

# Mortgages and Monetary Policy\*

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## Abstract

Mortgages are long-term nominal loans. Under incomplete asset markets, monetary policy is shown to affect housing investment and the economy through the cost of new mortgage borrowing and the value of payments on outstanding debt. These channels, distinct from traditional transmission of monetary policy, are evaluated within a general equilibrium model. Persistent monetary policy shocks, resembling the level factor in the nominal yield curve, have larger effects than transitory shocks, manifesting themselves as long-short spread. The transmission is stronger under adjustable- than fixed-rate mortgages. Higher, persistent, inflation benefits homeowners under FRMs, but hurts them under ARMs.

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# 1 Introduction

Mortgage is the main financial liability for most households. Furthermore, it is a long-term liability set in nominal terms. To be precise, standard mortgage loans require homeowners to make nominal installments—regular interest and amortization payments—during the life of the loan. The installments are set so as to guarantee that, given the mortgage interest rate, the principal is gradually repaid in full by the end of the mortgage term, typically 20 to 30 years. A standard fixed-rate mortgage (FRM), characteristic for instance for the United States, has a fixed nominal interest rate and constant nominal installments, set at origination, for the entire term of the loan. An adjustable-rate mortgage (ARM), typical for instance for the United Kingdom, sets nominal installments on a period-by-period basis so as, given the current short-term nominal interest rate, the loan is expected to be repaid in full during its remaining term (the mortgage interest rate is usually linked to a short-term government bond yield). Most mortgage loans are variants of these two basic contracts.<sup>1</sup>

While the long-term nominal nature of mortgages has been studied at the level of an individual household (e.g., Campbell and Cocco, 2003), little is known about its implications for the aggregate economy in general and the transmission of monetary policy in particular (see Campbell, 2012). The typical macro model used for monetary policy analysis features nominal rigidities in the form of prices set for a number of periods ahead, but it ignores the potential role of the nominal rigidity built into mortgage contracts. This paper aims to provide a step towards understanding this connection, focusing not only on residential investment and the aggregate economy, but also on the redistribution of income between homeowners and mortgage investors. By the nature of our question, these effects are studied

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<sup>1</sup>Most countries have typically one of the two types dominating. In the United States, FRMs account on average (1982-2006) for 70% of mortgage originations (Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10); before 1982, they were essentially the only contract available. Other countries in which FRMs—with interest rates fixed for at least 10 years—have traditionally dominated the mortgage market include Belgium, Denmark, and France; in most other advanced economies, either ARMs or FRMs with interest rates fixed for less than 5 years prevail; see Scanlon and Whitehead (2004) and European Mortgage Federation (2012a). Countries also differ in terms of prepayment penalties, costs of refinancing, recourse, and other details of the contracts. Research is still inconclusive on the causes of the cross-country heterogeneity (e.g., Miles, 2004; Green and Wachter, 2005; Campbell, 2012). We take the form of the contracts as given and abstract from many institutional details.

in general equilibrium (the contracts are taken as given but all prices are endogenous). To provide a clear characterization of the transmission mechanism, the paper abstracts from all other nominal frictions, as well as other channels through which housing finance affects the macroeconomy.<sup>2</sup>

Basic facts suggest that the role of mortgage finance in the transmission mechanism should not be ignored. Mortgage payments (interest and amortization) as a fraction of income—the so called ‘debt-servicing costs’—are nontrivial. Our estimates suggest that, on average over the past 30-40 years, they were equivalent to 15-22% of the pre-tax income of the 3rd and 4th quintiles of the U.S. wealth distribution, representing the typical homeowner (Campbell and Cocco, 2003). Hancock and Wood (2004) report that in the United Kingdom this ratio fluctuated between 15% and 20% over the period 1991-2001. In Germany, mortgage debt servicing costs are reported to be around 27% of disposable income (European Mortgage Federation, 2012b), in Denmark 36.5% (for first-time homeowners; European Mortgage Federation, 2012c), and in France 30% (for first-time homeowners; European Mortgage Federation, 2009). In terms of mortgage debt, the mortgage debt to (annual) GDP ratio in advanced economies has reached on average 70% in 2009.<sup>3</sup> In some countries outstanding mortgage debt is even larger than government debt and its maturity is longer.<sup>4</sup>

We show that, under incomplete asset markets, the nominal rigidity inherent in mortgage contracts leads to two channels of monetary policy transmission. One channel works through new borrowing (a price effect) and is *qualitatively the same* under both FRMs and ARMs. The other channel works through outstanding mortgage debt (current and expected future wealth effects) and is even *qualitatively different* under the two contracts. Both channels

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<sup>2</sup>For instance, following Iacoviello (2005), a large literature focuses on the role of housing as a collateral facilitating borrowing for consumption purposes. In this literature, loans are one-period loans and monetary policy has real effects due to nominal price rigidities, which allow the monetary authority to affect the ex-ante short-term real interest rate. We abstract from this channel.

<sup>3</sup>There is, however, substantial cross-country variation in this ratio, from 22% in Italy to 105% in the Netherlands; International Monetary Fund (2011), Chapter 3.

<sup>4</sup>Hilscher, Raviv, and Reis (2014) report that in the United States government debt is predominantly of short-term maturity.

are distinct from the standard monetary transmission through real interest rates—in our framework, monetary policy has real effects even if it has no direct effect on the ex-ante short-term real interest rate.<sup>5</sup>

Regarding the first channel: under incomplete asset markets, households value future cash flows differently than mortgage investors, for whom (in the absence of arbitrage opportunities) the present value of a new one-dollar loan is one dollar. As a result, even in the absence of movements in the ex-ante real interest rate, changes in the expected path of future short-term nominal interest rates, and thus inflation, affect the expected distribution of debt-servicing costs over the term of a new loan and hence the present value of its mortgage payments from the household’s perspective. We refer to this channel as the ‘price effect’, as it constitutes a part of the effective price of new housing. It is a dynamic version of the tilt/frontloading effect (e.g., Schwab, 1982) and, qualitatively, it is similar across FRMs and ARMs.

In the second channel, monetary policy affects the current and expected future real payments on outstanding mortgage debt (i.e., on loans that have already been taken out) and thus disposable income. We refer to these effects as current and future ‘wealth effects’ and the nature of the contract is critical for their direction. In the case of FRM, only the inflation rate affects real mortgage payments. Higher inflation reduces the real value of the nominal payments homeowners have to make and the strength of this effect increases with inflation persistence—persistently higher inflation deflates the payments more and more, producing substantial positive wealth effects for the homeowner towards the end of the life of the loan (the effects are *back-loaded*). In the case of ARM, the short-term nominal interest rate matters in addition to the inflation rate. The immediate effect of a persistent increase in the two rates is an increase in the real value of interest payments—the effect of a higher nominal interest rate dominates the counterbalancing effect of higher inflation. As a result, real mortgage payments increase immediately (the effect is *front-loaded*) as if

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<sup>5</sup>In our model, the real interest rate responds to monetary policy only indirectly through general equilibrium effects.

monetary policy had control over the short-term real interest rate. Over time, the cumulated effect of persistently high inflation becomes stronger, gradually reducing the real value of the payments over the life of the loan. Because most of the wealth effects are front-loaded under ARM but back-loaded under FRM, they have larger effect under ARM than under FRM when households discount the future (and future income cannot be brought fully forward through other financial instruments). The form of the mortgage contract is thus critical for both the sign and size of the redistributive effects of inflation (the size depends, of course, also on the size of outstanding mortgage debt).

