_1 = 0.9974$, and $\alpha_2 = 0.7463$) v.s. real-world mortgage (Yahoo mortgage calculator). Solid line=model, dashed line=real-world mortgage. Here, $l_0 = \$250,000$ and $i = 9.28\%/4$. The approximation error is expressed as the present value (using $1/i$) of the difference between the installments in the model and in the mortgage calculator, divided by the size of the loan.

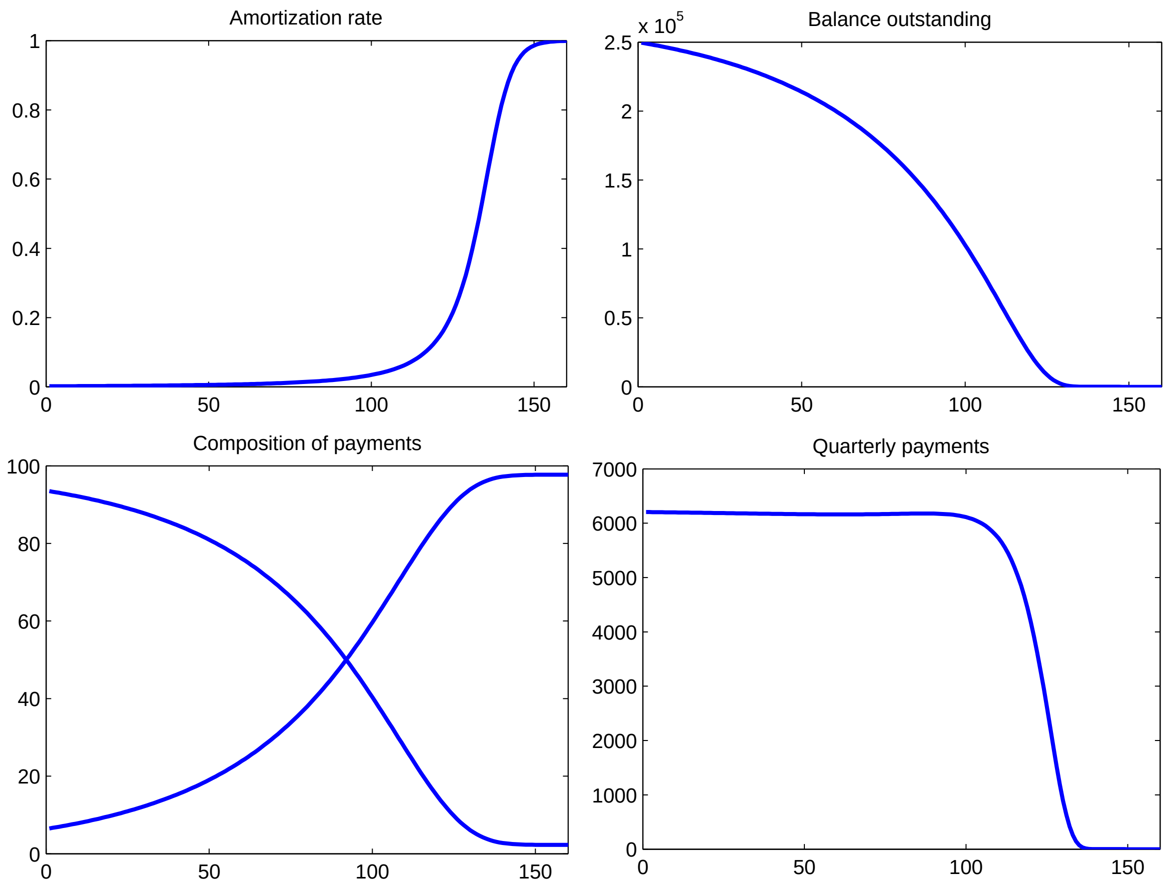

Figure A.3: Mortgage loan with $\kappa = 0.00162$, $\alpha_1 = 0.9974$, and $\alpha_2 = 0.7463$, payments over 40 years.

D. Loan-to-value ratio

This appendix elaborates further on the motivation for the assumption that the loan-to-value ratio in the model is treated as a parameter. Figures A.4 and A.5 show that the loan-to-value ratio in the data stayed relatively constant, despite large changes in the nominal interest rate. It needs to be stressed, however, that the data are for first mortgages—the concept of mortgage loans in the model—and thus the graph does not capture the rise in the importance of second mortgages and home equity loans to boost leverage in the period 2002-2007 and their subsequent decline (see Favilukis et al., 2015).

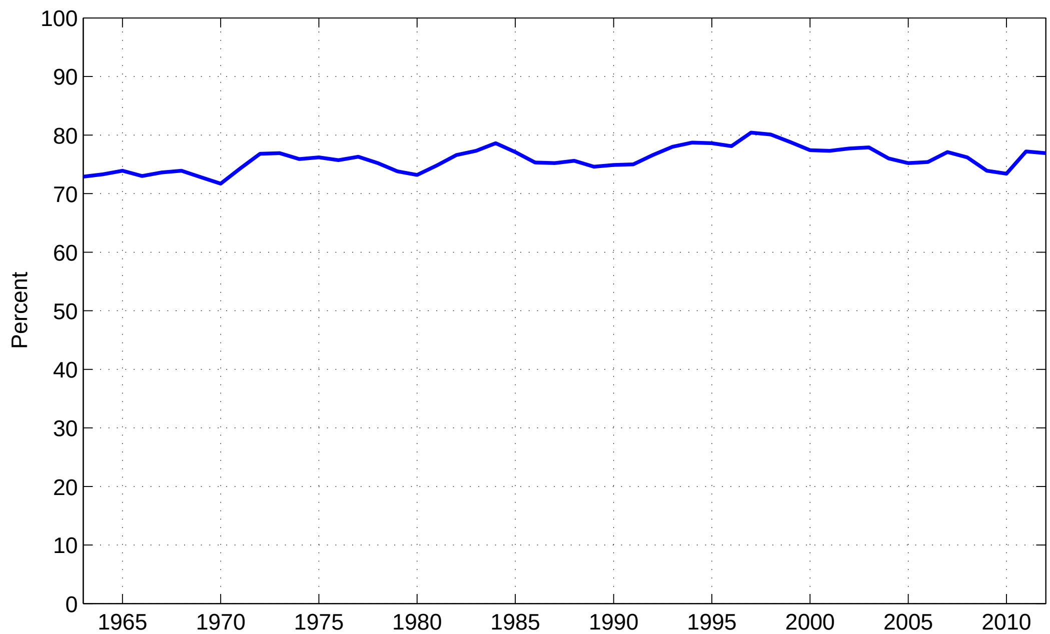

Figure A.4: Data equivalent to θ. The cross-sectional mean of the loan-to-value ratio for conventional single family newly-built home mortgages. Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10.