These channels are illustrated analytically in a simple partial equilibrium model of a homeowner, before being embedded in a stylized general equilibrium model. The model has incomplete asset markets and a continuum of two agent types, homeowners and capital owners/investors. There is a representative agent of each type. Such a split of the population is motivated by Campbell and Cocco (2003): the typical homeowner comes from the 3rd and 4th quintiles of the wealth distribution, while the 5th quintile represents capital owners. As such, homeowners derive income from labor, which they supply elastically, and invest in housing capital, financing a given fraction of housing investment with mortgages. Capital owners do not work and invest in capital used in production, one-period nominal bonds, and mortgages, pricing the assets competitively by arbitrage (in an alternative version of the model homeowners can also access the noncontingent one-period bond market).<sup>6</sup> The production side of the economy is standard, consisting of perfectly competitive producers and homebuilders. Monetary policy is characterized by an interest rate rule. As the two agent types do not trade a full set of state-contingent securities, their stochastic discount factors are not equalized in every state. No-arbitrage pricing implies that capital owners are indifferent across the three assets and new mortgage lending is thus determined by housing demand, affected by monetary policy, through the channels described above. Market incompleteness

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<sup>6</sup>In the data, the 3rd and 4th quintiles have one major asset, a house, and one major liability, a mortgage. Their main source of income is labor income. In contrast, the 5th quintile hold almost the entire corporate equity in the economy and housing is a less important component of their asset composition; labor income is also a less important source of their income. The 1st and 2nd quintiles are essentially renters with no assets and little liabilities and are not included in the model.

in this model is a necessary, but not sufficient, condition for monetary policy to have real effects. Even with market incompleteness, monetary policy is neutral when housing finance takes the form of one-period nominal loans or long-term mortgages denominated in real terms (i.e., inflation indexed).<sup>7</sup>

Three key properties of the mortgage transmission mechanism, suggested by the partial equilibrium model, are confirmed by the general equilibrium model with all prices endogenously determined (in particular, the real interest rate). First, the size of the real effects of monetary policy shocks increases with the persistence of the shocks: persistent monetary policy shocks that manifest themselves as the level factor in the nominal yield curve (they shift both short and long rates, as well as inflation) have a larger impact than transitory shocks, manifesting themselves as the long-short spread. This is because homeowners care about the entire path of these variables over the term of the loan. Second, monetary policy shocks have larger real effects under ARM than FRM. Broadly speaking, this is because the price and wealth effects reinforce each other under ARM, but tend to offset each other under FRM. And third, higher inflation redistributes income from capital owners to homeowners under FRM, but (at least initially) from homeowners to capital owners under ARM. The direction of the redistribute effects thus crucially depends on the form of mortgage debt, something that existing literature, reviewed in the next section, does not consider. For all three results, the real effects are stronger the more difficult it is for homeowners to smooth consumption over time through other financial instruments. A recent micro-level empirical study by Di Maggio, Kermani, and Ramcharan (2014) is consistent with some of these predictions. Specifically, in the data, U.S. homeowners respond more to interest rate changes in counties with a larger fraction of ARMs than FRMs and households that are more financially constraint respond more.

The paper proceeds as follows. Section 2 relates the paper to a broader literature. Section 3 explains the nature of the nominal rigidity. Section 4 describes the general equilibrium

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<sup>7</sup>More precisely, in the case of one-period nominal loans, monetary policy is ‘almost neutral’—it has real effects only due to current-period inflation surprises and these effects are tiny.

model. Section 5 explains how the model can be mapped into data. Section 6 reports the findings. Section 7 concludes and offers suggestions for future research. A supplemental material contains (i) a list of the model’s equilibrium conditions, (ii) computation, (iii) description of the data counterparts to the variables in the model, (iv) estimates of mortgage debt servicing costs for the United States, (v) sensitivity analysis, (vi) the model’s business cycle properties.

## 2 Related studies

The paper is related to different strands of the literature. First, following Iacoviello (2005), a number of studies focus on the interaction between sticky prices and the collateral value of housing, whereby housing facilitates borrowing for general consumption purposes, similar to home equity lines of credit (Iacoviello, 2010, contains a brief summary of this line of research). Loans in these models are one-period loans and, due to sticky prices, monetary policy transmits to the real economy by affecting their ex-ante real interest rate. The presence of borrowing constraints and housing as a collateral then further amplifies the real rate mechanism.<sup>8</sup> We abstract from these channels, focusing instead on the role of mortgages as loans for house purchase (i.e., first mortgages, as opposed to home equity lines of credit) and stressing their long-term nominal nature.<sup>9</sup> Both our paper and the above literature abstract from the possibility that monetary policy affects term premia, mortgage markups, or the primary-secondary spread.<sup>10</sup>

Second, recent studies investigate the redistributive effects of monetary policy in economies

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<sup>8</sup>Rubio (2011) extends the Iacoviello (2005) framework by considering one-period loans with interest rates evolving in a sluggish manner, as a weighted average of past interest rates. Graham and Wright (2007) study a similar setup, albeit without housing. Calza, Monacelli, and Stracca (2013) distinguish between ARMs, modeled as one-period loans, and FRMs, modeled as two-period loans.

<sup>9</sup>Transmission into housing demand through the standard real interest rate channel is also studied outside of the Iacoviello (2005)-framework; see Edge (2000), Li and Chang (2004) and Dressler and Li (2009). Ghent (2012) considers multi-period FRMs denominated in real, rather than nominal, terms. The real effects of monetary policy in her model are small as the long-term real interest rate responds only little to monetary policy shocks.

<sup>10</sup>Koijen, Van Hemert, and Van Nieuwerburgh (2009) argue that term premia are a key determinant of mortgage choice by U.S. households over the business cycle.

with incomplete asset markets and nominal debt contracts (Doepke and Schneider, 2006; Meh, Rios-Rull, and Terajima, 2010; Sheedy, 2013; Doepke, Schneider, and Selezneva, 2015). In these models, higher inflation benefits borrowers. We show that, in the case of mortgage debt, the redistributive consequences critically depend, both qualitatively and quantitatively, on whether mortgages are ARMs or FRMs, something the literature does not consider.<sup>11</sup>

Third, following Campbell and Cocco (2003), mortgages (or other long-term loans) have been considered in relation to mortgage choice (Koiijen et al., 2009), homeownership rates (Chambers, Garriga, and Schlagenhaut, 2009a,b) and default (Garriga and Schlagenhaut, 2009; Chatterjee and Eyigungor, 2011; Corbae and Quintin, 2011; Campbell and Cocco, 2015). The focus of these studies on steady-state equilibria or dynamics with exogenous price processes allows the inclusion of various details of real-world mortgage contracts that our model abstracts from.<sup>12</sup> Mortgages also play a key role in accounting for the business cycle behavior of housing construction in Kydland, Rupert, and Sustek (forthcoming). Their model requires an estimated reduced-form process for short- and long-term nominal interest rates, inflation, and TFP and thus does not allow the structural analysis carried out here.

Fourth, the relationship between monetary policy and housing has been studied empirically in various regression models (Kearl, Rosen, and Swan, 1975; Kearl, 1979) and structural VARs (e.g., Bernanke and Gertler, 1995; Iacoviello and Minetti, 2008; Calza et al., 2013). In our model, the more important monetary policy shocks manifest themselves as the level factor in the nominal yield curve. As such, they are not comparable to the standard monetary policy shocks in this literature. To mention just one difference, identification issues aside, the level factor is highly positively correlated with inflation, whereas the conventional response of inflation to a positive monetary policy shock in structural VARs is negative. Atkeson and Kehoe (2009) make a case for a greater focus of research on level factor shocks in understanding monetary policy. Our model provides a mechanism for transmitting such

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<sup>11</sup>Auclert (2014) also studies a redistributive channel of monetary policy, but focusing on real interest rate effects. Redistributive effects of monetary policy are also at the heart of the transmission mechanism proposed by Sterk and Tenreyro (2013).

<sup>12</sup>Favilukis, Ludvigson, and Van Nieuwerburgh (2011) consider a number of details at the micro level in a general equilibrium model with aggregate shocks, but mortgages are modeled as one-period loans.



shocks to the real economy.

Finally, a number of earlier studies investigate theoretically the effect of inflation on the housing market in the context of mortgage contract design (Lessard and Modigliani, 1975), a supply-demand econometric model (Kearl, 1979), and a consumer's optimal housing choice under different steady-state inflation rates (Schwab, 1982; Alm and Follain, 1984).<sup>13</sup> More recently, Brunnermeier and Julliard (2008) argue that inflation affects housing through money illusion, whereby households make home purchase decisions while ignoring the effects of inflation on future real mortgage payments. Aruoba, Davis, and Wright (2012) investigate the effect of inflation on housing investment in a money-search model with home production, to which housing is an important input.

### 3 The nominal rigidity in mortgage contracts

The nominal rigidity, and the resulting two channels of monetary policy transmission, is explained in partial equilibrium using a deterministic three-period model. An extension to infinite horizon and shocks is straightforward but at the cost of extra notation and more complicated expressions. Throughout this section, the real interest rate and real labor income are held constant and the one-period nominal interest rate is varied exogenously. All three variables are endogenized in the next section. The main insights, however, can be gained from the simple model used here. Our focus is on plain vanilla FRM and ARM loans.

#### 3.1 Three-period model

Time is denoted by  $t = 1, 2, 3$ . Each period a household is endowed with constant real income  $w$  and in  $t = 1$  has no outstanding mortgage debt (outstanding debt is introduced later). In  $t = 1$ , the household makes a once-and-for-all house purchase decision, financing

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<sup>13</sup>In addition, Poterba (1984) argues that, as the U.S. income tax brackets are set in nominal terms, mortgage finance and inflation interact due to the tax deductibility of mortgage interest payments. This feature, specific to the United States and a few other countries, adds an additional layer of nominal rigidity to a mortgage contract and is abstracted from in this paper.

a fraction  $\theta$  of the purchase with a loan and a fraction  $1 - \theta$  with income. The loan can be used only for house purchase and the house lasts for  $t = 2, 3$ . The life-time utility function of the household is  $V = \sum_{t=1}^3 \beta^{t-1} u(c_t) + \sum_{t=2}^3 \beta^{t-1} g(h)$ , where  $\beta$  is a discount factor,  $c_t$  is period- $t$  nonhousing consumption,  $h$  is housing, and  $u(\cdot)$  and  $g(\cdot)$  have standard properties. The household maximizes utility with respect to  $c_1, c_2, c_3$ , and  $h$ , subject to three per-period budget constraints:  $c_1 + h = w + l/p_1$ ,  $c_2 = w - m_2/p_2$ , and  $c_3 = w - m_3/p_3$ , where  $l = \theta p_1 h$  is the nominal value of the loan,  $m_2$  and  $m_3$  are nominal loan installments (to be specified below), and  $p_t$  is the aggregate price level (i.e., the price of goods in terms of an abstract unit of account; this section abstracts from the relative price of housing). Assume there is a financial market that prices assets by the no-arbitrage principle but in which the household does not participate due to, for instance, high entry costs (in the full model this assumption is partially relaxed). Assume also that monetary policy controls the one-period nominal interest rate  $i_t$ . No-arbitrage pricing restricts  $i_t$  to satisfy  $1 + r = (1 + i_t)/(1 + \pi_{t+1})$  (i.e., the Fisher effect), where  $1 + r$  is a gross real interest rate, assumed here to be constant and given by some exogenous pricing kernel  $\mu^* = (1 + r)^{-1}$ , and  $\pi_{t+1} \equiv p_{t+1}/p_t - 1$  is the inflation rate between periods  $t$  and  $t + 1$ .

## 3.2 Mortgages

The installments have a general specification,  $m_2 \equiv (i_2^M + \gamma)l$  and  $m_3 \equiv (i_3^M + 1)(1 - \gamma)l$ . Here,  $i_t^M$  denotes the mortgage interest rate (henceforth referred to as the ‘mortgage rate’). Under FRM,  $i_2^M = i_3^M = i^F$ ; under ARM,  $i_2^M$  and  $i_3^M$  may be different. Further,  $\gamma$  is the amortization rate in the first period of the life of the mortgage, when the outstanding nominal debt is  $l$ . In the second period, the outstanding nominal debt is  $(1 - \gamma)l$  and the amortization rate is equal to one (i.e., the mortgage is repaid in full).<sup>14</sup>

Under FRM,  $m_2 = m_3$ . The amortization rate therefore solves  $i^F + \gamma = (i^F + 1)(1 - \gamma)$ , which yields  $\gamma = 1/(2 + i^F) \in (0, 0.5)$ , for  $i^F > 0$ . As  $d\gamma/di^F = -1/(2 + i^F)^2 \in (-0.25, 0)$ ,

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<sup>14</sup>Mortgage installments can be calculated either from an annuity formula or by specifying a sequence of amortization rates. For our discussion, the second method is more suitable.

the installments  $m_2$  and  $m_3$  increase (for a given  $l$ ) when  $i^F$  increases.

Under ARM,  $\gamma = 1/(2 + i_2^M) \in (0, 0.5)$ , for  $i_2^M > 0$ . That is,  $\gamma$  is set such that the installments are equalized under the assumption that the mortgage rate stays constant at  $i_2^M$  during the life of the loan. In period 3, if the mortgage rate increases to  $i_3^M > i_2^M$  then mortgage installments increase to  $m_3 = (i_3^M + 1)(1 - \gamma)l > m_2$ . The opposite is true if the mortgage rate declines. It is also the case that  $d\gamma/di_2^M \in (-0.25, 0)$  and therefore that  $m_2$  increases when  $i_2^M$  increases. Notice that ARM is not the same thing as a one-period loan, a modeling shortcut sometimes taken in the literature (see the previous section). One-period loan is a loan with  $\gamma = 1$  and thus with  $m_2 = (1 + i_2^M)l$  and  $m_3 = 0$ . This distinction has important implications, as will become apparent in Section 3.2.4.

### 3.2.1 Mortgage pricing and housing investment under FRM

In the absence of arbitrage,  $i^F$  has to satisfy

$$1 = Q_1^{(1)}(i^F + \gamma) + Q_1^{(2)}(1 - \gamma)(i^F + 1), \quad (1)$$

where  $Q_1^{(1)} = (1 + i_1)^{-1}$  and  $Q_1^{(2)} = [(1 + i_1)(1 + i_2)]^{-1}$  are the period-1 prices of one- and two-period zero-coupon bonds, determined according to the expectations hypothesis. Condition (1) states that the present value of installments for a mortgage of size one is equal to one. It is straightforward to show that, for  $\gamma \in [0, 1)$ ,  $i_1 < i_2$  implies  $i_1 < i^F < i_2$  and vice versa.

The household's first-order condition is  $u'(c_1)(1 + \tau_H) = \beta(1 + \beta)g'(h)$ , where

$$\tau_H = -\theta \left\{ 1 - \left[ \mu_{12} \frac{i^F + \gamma}{1 + \pi_2} + \mu_{12}\mu_{23} \frac{(1 + i^F)(1 - \gamma)}{(1 + \pi_2)(1 + \pi_3)} \right] \right\} \quad (2)$$

is a wedge between the marginal utility of period-1 nonhousing consumption and the marginal lifetime utility of housing, and where  $\mu_{t,t+1} \equiv \beta u'(c_{t+1})/u'(c_t)$  is the household's 'stochastic' discount factor. Notice that the wedge works like an ad-valorem tax/subsidy on housing investment and that the expression within the square brackets is the present value of the real

mortgage installments from the household's perspective (i.e., the installments are evaluated at the household's stochastic discount factor,  $\mu_{t,t+1}$ , rather than the pricing kernel of the financial market,  $\mu^*$ ). The present value represents the cost of the mortgage to the household. Because the household does not trade in the financial market, in general,  $\mu_{t,t+1} \neq \mu^*$  and the present value is different from one. Equation (2) shows that, in general, the wedge depends on nominal variables,  $i^F$ ,  $\pi_2$ , and  $\pi_3$ . By controlling  $i_1$  and  $i_2$ —and thus, through the no-arbitrage conditions,  $i^F$ ,  $\pi_2$ , and  $\pi_3$ —monetary policy affects  $\tau_H$  and the household's optimal choice of  $h$ . This is the *price effect*. Section 3.3 contains numerical examples of the price effect for a 30-year FRM.

### 3.2.2 Mortgage pricing and housing investment under ARM

Under ARM,  $i_2^M = i_1$  and  $i_3^M = i_2$  implies a no-arbitrage pricing of the loan:

$$Q_1^{(1)}(i_2^M + \gamma) + Q_1^{(2)}(1 - \gamma)(i_3^M + 1) = \frac{i_1 + \gamma}{1 + i_1} + \frac{(1 - \gamma)}{(1 + i_1)} \left[ \frac{(i_2 + 1)}{(1 + i_2)} \right] = 1.$$

The household's first-order condition takes the same form as under FRM, but with a wedge

$$\tau_H = -\theta \left\{ 1 - \left[ \mu_{12} \frac{i_1 + \gamma}{1 + \pi_2} + \mu_{12} \mu_{23} (\mu^*)^{-1} \frac{1 - \gamma}{1 + \pi_2} \right] \right\}, \quad (3)$$

where we have substituted the Fisher effect  $(1 + i_2)/(1 + \pi_3) = (\mu^*)^{-1}$ .

Despite the added flexibility in adjusting the mortgage rate—which according to the Fisher effect moves one-for-one with the inflation rate—the wedge again depends on nominal variables,  $i_1$  and  $\pi_2$ . A decline in  $i_1$ , for instance, reduces the real installments in  $t = 2$ : through the Fisher effect,  $\pi_2$  declines one-for-one with  $i_1$  but—as  $\gamma \in (0, 0.5)$  and  $d\gamma/di_2^M \in (-0.25, 0)$ —the effect on the numerator in the first term in equation (3) is stronger than the effect on the denominator. The decline in  $\pi_2$ , however, increases real mortgage installments in  $t = 3$ , thus increasing the second term in equation (3). If the household's stochastic discount factor assigns a sufficiently large weight on payments in  $t = 2$ , relative to  $t = 3$ , the

wedge declines. (Applying the same argument, the wedge declines also in the FRM case, as long as  $i^F$  falls enough in response to the decline in  $i_1$ .) Section 3.3 contains some numerical examples of the price effect for a 30-year ARM.