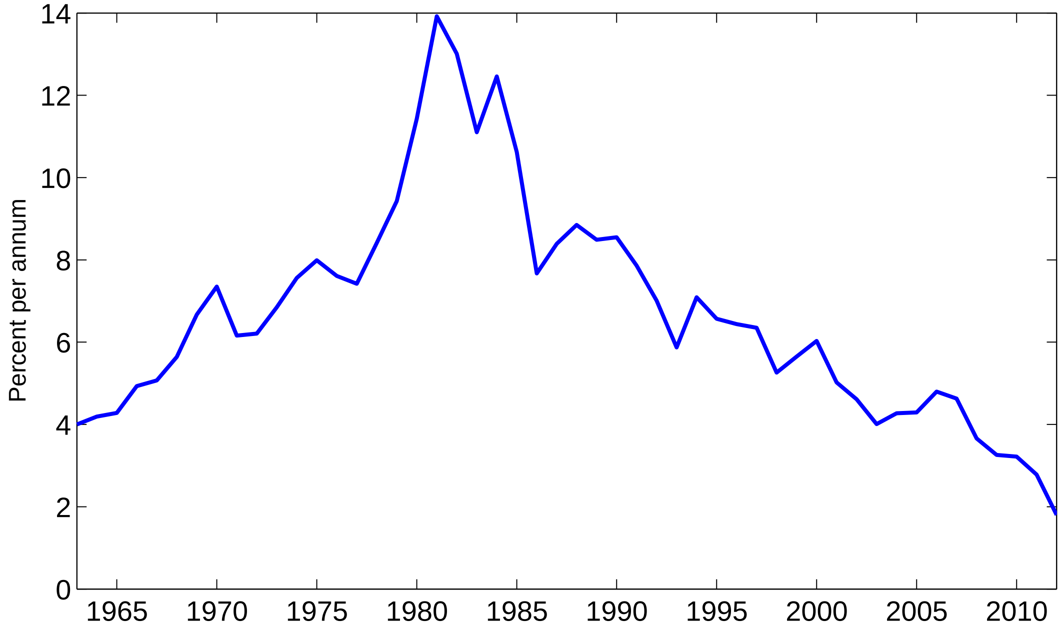

Figure A.5: 10-year U.S. nominal interest rate.

Nevertheless, in order to assess the quantitative impact of the observed small variation in the loan-to-value ratio in Figure A.4, we make θ in the model stochastic, following a four-variable VAR process with TFP, the FRM rate, and the inflation rate; $z_t = [\log A_{Mt}, i_t^{FRM}, \pi_t, \theta_t]^\top$. Below we report the estimated process (1984.Q1-2006.Q4) and present the results in Table A.1. Panel (a) repeats the results from the baseline case discussed in the main text. In the case of stochastic θ, case (b), σ is calibrated so as to match the relative volatility of x_{Ht}, yielding $\sigma = 6.3$. The value of σ is held fixed across cases (b) and (c). In case (c), θ is again treated as a parameter, as in case (a), but the four-variable stochastic process is the same as in case (b). This ensures that only the direct distortionary effect of θ on the first-order condition for housing, not the underlying probability space, changes across experiments (b) and (c). As can be seen from the table, the effect of the small variation in θ observed in Figure A.4 (standard deviation of one percentage point) on the model lead-lag properties is also small. Again, one needs to bear in mind, when interpreting these results in the context of the literature on the recent housing boom and bust, that both the data and our model speak to first mortgages for housing purchases only.

$$z_{t+1} = \begin{pmatrix} 0.9616 & -0.7278 & -0.1613 & 0.0838 \\ 0.0235 & 0.9625 & 0.0399 & 0.0055 \\ -0.0018 & 0.5976 & 0.1399 & -0.0412 \\ -0.1961 & -0.2529 & -0.0135 & 0.8132 \end{pmatrix} z_t + \begin{pmatrix} 0.1025 & 0.2999 & 0.1793 & -0.0752 \\ -0.0157 & -0.0896 & 0.0324 & 0.0001 \\ 0.1007 & -0.0719 & 0.0602 & 0.0319 \\ -0.4444 & 0.1773 & -0.9597 & -0.0847 \end{pmatrix} z_{t-1}$$

$$
+\begin{pmatrix} -0.2350 & -0.9941 & 0.1643 & -0.0943 \\ 0.0023 & 0.0017 & 0.0437 & -0.0211 \\ -0.1456 & 0.3151 & 0.0546 & 0.0045 \\ 0.8494 & 0.3525 & 0.2262 & 0.1961 \end{pmatrix} z_{t-2} + \begin{pmatrix} 0.1087 & 1.2912 & 0.0575 & 0.1058 \\ 0.0196 & -0.0373 & -0.0575 & 0.0153 \\ 0.0515 & -0.6561 & -0.2156 & -0.0765 \\ -0.2995 & -0.4527 & 0.1948 & -0.1108 \end{pmatrix} z_{t-3}
$$

$$
+\begin{pmatrix} 0.0045 & 0 & 0 & 0 \\ 0.0002 & 0.0009 & 0 & 0 \\ -0.0010 & 0.0008 & 0.0024 & 0 \\ 0.0004 & -0.0003 & 0.0003 & 0.0070 \end{pmatrix} \epsilon_{t+1}, \qquad \epsilon_{t+1} \sim N(0, I)
$$