### 3.2.3 Front-end effects of the nominal interest rate

In the case of a 30-year mortgage, the amortization rate at the front end of the loan is close to zero (in the numerical example in Section 3.3 it is 0.00162 per quarter) and is not particularly sensitive to changes in the mortgage rate. For a long-term mortgage, the real installments at the front end can thus be approximated as  $(i_t^M + \gamma)/(1 + \pi_{t+1}) \approx i_t^M/(1 + \pi_{t+1}) \approx i_t^M$ , where the last approximation holds for a small  $\pi_{t+1}$ . Thus, at the front end of the loan, the movements in real mortgage installments occur essentially due to movements in the nominal interest rate, rather than inflation. As a result, the effect on the real installments is the same regardless of whether  $i_t^M$  changes in line with  $\pi_{t+1}$ , as in our model, or in line with changes in the real rate,  $r$ , as would be the case, for instance, in models with nominal price rigidities.

Over time, however, the interest rate effect gets dominated by inflation effects, as all past inflation from the start of the life of the loan will matter for the real value of the installments:  $(i_t^M + \gamma_k)/[(1 + \pi_{t+1})(1 + \pi_t) \cdots (1 + \pi_{t-k})]$ , where  $k$  is the number of periods since origination. The expected real mortgage installments at the back end are therefore essentially determined by inflation.

### 3.2.4 Discussion: monetary policy neutrality

When  $\mu_{t,t+1} = \mu^*$ ,  $\tau_H = 0$  and monetary policy is neutral (in fact superneutral, but the shorter ‘neutral’ is used throughout the paper). Market incompleteness is thus a necessary condition for any real effects of monetary policy in this environment. In this sense, financial market imperfections plays a similar role in our framework as goods market imperfections in models with nominal price rigidities.

When  $\mu_{t,t+1} \neq \mu^*$ , there is no price effect under certain types of loans. First, there is no price effect if  $\gamma = 1$  (one-period loan). In this case,  $\tau_H = -\theta \{1 - \mu_{12}[(1 + i_1)/(1 + \pi_2)]\}$ ,

where one can substitute in the Fisher effect  $(1 + i_1)/(1 + \pi_2) = 1 + r = (\mu^*)^{-1}$ , and  $\mu_{12}$  is evaluated at  $c_2 = w - \theta(1 + r)h$ . A sequence of such one-period loans is typical for the literature on the collateral value of housing, reviewed in Section 2; in that literature monetary policy has real effects through a direct effect on  $r$  due to nominal price rigidities.<sup>15</sup>

Neutrality also results when the loan is an index-linked mortgage (also known as a price-level adjusted or real mortgage), which adjusts the principal for changes in the price level. The nominal installments are thus  $m_2 = (i_2^M + \gamma)[(1 + \pi_2)l]$  and  $m_3 = (i_3^M + 1)(1 - \gamma)[(1 + \pi_2)(1 + \pi_3)l]$ . No-arbitrage pricing implies  $i_2^M = i_3^M = r$ . As a result, real installments are  $\tilde{m}_2 = (r + \gamma)\tilde{l}$  and  $\tilde{m}_3 = (r + 1)(1 - \gamma)\tilde{l}$ , rendering monetary policy neutral. The wedge in this case is  $\tau_H = -\theta \{1 - [\mu_{12}(\gamma + r) + \mu_{12}\mu_{23}(r + 1)(1 - \gamma)]\}$ . The key difference with respect to an ARM is that it is the principal, not the mortgage rate, that gets adjusted in response to inflation.

Finally, the absence of the price effect results also when the loan is a two-period nominal zero-coupon bond; i.e.,  $m_2 = 0$  and  $m_3 = (1 + i_1)(1 + i_2)l$ . Here, implicitly,  $\gamma = -i_1 < 0$ . It is straightforward to show that in this case  $\tau_H = (\mu_{12}\mu_{23})/(\mu^*)^2$ , where  $\mu_{12}$  and  $\mu_{23}$  are evaluated, respectively, at  $w$  and  $w - (1 + r)^2\theta h$ .<sup>16</sup>

The key ingredient for the existence of the price effect are periodic nominal payments. That is,  $\gamma \in (-i_1, 1)$ . In this case, the nominal variables in the wedge do not get substituted out through the Fisher effect and monetary policy affects  $\tau_H$ . The value of  $\gamma$  controls the form of the nominal rigidity. For instance, in the case of  $\gamma = 0$  (a coupon bond), the nominal payments are concentrated in  $t = 3$  and the quantitative effect of monetary policy works primarily by changing the real value of the repayment of the principal. In the case of a mortgage (both FRM and ARM), the repayment of the principal occurs throughout the term of the loan, in such a way as to keep the nominal payments the same every period (conditional

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<sup>15</sup>The constraint in that literature is slightly different from our version of it. Usually, it takes the form  $[(1 + i_t)/(1 + \pi_{t+1})](l_t/p_t) \leq \theta h_{t+1}$ . That is, repayment of the one-period loan with interest, in real terms, must be less or equal to a fraction of the value of the house next period. The constraint is usually assumed to hold with equality in all states of the world. These details are unimportant for the point being made here.

<sup>16</sup>The one-period nominal loan and the two-period nominal zero-coupon bond, while immune to the price effect, are subject to wealth effects (like any other fixed income security) if there are ex-post inflation surprises, discussed below. The real mortgage is immune to both effects.

on a given mortgage rate), making the nominal rigidity more evenly distributed.<sup>17</sup>

### 3.2.5 Outstanding mortgage debt

Let us now abstract from the housing investment decision and focus instead on how monetary policy affects the real value of payments on outstanding mortgage debt. Suppose that in  $t = 1$  the household has some outstanding two-period mortgage loan  $l_0$ , taken out in  $t = 0$  and maturing in  $t = 2$ . The household's budget constraint in  $t = 1$  is  $c_1 = w - \tilde{m}_1$ , where  $\tilde{m}_1 \equiv m_1/p_1 = [(i_1^M + \gamma)/(1 + \pi_1)]\tilde{l}_0$ , with  $\tilde{l}_0 \equiv l_0/p_0$ . The mortgage rate  $i_1^M$  is predetermined in  $t = 1$ ; it is equal to some  $i_0^F$  under FRM and to  $i_0$ , the period-0 short rate, under ARM. Clearly, a lower  $\pi_1$  generates a negative *current wealth effect* for the household in  $t = 1$ . This is the standard wealth effect due to inflation 'surprises' considered in the literature reviewed in Section 2. Clearly, this effect is present also in the case of one-period loans ( $\gamma = 1$ ). In  $t = 2$ , the real payments on the 2-period loan are, respectively under FRM and ARM,

$$\tilde{m}_2 = \frac{i_0^F + 1}{(1 + \pi_1)(1 + \pi_2)}(1 - \gamma)\tilde{l}_0 \quad \text{and} \quad \tilde{m}_2 = \frac{1 + r}{1 + \pi_1}(1 - \gamma)\tilde{l}_0,$$

where in the second equation we have used  $1 + r = (i_1 + 1)/(1 + \pi_2)$ . Thus, a lower  $\pi_1$  generates not only a negative wealth effect in  $t = 1$ , but also a negative *expected future wealth effect*, as it increases real payments in the final period  $t = 2$ .

The fact that the ARM loan matures in  $t = 2$  (i.e., the amortization rate in  $t = 2$  equals one), allowed us to substitute the Fisher effect in the ARM expression for  $\tilde{m}_2$  above. Things are, however, a little bit more complicated with ARM loans. To gain further insight into the differences in wealth effects between FRM and ARM loans, it is necessary to consider loans with a three-period term, maturing in  $t = 3$ . While the preceding discussion of wealth effects

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<sup>17</sup>The focus of the paper is on mortgages, as opposed to coupon bonds, typically issued by corporations, as long-term corporate assets are less debt-dependent than housing (long-term corporate assets are typically more than 75% financed through retained earnings and other forms of equity; Rajan and Zingales, 1995). Nonresidential real estate is also typically financed by coupon bonds. For a model with nominal corporate debt see Gomes, Jermann, and Schmid (2013). The issues discussed here in relation to mortgages apply also to car loans. We abstract from car loans as mortgage debt has a longer term and makes up a much larger fraction of household debt than car loans.

in  $t = 1$  applies in this case as well, things are different in  $t = 2$ . Real mortgage payments in  $t = 2$  under both contracts now take the form

$$\tilde{m}_2 = \frac{i_2^M + \gamma_2}{(1 + \pi_1)(1 + \pi_2)}(1 - \gamma_1)\tilde{l}_0, \quad (4)$$

where  $\gamma_2$  is a period-2 amortization rate and  $i_2^M = i_0^F$  under FRM and  $i_2^M = i_1$  under ARM. Now, because  $\gamma < 1$ , it is not possible to carry out the Fisher effect substitution as before and both  $i_1$  and  $\pi_2$  matter in the case of ARM. Consider, for example, a decline in  $i_1$  in  $t = 1$  and the resulting equiproportionate decline in  $\pi_2$  due to the Fisher effect. Under FRM, the lower  $\pi_2$  increases  $\tilde{m}_2$ . But under ARM,  $\tilde{m}_2$  declines. It is straightforward to check that, as  $\gamma_2 \in (0, 0.5)$  and  $d\gamma_2/di_1 \in (-0.25, 0)$ , a decline in  $i_1$ , accompanied by an equiproportionate decline in  $\pi_2$ , reduces  $\tilde{m}_2$  as the effect on the numerator in equation (4) is greater than the effect on the denominator (in the case of a 30-year mortgage, the argument from Section 3.2.3 on the front-end effect of the nominal interest rate applies in this case as well).<sup>18</sup> In  $t = 3$ , the payments are

$$\tilde{m}_3 = \frac{i_3^M + 1}{(1 + \pi_1)(1 + \pi_2)(1 + \pi_3)}(1 - \gamma_2)(1 - \gamma_1)\tilde{l}_0,$$

where  $i_3^M = i_0^F$  under FRM and  $i_3^M = i_2$  under ARM, with  $(i_2 + 1)/(1 + \pi_3) = 1 + r$ . Under both contracts, the payments in  $t = 3$  consist mainly of amortization and a lower  $\pi_2$  increases  $\tilde{m}_3$  under both contracts.

In sum, under FRM, a lower  $i_1$  (and thus a lower  $\pi_2$ ) increases real mortgage payments on outstanding debt in both  $t = 2$  and  $t = 3$ , under ARM it also increases the payments in  $t = 3$  but reduces them in  $t = 2$ . The next section demonstrates the wealth effects in the case of 30-year mortgages.

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<sup>18</sup>The properties of  $\gamma_2$  listed here are derived from the equation  $(i_1 + \gamma_2)(1 - \gamma_1) = (i_1 + 1)(1 - \gamma_2)(1 - \gamma_1)$ , which states that the installments in periods 2 and 3 have to be equal, conditional on  $i_1$ . This yields  $\gamma_2 \approx (1 - \gamma_1)/(2 + i_1 - \gamma_1)$ , which, for some  $\gamma_1 \in (0, 1)$ , is in the interval  $(0, 0.5)$ . Taking the derivative with respect to  $i_1$  then confirms that  $d\gamma_2/di_1 \in (-0.25, 0)$ , for  $\gamma_1 \in (0, 1)$ .



### 3.3 Numerical examples: 30-year mortgage

Figure 1 provides a numerical example to illustrate the price and expected future wealth effects in the case of a 30-year mortgage. Specifically, it plots debt-servicing costs,  $\tilde{m}_t/w$ , over the term of the loan under two alternative paths of  $i_t$ ; a constant ‘steady-state’  $i_t = 4\%$  and a mean-reverting decline of  $i_t$  to 1% in period 1, which we refer to as ‘monetary policy easing’. The persistence of the decline is 0.95, which is the average autocorrelation of the short rate in the data. All the assumptions of the 3-period model—constant  $r$  and  $w$  and no-arbitrage pricing, with equation (1) extended to 120 quarters—are maintained here. The parameterization is  $r = 1\%$  per annum and  $\tilde{l} = 16w$ , i.e., four times annual income. Indeed, in the model, the household chooses  $\tilde{l}$  optimally (by choosing  $h$ ). The point here is simply to illustrate the size of these effects for one particular loan size.<sup>19</sup>

At the steady-state interest rate, debt-servicing costs are front-loaded and decline monotonically over the life of the mortgage from 29% to 6.5%. This is the standard ‘tilt effect’ (e.g., Schwab, 1982), occurring due to a positive inflation rate (in this case 3%). This profile is a baseline against which to compare the debt-servicing costs under monetary policy easing.

Starting with the case of a new loan, under both ARM and FRM, monetary policy easing reduces debt-servicing costs at the front end, where they are the highest, and somewhat increases them at the back end, where they are the smallest. The decline under FRM is smaller than under ARM because the FRM interest rate, due to the mean-reverting nature of the short rate in this example, declines by less than the short rate itself. The flattening of the path of debt-servicing costs results in smoother consumption and thus a decline in  $\tau_H$  under a sufficiently concave utility function (and/or sufficiently small  $\beta$ ). Using a log utility function and  $\beta = 0.9883$ , a baseline parameterization of the model of the next section,  $\tau_H$  declines by 1.66 percentage points in the case of FRM and by 3.83 percentage points in the

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<sup>19</sup>The parameterization of the loan size is based on the average ratio, 1975-2010, of the median price of a new home (assuming a loan-to-value ratio of 76%) to the median household net income (assuming an income tax rate of 23.5%). The data on house prices and gross incomes are from the U.S. Census Bureau. The loan-to-value ratio is the average ratio for single family newly-built home mortgages (Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10); the tax rate is a NIPA-based estimate. A historical 2% markup is added to the interest rate.

case of ARM. If the risk-aversion coefficient is increased from 1 to 2, the declines of  $\tau_H$  are 3.2 and 7.76 percentage points, respectively. (Recall that the wedge has a direct interpretation as an ad-valorem tax on new housing.)

For the case of an existing loan, we consider a loan with 119 periods remaining (the magnitudes of the expected future wealth effects decline as the remaining term of the loan gets shorter). In the case of ARM, as the loan is only one period into its life, the expected path of debt-servicing costs is essentially the same as that for the new loan. That is, there is a sharp immediate decline in debt-servicing costs, followed by their increase several periods later. Under FRM, however, the persistently low inflation leads to a gradual increase in debt-servicing costs for the remainder of the term of the loan. Thus, while in the case of ARM, the price and expected future wealth effects work in the same direction, in the case of FRM they work in opposite directions, in line with our discussion in the previous section.

Figure 2 plots the results of the same experiment, but for two alternative degrees of persistence: 0.99 and 0.5. In the 0.99 case, the magnitudes are much larger than in the 0.95 case. Furthermore, for the new loans, the results under FRM and ARM are more similar to each other than in the 0.95 case, as the long rate drops almost as much as the short rate. For existing loans, however, the effects under FRM and ARM diverge further apart. When the persistence is 0.5, the effects on both new and existing loans are small, in fact hardly noticeable in the FRM case.

## 4 General equilibrium model

The general equilibrium model extends the model of the previous section to infinite horizon and shocks and endogenizes the variables that were either held constant (real labor income and the real interest rate) or were treated as exogenous (the short-term nominal interest rate).

## 4.1 Environment

The economy’s population is split into two groups, ‘homeowners’ and ‘capital owners’, with measures  $\Psi$  and  $(1 - \Psi)$ , respectively. Within each group, agents are identical. An aggregate production function combines capital and labor to produce a single good, which can be used for consumption, accumulation of capital, or as housing structures; houses require land in addition to structures. Capital owners own the economy’s capital stock, whereas homeowners supply labor and own the economy’s housing stock. Such abstraction is motivated by the cross-sectional observations discussed in the Introduction. Capital owners play here the role of mortgage investors, kept outside of the three-period model. Real labor income is endogenized by homeowners’ labor supply decisions in competitive factor markets, the real interest rate is endogenized by the marginal product of capital, and the nominal interest rate is endogenized by a monetary policy feedback rule. We also relax the extreme assumption of no participation of homeowners in the financial market maintained in the three-period model by giving homeowners access, at a cost, to a one-period bond market. The case of no participation, however, is considered as a benchmark, as it has quite stark implications. The model is solved under either FRM loans or ARM loans.<sup>20</sup> Where applicable, the notation is the same as in Section 3. Only new variables and functions are therefore defined. When a variable’s notation is the same for both agent types, an asterisk (\*) denotes the variable pertaining to capital owners.