Table A.1: The effects of stochastic θ

	Rel.	Correlations of y in period t with variable υ in period $t+j$:							
υ_{t+j}	st.dev.	j = -4	-3	-2	-1	0	1	2	3	4
(a) Baseline model; $\sigma = 6.4$; three-variable VAR										
x	4.42	0.07	0.29	0.56	0.78	**0.93**	0.71	0.43	0.10	-0.18
x_H	8.45	0.19	0.34	0.50	0.55	**0.51**	0.31	0.11	-0.13	-0.32
x_M	4.33	-0.12	0.03	0.25	0.50	**0.78**	0.70	0.52	0.31	0.12
τ_H	3.26	-0.21	-0.33	-0.43	-0.43	**-0.32**	-0.17	-0.02	0.18	0.34
(b) Stochastic θ; $\sigma = 6.3$; four-variable VAR										
x	4.65	0.11	0.30	0.56	0.77	**0.95**	0.73	0.46	0.14	-0.10
x_H	8.45	0.27	0.48	0.59	0.64	**0.56**	0.41	0.24	0.01	-0.22
x_M	4.54	-0.14	-0.10	0.16	0.43	**0.76**	0.62	0.42	0.20	0.11
τ_H	3.13	-0.28	-0.49	-0.56	-0.54	**-0.44**	-0.32	-0.18	0.03	0.25
(c) No time-varying distortionary effects of θ; $\sigma = 6.3$; four-variable VAR										
x	4.66	0.11	0.31	0.57	0.77	**0.95**	0.73	0.47	0.14	-0.10
x_H	8.53	0.27	0.49	0.61	0.63	**0.58**	0.41	0.23	-0.01	-0.23
x_M	4.55	-0.15	-0.10	0.16	0.43	**0.76**	0.63	0.43	0.22	0.12
τ_H	3.15	-0.29	-0.50	-0.56	-0.56	**-0.45**	-0.32	-0.17	0.03	0.26

E. Alternative amortization schedules

This appendix studies the sensitivity of the results to alternative amortization schedules. In particular, we consider nominal fixed-rate and adjustable-rate loans with constant amortization schedules, employed (in the case of adjustable-rate loans set in real terms) by Campbell and Hercowitz (2006) and Garriga et al. (2014). This results as a special case when in the law of motion for δ_{Dt}, $\alpha = 1$ and the economy is started off from the steady state. Then, $\delta_{D,t+1} = \delta_{Dt} = \kappa \ \forall t$.

Tables A.2 and A.3 report the results, with panel (a) repeating the results from the baseline case in the main text. We consider different parameterizations of κ, translating in the tables the values of κ to the half-life of the loan, in terms of quarters and years; half-life T is defined as $0.5d_0 = (1-\kappa)^T d_0$. As in the main text, σ is recalibrated so as to keep the volatility of x_{Ht} constant across experiments.

Recall that in the baseline case, $\alpha = 0.9946$ and $\kappa = 0.00162$, resulting in nominal installments approximating those of mortgage loans. These values give a steady-state amortization rate equal to 0.0144. We therefore start in panel (b) with $\kappa = 0.0144$ and keep reducing the half-life (increasing κ). As should be expected from our discussion in Section 4.2, the strength of the transmission from nominal interest rates to the wedge declines as κ increases, resulting in a weakening of the incentive that makes x_{Ht} to lead in the model.