### 4.1.1 Capital owners

A representative capital owner maximizes expected life-time utility

$$E_t \sum_{t=0}^{\infty} \beta^t u(c_t^*), \quad \beta \in (0, 1),$$

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<sup>20</sup>Because there is a representative homeowner, allowing for a choice between FRM and ARM leads to a bang-bang solution. If the fraction of loans of each type is set exogenously, then the results are a convex combination of the impulse-responses reported in Section 6 for FRM and ARM separately. A way to think about our assumption is as considering a prototypical FRM country (like the U.S., Denmark, France, or Belgium) and a prototypical ARM economy (like the U.K. and most other countries).

where  $u(\cdot)$  has standard properties, subject to a sequence of budget constraints

$$c_t^* + x_{Kt} + \frac{b_{t+1}^*}{p_t} + \frac{l_t^*}{p_t} = [(1 - \tau_K)r_t + \tau_K\delta_K]k_t + (1 + i_{t-1})\frac{b_t^*}{p_t} + \frac{m_t^*}{p_t} + \tau_t^* + \frac{p_{Lt}}{1 - \Psi}. \quad (5)$$

Here,  $x_{Kt}$  is investment in capital,  $b_{t+1}^*$  is holdings of a one-period nominal bond between periods  $t$  and  $t + 1$ ,  $\tau_K$  is a capital income tax rate,  $\delta_K \in (0, 1)$  is a depreciation rate,  $k_t$  is capital, and  $\tau_t^*$  is a lump-sum transfer. In addition,  $1/(1 - \Psi)$  is new residential land, which the capital owner receives each period as an endowment, and  $p_{Lt}$  denotes its price in terms of consumption. The capital stock evolves as

$$k_{t+1} = (1 - \delta_K)k_t + x_{Kt}. \quad (6)$$

As explained in the next section, the capital income tax rate, transfers, and tax deductible depreciation are included in order to allow a sensible calibration of the model. Other than that, these elements are unimportant.<sup>21</sup>

The modeling of mortgages follows Kydland et al. (forthcoming), who consider infinitely-lived loans, which nevertheless have the key characteristics of standard (finitely-lived) mortgage loans. Denoting by  $d_t^*$  the period- $t$  stock of all outstanding nominal mortgage debt owed to the capital owner, the nominal mortgage payments received by the capital owner in period  $t$  are

$$m_t^* = (R_t^* + \gamma_t^*)d_t^*, \quad (7)$$

where  $R_t^*$  and  $\gamma_t^*$  are, respectively, the interest and amortization rates of the outstanding debt. The variables comprising  $m_t^*$  are state variables evolving as

$$d_{t+1}^* = (1 - \gamma_t^*)d_t^* + l_t^*, \quad (8)$$

$$\gamma_{t+1}^* = (1 - \phi_t^*)(\gamma_t^*)^\alpha + \phi_t^*\kappa, \quad (9)$$

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<sup>21</sup>For the reasons mentioned in Section 3.2.4, we abstract from debt finance in the case of capital.

$$R_{t+1}^* = \begin{cases} (1 - \phi_t^*)R_t^* + \phi_t^*i_t^F, & \text{if FRM,} \\ i_t, & \text{if ARM,} \end{cases} \quad (10)$$

where  $\phi_t^* \equiv l_t^*/d_{t+1}^*$  is the fraction of new loans in the outstanding debt next period. The amortization rate  $\gamma_{t+1}^*$  and the interest rate  $R_{t+1}^*$  in the FRM case thus evolve as weighted averages of the amortization and interest rates, respectively, on the existing stock and new loans.  $\kappa, \alpha \in (0, 1)$  are parameters. Specifically,  $\kappa$  is the initial amortization rate of a new loan and  $\alpha$  controls the evolution of the amortization rate over time. Notice that setting  $\alpha = 0$  and  $\kappa = 1$  implies  $\gamma_t = 1 \forall t$ . That is,  $l_t^*$  becomes a one-period loan. Setting  $\alpha = 1$  results in a constant amortization rate  $\gamma_t = \kappa$  and thus declining nominal mortgage installments over the (infinite) life of the loan. Recall from the previous section that in order to keep the mortgage installments constant, the amortization rate has to be increasing. When  $\kappa, \alpha \in (0, 1)$ , the amortization rate increases, converging to one, over the life of the loan. Kydland et al. (forthcoming) show that  $\kappa$  and  $\alpha$  can be chosen so as to approximate the payments of standard 30-year mortgages.<sup>22</sup> Notice that, even though new loans are extended every period, each new loan  $l_t^*$  (both FRM and ARM) is a long-term loan, starting with an amortization rate  $\kappa < \gamma_t^*$ . Furthermore, the loans are held until maturity. Refinancing by homeowners is dealt with in Section 6.

Under FRM, the first-order condition for  $l_t^*$  ensures that  $i_t^F$  is such that the capital owner is indifferent between new mortgages and rolling over the one-period bond from period  $t$  on. The first-order condition is an infinite-horizon counterpart to equation (1); see Appendix A. Under ARM, the current one-period interest rate  $i_t$  is applied to both new and outstanding mortgage loans (equation (10)), making the capital owner again indifferent between mortgages and rolling over the bond. As in equilibrium the capital owner is indifferent across investing in mortgages, bonds, and capital, his composition of period- $t$  investment is pinned down by homeowners' demand for new mortgages and the one-period bond.

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<sup>22</sup>Under appropriate choice of  $\kappa$  and  $\alpha$ , even though the loan has an infinite life, it gets essentially repayed within 30 years and the nominal installments are approximately constant for most of these 30 years. Such modeling of mortgages is convenient, as both the agents and the loans have an infinite life, thus allowing a simple recursive representation of the model with only a few state variables.

### 4.1.2 Homeowners

A representative homeowner maximizes expected life-time utility

$$E_t \sum_{t=0}^{\infty} \beta^t v(c_t, 1 - n_t, h_t),$$

where  $n_t$  is labor and  $v(., ., .)$  has standard properties. The maximization is subject to a sequence of constraints

$$c_t + p_{Ht}x_{Ht} - \frac{l_t}{p_t} + \frac{b_{t+1}}{p_t} = (1 - \tau_N)w_t n_t - \tau_t + (1 + i_{t-1} + \Upsilon_{t-1})\frac{b_t}{p_t} - \frac{m_t}{p_t}, \quad (11)$$

$$\frac{l_t}{p_t} = \theta p_{Ht}x_{Ht}, \quad (12)$$

$$h_{t+1} = (1 - \delta_H)h_t + x_{Ht}. \quad (13)$$

Here,  $x_{Ht}$  is newly purchased homes,  $p_{Ht}$  is their relative price,  $w_t$  is a real wage rate,  $\tau_N$  is a labor income tax rate,  $\tau_t$  is a lump-sum tax, and  $\delta_H \in (0, 1)$  is a depreciation rate. Again, taxes are included purely for calibration purposes.<sup>23</sup>  $\Upsilon_{t-1}$  is a bond market participation cost, governed by a function  $\Upsilon(-\tilde{b}_t)$ , where  $\tilde{b}_t \equiv b_t/p_{t-1}$  is the homeowner's real holdings of the bond. The function  $\Upsilon(\cdot)$  is assumed to be increasing and convex and it allows us to control the extent to which homeowners can smooth consumption.<sup>24</sup> In a nonstochastic steady state,  $\tilde{b} = 0$ . In order to avoid the cost affecting the definition of aggregate output, it is rebated to the homeowner in a lump-sum way as a part of  $\tau_t$ . As in the case of the capital

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<sup>23</sup>As in the three-period model,  $\theta$  is a parameter. Chambers et al. (2009a) make a similar assumption and empirical evidence supports this assumption: over the period 1973-2006, there has been very little variation in the cross-sectional average of the loan-to-value ratio for single family newly-built home first mortgages, despite large changes in interest rates and other macroeconomic conditions (Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10).

<sup>24</sup>It is further assumed that  $\Upsilon(\cdot) = 0$  when  $\tilde{b}_t = 0$ ,  $\Upsilon(\cdot) > 0$  when  $\tilde{b}_t < 0$  (the homeowner is borrowing), and  $\Upsilon(\cdot) < 0$  when  $\tilde{b}_t > 0$  (the homeowner is saving). We think of  $\Upsilon(\cdot) > 0$  as capturing a premium for unsecured consumer credit, which is increasing in the amount borrowed.  $\Upsilon(\cdot) < 0$  can be interpreted as higher intermediation costs for homeowners than capital owners, which reduces the homeowners' returns on savings below those of capital owners. A technical role of the cost function is that, as in two-country business cycle models with incomplete markets, it prevents the one-period debt from becoming a random walk in a log-linear solution of the model.

owner, mortgage payments are given by

$$m_t = (R_t + \gamma_t)d_t, \quad (14)$$

where

$$d_{t+1} = (1 - \gamma_t)d_t + l_t, \quad (15)$$

$$\gamma_{t+1} = (1 - \phi_t)(\gamma_t)^\alpha + \phi_t\kappa, \quad (16)$$

$$R_{t+1} = \begin{cases} (1 - \phi_t)R_t + \phi_t i_t^F, & \text{if FRM,} \\ i_t, & \text{if ARM,} \end{cases} \quad (17)$$

with  $\phi_t \equiv l_t/d_{t+1}$ .

### 4.1.3 Technology

An aggregate production function, operated by perfectly competitive producers, is given by  $Y_t = A_t f(K_t, N_t)$ , where  $K_t$  is the aggregate capital stock,  $N_t$  is aggregate labor, and  $f(., .)$  has the standard neoclassical properties. Total factor productivity (TFP) evolves as  $\log A_{t+1} = (1 - \rho_A) \log A + \rho_A \log A_t + \epsilon_{A,t+1}$ , where  $\rho_A \in (0, 1)$ ,  $A$  is the unconditional mean, and  $\epsilon_{A_t} \sim iidN(0, \sigma_A)$ . The total factor productivity shock is the more important shock for the business cycle properties of the model, reported in Appendix F of the supplemental material. The real rate of return on capital,  $r_t$ , and the real wage rate,  $w_t$ , are determined by the marginal products of capital and labor, respectively. The resource constraint of the economy is  $C_t + X_{Kt} + q_t X_{St} + G = Y_t$ , where  $C_t$  is aggregate consumption,  $X_{Kt}$  is aggregate investment in capital,  $X_{St}$  is new residential structures, and  $G$  is (constant) government expenditures, introduced for calibration purposes only. Here,  $q_t$  is the marginal rate of transformation between new residential structures and the other uses of output, and hence the relative price of new residential structures. It is given by a strictly increasing convex function  $q(X_{St})$ , which makes the economy's production possibilities frontier concave in the space of  $(C_t + X_{Kt} + G)$  and  $(X_{St})$ —a specification akin to that of Huffman and Wynne

(1999), a stand-in for the costs of moving factors of production across different sectors of the economy. The purpose of  $q(\cdot)$  is to ensure realistic volatility of new residential structures in response to shocks; if the production possibilities frontier was linear, given the calibration of the shocks, the volatility would be way too high.

As in Davis and Heathcote (2005), new homes consist of new residential structures and land and are produced by perfectly competitive homebuilders according to an aggregate production function  $X_{Ht} = g(X_{St}, X_{Lt})$ . Here,  $X_{Ht}$  is the aggregate number of new homes constructed in period  $t$ ,  $X_{Lt}$  is the aggregate new residential land, and  $g$  has the standard neoclassical properties.

#### 4.1.4 Monetary policy shocks

A monetary authority follows an interest rate feedback rule with a stochastic inflation target (e.g., Ireland, 2007)

$$i_t = (i + \bar{\pi}_t - \bar{\pi}) + \nu_\pi(\pi_t - \bar{\pi}_t), \quad \nu_\pi > 1, \quad (18)$$

where  $i$  is the nonstochastic steady-state short-term nominal interest rate, and  $\bar{\pi}_t$  is an inflation target. The inflation target follows an AR(1) process  $\bar{\pi}_{t+1} = (1 - \rho_\pi)\bar{\pi} + \rho_\pi\bar{\pi}_t + \epsilon_{\pi,t+1}$ , where  $\rho_\pi \in [0, 1)$ ,  $\bar{\pi}$  is the nonstochastic steady-state inflation rate, and  $\epsilon_{\pi,t+1} \sim iidN(0, \sigma_\pi)$ . Notice that the interest rate rule has a more standard representation:  $i_t = i + \nu_\pi(\pi_t - \bar{\pi}) + \xi_t$ , where  $\xi_t \equiv -(\nu_\pi - 1)(\bar{\pi}_t - \bar{\pi})$ .<sup>25</sup>

As will be shown in Section 4.3, when  $\rho_\pi$  is close to one, the inflation target shock works in equilibrium like a ‘level factor’, moving short and long rates equally, and allows the model to reproduce the observed volatility and persistence of the 30-year mortgage rate. A number of studies document that the level factor accounts for over 90% of the volatility of nominal yields across maturities (see, e.g., Piazzesi, 2006). There has been little success

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<sup>25</sup>Using an interest rate feedback rule, rather than treating  $i_t$  as exogenous, has the advantage that it endogenously determines the current inflation rate  $\pi_t$ , which otherwise would be undetermined (Woodford, 2003). An alternative strategy of assuming a money growth rule has the problem that money growth rules, in an environment like ours, fail to generate persistent movements in nominal interest rates and inflation, even in the presence of persistent money growth shocks (e.g., Gavin, Keen, and Pakko, 2005).



so far in providing a structural interpretation of the level factor, though it is generally regarded to be related to monetary policy (see Atkeson and Kehoe, 2009, for a review). In the absence of an off-the-shelf theory, we follow Gallmeyer, Hollifield, Palomino, and Zin (2007) and Atkeson and Kehoe (2009) and simply model the level factor as arising from very persistent monetary policy shocks, taking the form of shocks to the inflation target. Nevertheless, smaller degrees of persistence of the monetary policy shock are also considered in the computational experiments in Section 6. Given the numerical examples in Section 3.3, one would expect the persistence of the shock to play an important role.

## 4.2 Equilibrium

The equilibrium concept is a recursive competitive equilibrium. First, let  $z_t \equiv [\log A_t, \bar{\pi}_t, p_{t-1}]$  be the vector of exogenous state variables and the lagged endogenous variable  $p_{t-1}$ ,  $s_t^* \equiv [k_t, b_t^*, d_t^*, \gamma_t^*, R_t^*]$  the vector of the capital owner's state variables,  $s_t \equiv [h_t, b_t, d_t, \gamma_t, R_t]$  the vector of the homeowner's state variables, and  $S_t \equiv [K_t, H_t, B_t, D_t, \Gamma_t, \mathfrak{R}_t]$  the vector of aggregate endogenous state variables, where the elements are, respectively, aggregate capital, housing stock, bonds, outstanding mortgage debt, and its amortization and interest rates. Next, write the capital owner's optimization problem as

$$U(z, S, s^*) = \max_{[x_K, (b^*)', l^*]} \left\{ u(c^*) + \beta E[U(z', S', (s^*)') | z] \right\}, \quad (19)$$

where a prime denotes a value next period and the constraints (5)-(10) are thought to have been substituted in the utility and value functions. Similarly, write the homeowner's problem as

$$V(z, S, s) = \max_{[x_H, b', n]} \left\{ v(c, 1 - n, h) + \beta E[V(z', S', s') | z] \right\}, \quad (20)$$

where the constraints (11)-(17) are thought to have been substituted in the utility and value functions. Let  $W_t \equiv [X_{Kt}, p_t, i_t^M, X_{Ht}, B_{t+1}, N_t]$  be the vector of aggregate decision variables and prices, where  $i_t^M = i_t^F$  under FRM and  $i_t^M = i_t$  under ARM. Define a function

$$W_t = W(z_t, S_t).$$

A recursive competitive equilibrium consists of the functions  $U$ ,  $V$ , and  $W$  such that: (i)  $U$  and  $V$  solve (19) and (20), respectively; (ii)  $r_t$  and  $w_t$  are given by the respective marginal products of capital and labor,  $p_{Ht}$  and  $p_{Lt}$  are given by the respective marginal products of structures and land, and  $q_t = q(X_{St})$ ; (iii)  $i_t$  is given by the monetary policy rule (18); (iv) the bond, mortgage, housing, and land markets clear:  $(1 - \Psi)b_{t+1}^* + \Psi b_{t+1} = 0$ ,  $(1 - \Psi)(l_t^*/p_t) = \Psi\theta p_{Ht}x_{Ht}$ ,  $\Psi x_{Ht} = g(X_{St}, X_{Lt})$ , and  $X_{Lt} = 1$ ; (v) aggregate consistency is ensured:  $K_t = (1 - \Psi)k_t$ ,  $X_{Kt} = (1 - \Psi)x_{Kt}$ ,  $X_{Ht} = \Psi x_{Ht}$ ,  $N_t = \Psi n_t$ ,  $B_t = \Psi b_t$ ,  $H_t = \Psi h_t$ ,  $(1 - \Psi)m_t^* = \Psi m_t$ ,  $(1 - \Psi)d_t^* = \Psi d_t = D_t$ ,  $\gamma_t^* = \gamma_t = \Gamma_t$ ,  $R_t^* = R_t = \mathfrak{R}_t$ , and  $G + (1 - \Psi)\tau_t^* = \tau_K(r_t - \delta_K)K_t + \tau_N w_t N_t + \Psi(\tau_t - \Omega_t)$ , where  $\Omega_t$  is the participation cost; (vi) the exogenous state variables follow their respective stochastic processes and the endogenous aggregate state variables evolve according to aggregate counterparts to the laws of motion for the respective individual state variables; and (vii) the individual optimal decision rules of the capital owner (for  $x_K$ ,  $(b^*)'$ , and  $l^*$ ) and the homeowner (for  $x_H$ ,  $b'$ , and  $n$ ) are consistent with  $W(z, S)$ , once the market clearing conditions (iv) and the aggregate consistency conditions (v) are imposed.<sup>26</sup>

It is straightforward to check that the goods market clears by Walras' Law:  $C_t + X_{Kt} + q_t X_{St} + G = Y_t$ , where  $C_t = (1 - \Psi)c_t^* + \Psi c_t$ . Equations characterizing the equilibrium are contained in Appendix A; a computational procedure resulting in log-linear approximation of  $W(z, S)$  around the model's non-stochastic steady state is described in Appendix B.