Table A.2: Alternative amortization schedules, FRM

	Rel.	Correlations of y in period t with variable υ in period $t+j$:								
υ_{t+j}	st.dev.	j = -4	-3	-2	-1	0	1	2	3	4
Approximate 30-yr mortgage										
(a) $\alpha = 0.9946$ and $\kappa = 0.00162$ ($\sigma = 6.4$)										
x_H	8.45	0.19	0.34	0.50	0.55	**0.51**	0.31	0.11	-0.13	-0.32
Constant amortization rates										
(b) $\alpha = 1$ and $\kappa = 0.0144 \Rightarrow$ half life = 48qtrs = 12yrs ($\sigma = 6.2$)										
x_H	8.45	0.17	0.33	0.48	0.56	**0.54**	0.34	0.14	-0.11	-0.30
(c) $\alpha = 1$ and $\kappa = 0.034 \Rightarrow$ half life = 20qtrs = 5yrs ($\sigma = 3.9$)										
x_H	8.45	0.15	0.31	0.48	0.56	**0.57**	0.35	0.14	-0.11	-0.31
(d) $\alpha = 1$ and $\kappa = 0.083 \Rightarrow$ half life = 8qtrs = 2yrs ($\sigma = 2.03$)										
x_H	8.45	0.11	0.29	0.49	0.61	**0.66**	0.44	0.21	-0.06	-0.27
(e) $\alpha = 1$ and $\kappa = 0.159 \Rightarrow$ half life = 4qtrs = 1yr ($\sigma = 1.02$)										
x_H	8.45	0.09	0.26	0.48	0.63	**0.73**	0.52	0.29	0.02	-0.20

Notes: In each case, σ is recalibrated so as to keep $\text{std}(x_{Ht})/\text{std}(y_t)$ constant.

Table A.3: Alternative amortization schedules, ARM

υ_{t+j}	Rel. st.dev.	Correlations of y in period t with variable υ in period $t+j$: j = -4	-3	-2	-1	0	1	2	3	4
APPROXIMATE 30-YR MORTGAGE										
(a) $\alpha = 0.9946$ and $\kappa = 0.00162$ ($\sigma = 0.96$)										
x_H	8.45	0.33	0.42	0.45	0.38	**0.24**	0.03	-0.17	-0.29	-0.36
CONSTANT AMORTIZATION RATES										
(b) $\alpha = 1$ and $\kappa = 0.0144 \Rightarrow$ half life = 48qtrs = 12yrs ($\sigma = 0.9$)										
x_H	8.45	0.33	0.44	0.50	0.46	**0.35**	0.12	-0.10	-0.25	-0.33
(c) $\alpha = 1$ and $\kappa = 0.034 \Rightarrow$ half life = 20qtrs = 5yrs ($\sigma = 0.865$)										
x_H	8.45	0.33	0.45	0.54	0.52	**0.44**	0.19	-0.05	-0.22	-0.32
(d) $\alpha = 1$ and $\kappa = 0.083 \Rightarrow$ half life = 8qtrs = 2yrs ($\sigma = 0.75$)										
x_H	8.45	0.27	0.42	0.55	0.60	**0.60**	0.32	0.06	-0.13	-0.26
(e) $\alpha = 1$ and $\kappa = 0.159 \Rightarrow$ half life = 4qtrs = 1yr ($\sigma = 0.72$)										
x_H	8.45	0.27	0.43	0.59	0.67	**0.72**	0.46	0.18	-0.04	-0.20

Notes: In each case, σ is recalibrated so as to keep $\mathrm{std}(x_{Ht})/\,\mathrm{std}(y_t)$ constant.

F. Refinancing

Finally, we check the sensitivity of the results to refinancing. Refinancing is introduced into the model in a simple way that preserves the applicability of solving the model by linear-quadratic approximation. In particular, it is assumed that a fraction ζ_t of the outstanding debt can be refinanced each period. The representative household chooses ζ_t each period, subject to a quadratic cost function in terms of time. Leisure in the utility function is thus given by $1 - h_{Mt} - h_{Ht} - h_{Ft}$, where $h_{Ft} = \varpi(\zeta_t - \zeta)^2$. Here, $\varpi > 0$ is a parameter and ζ is the fraction of debt that is refinanced in steady state. We specify the refinancing costs in terms of time so that the definition of market output is unaffected by the introduction of refinancing into the model. With refinancing, the laws of motion for the mortgage variables are as follows. First, debt evolves as

$$d_{t+1} = (1 - \zeta_t)(1 - \delta_{Dt})d + l_t.$$

Here, the implicit assumption is that the fraction of debt that is refinanced applies to all vintages of debt equally and that refinancing occurs after the current-period mortgage payments (and thus also amortization payments) have been made. New loans then consist of mortgages used for new house purchases and loans that are being refinanced

$$l_t = \theta p_t q_t x_{Ht} + \zeta_t(1 - \delta_{Dt})d_t.$$

Combining these two equations gives back the original law of motion for debt

$$d_{t+1} = (1 - \delta_{Dt})d + \theta p_t q_t x_{Ht}.$$

Second, the law of motion for the amortization rate stays the same

$$\delta_{D,t+1} = (1 - \nu_t)f(\delta_{Dt}) + \nu_t \kappa,$$

where ν_t is defined as before

$$\nu_t \equiv \frac{\theta p_t q_t x_{Ht}}{d_{t+1}}.$$

This implicitly assumes that debt that is being refinanced has the initial amortization rate the same as what would be applied to it if it wasn't refinanced (this captures the notion that, for instance, a loan that is refinanced 10 years before maturity is replaced with a 10-year loan, rather than a loan of the full length of 30 years).[31]

And third, the law of motion for the interest rate is

$$R_{t+1} = (1 - \vartheta_t)R_t + \vartheta_t i_t.$$

Notice that in our context it is sensible to talk about refinancing only in the case of FRMs.

[31] We have also carried out an experiment with the other case, which is that refinanced loans start with the amortization rate κ. This did not turn out to play a big role. The assumption we work with here has the advantage that, as explained below, it gives refinancing a very transparent role.

Here,

$$\vartheta_t \equiv \frac{l_t}{d_{t+1}}.$$

Notice that $\nu_t \neq \vartheta_t$, unless $\zeta_t = 0$.

Mortgage payments are given as before, $m_t = (R_t + \delta_{Dt})d_t$, and the form of the household's budget constraint stays the same. All other elements of the economy also stay the same. Thus, what refinancing does in the model is to change the weights on the old effective interest rate and the current market interest rate in calculating the effective interest rate on the new stock of debt, without tieing this change in weights to new housing investment.

We base the calibration of the refinancing parameters on Freddie Mac's Weekly Primary Mortgage Market Survey for the period 1987-2006. The survey reports the refinance share of first lien loan applications (weighted by dollar amounts). In our model, this corresponds to $\zeta_t(1 - \delta_{Dt})d_t/l_t$. On average, the refinance share in the data has been 39%. We therefore choose ζ_t in steady state so that the model is consistent with this observation. The cost parameter ϖ is then calibrated by matching the standard deviation of the refinance share (percentage point deviations from HP-filtered trend), relative to that of HP-filtered log output, equal to 13.9. This yields $\varpi = 2.05$. The cyclical properties of the model with refinancing are reported in Table A.4. As the table shows, the cyclical lead in residential investment is, qualitatively, unaffected by the presence of refinancing, but the correlations are weaker. Essentially, the added flexibility to respond to interest rate changes weakens the mechanism that transmits nominal interest rates into real housing costs. But for the calibration based on the Survey data, this is not sufficient to change the main result.

Table A.4: The effect of refinancing, FRM

	Rel.	Correlations of y in period t with variable υ in period $t+j$:								
υ_{t+j}	st.dev.	j = -4	-3	-2	-1	0	1	2	3	4
Baseline, no refinancing										
x_H	8.45	0.19	0.34	0.50	0.55	**0.51**	0.31	0.11	-0.13	-0.32
Refinancing										
x_H	8.45	0.18	0.32	0.46	0.51	**0.47**	0.28	0.09	-0.14	-0.32