### 4.3 The equilibrium nominal interest rate and inflation

The capital owner's first-order conditions for  $b_{t+1}^*$  and  $x_{Kt}$  yield the Fisher equation. In a linearized form:  $i_t \approx E_t \pi_{t+1} + E_t r_{t+1}$ , where (abusing notation) the variables are in percentage point deviations from steady state. Given a stochastic process for  $r_t$ , the Fisher equation and

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<sup>26</sup>In the case of ARM,  $i_t^M = i_t$  makes the capital owner indifferent between new mortgages and bonds and the first-order condition for  $l_t^*$  can be dropped from the description of the equilibrium. In the case of FRM, the first-order condition is needed to determine  $i_t^F$ .

the monetary policy rule (18) determine  $i_t$  and  $\pi_t$ . For  $\rho_\pi$  close to one, excluding explosive paths for inflation (a common assumption), the resulting expression for  $i_t$  is

$$i_t \approx \sum_{j=0}^{\infty} \left( \frac{1}{\nu_\pi} \right)^j E_t r_{t+1+j} + \bar{\pi}_t. \quad (21)$$

The short rate is thus equal to the sum of the exogenous shock  $\bar{\pi}_t$  and the endogenous expected future path of the real interest rate  $r_t$ , given by a linearized version of the marginal product of capital,  $A_t f_K(K_t, N_t)$ . Substituting  $i_t$  from equation (21) back into the policy rule (18) gives the equilibrium inflation rate

$$\pi_t \approx \frac{1}{\nu_\pi} \sum_{j=0}^{\infty} \left( \frac{1}{\nu_\pi} \right)^j E_t r_{t+1+j} + \bar{\pi}_t. \quad (22)$$

Thus, under the assumption yielding the above expressions—that  $\rho_\pi$  is close to one—the equilibrium short-term nominal interest rate and inflation will move, subject to general equilibrium adjustments in  $r_t$ , one for one with the highly persistent shock  $\bar{\pi}_t$ . Because the movements in  $i_t$  are highly persistent, the long rate  $i_t^F$  will move almost as much as the short rate  $i_t$ . In this sense,  $\bar{\pi}_t$  works like a level factor, moving all interest rates and inflation approximately equally.<sup>27</sup>

## 5 Calibration

The model is quarterly and most parameter values are obtained by requiring the model to reproduce long-run averages of the data in a nonstochastic steady state. Some second moments are also used. As most of the required historical data are readily available for the United States, the calibration is based on U.S. data, even though the mechanism under

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<sup>27</sup>In contrast, the first term in equation (21), mainly driven by shocks to  $A_t$ , is much less persistent (given the estimates of the persistence of  $A_t$  from the data). Therefore, it produces only temporary movements in  $i_t$  and thus smaller movements in  $i_t^F$  than in  $i_t$ . As a result, it moves the long-short spread,  $i_t^F - i_t$ . In this sense it works like a slope factor. Furthermore, it makes the long-short spread negatively correlated with output, as in the data. Of course, as the model abstracts from term premia, it does not capture the part of the movements in the long-short spread in the data due to movements in term premia.

investigation applies more generally (Appendix C contains the description of the U.S. data and their adjustments to conform with the notion of the variables in the model). We conduct sensitivity analysis with respect to some of the choices described here.

## 5.1 Functional forms

The capital owner's per-period utility function is  $u(c^*) = \log c^*$ ; the homeowner's utility function is  $v(\bar{c}, n) = \omega \log \bar{c} + (1 - \omega) \log(1 - n)$ , where  $\bar{c}$  is a composite consumption good  $\bar{c}(c, h) = c^\xi h^{1-\xi}$ . The additive separability of the homeowner's utility function facilitates a transparent interpretation of the results as marginal utilities are independent of the consumption of other goods. Further, the goods production function is  $f(K, N) = K^\varsigma N^{1-\varsigma}$  and the housing production function is  $g(X_S, X_L) = X_S^{1-\varphi} X_L^\varphi$ . As in Kydland et al. (forthcoming),  $q(X_{St}) = \exp(\zeta(X_{St} - X_S))$ , where  $\zeta > 0$  and  $X_S$  is the steady-state ratio of new residential structures to output ( $Y$  is normalized to be equal to one in steady state). A similar functional form is used also for the bond market participation cost:  $\Upsilon(-\tilde{B}) = \exp(-\vartheta \tilde{B}_t) - 1$ , where  $\vartheta > 0$  and  $\tilde{B}_t = 0$  in steady state. It is straightforward to check that this function satisfies the properties set out in Section 4.1.2.

## 5.2 Debt-servicing costs

A particular challenge in calibrating the model arises due to the need to match debt-servicing costs of homeowners. This requires the model to be consistent with the cross-sectional distribution of income, in addition to standard aggregate ratios  $X_K/Y = 0.156$ ,  $X_S/Y = 0.054$ ,  $K/Y = 7.06$ ,  $H/Y = 5.28$ ,  $rK/Y = 0.283$ , and  $N = 0.255$ . The last ratio is from the American Time-Use Survey 2003, population 16+, the others are averages for 1958-2006.

Official data for mortgage debt servicing costs are not published for the United States. Estimates, however, can be obtained from different sources (see Appendix D), resulting in long-run averages (1972-2006) in the ballpark of 18.5% of homeowners' pre-tax income (i.e., before the income tax rate is applied). The model's steady-state counterpart to this aggregate

ratio is  $\widetilde{M}/(wN - \Psi\tau)$ , where  $\widetilde{M} = (R + \gamma)\widetilde{D}/(1 + \pi)$ , with  $\widetilde{D}$  being real mortgage debt.

Consistency with the observed cross-sectional distribution of income is achieved through the transfer  $\tau$  (the part of homeowners' income due to labor,  $wN$ , is constrained by requiring the model to be consistent with the capital share of output; see below). Recall that homeowners in the model are an abstraction for the 3rd and 4th quintiles of the U.S. wealth distribution, while capital owners are an abstraction for the 5th quintile. In the data, the 5th quintile derive 40% of income from capital and the rest from labor and transfers; in the case of the 3rd and 4th quintiles, 81% comes from labor (SCF, 1998). As a result, if the only source of income of capital owners in the model was capital, and given that the model is required to match the observed average capital share of output ( $rK/Y = 0.283$ ), capital owners would account for too small fraction of aggregate income (28.3% in the model v.s. 48% in the data), while homeowners' share would be too large (71.7% v.s. 34%). As a result, the steady-state debt-servicing costs would be too low (or the debt-to-GDP ratio would have to be too high, thus being inconsistent with the observed loan-to-value ratio  $\theta$  and amortization schedules). The parameter  $\tau$  adjusts for this discrepancy by transferring, in a lump-sum way, some of the labor income from homeowners to capital owners so as to match the distribution of income, without affecting the model's ability to match the other calibration targets.

### 5.3 Parameter values

The baseline parameter values are listed in Table 1, where the parameters are organized into eight categories:  $\Psi$  (population);  $\delta_K, \delta_H, \varsigma, A, \zeta, \varphi$  (technology);  $\tau_K, \tau_N, G, \tau$  (fiscal);  $\theta, \alpha, \kappa$  (mortgages);  $\vartheta$  (bond market);  $\bar{\pi}, \nu_\pi$  (monetary policy);  $\beta, \omega, \xi$  (preferences); and  $\rho_A, \sigma_A, \rho_\pi, \sigma_\pi$  (stochastic processes). Most parameters can be assigned values without solving a system of steady-state equations, four parameters ( $\omega, \xi, \tau_K, \tau$ ) have to be obtained jointly from such steady-state relations, and three parameters ( $\zeta, \rho_\pi, \sigma_\pi$ ) are assigned values by matching second moments of the data. Calibration of the three types of parameters is

described in turn.

In order to be consistent with the notion of homeowners and capital owners in the data,  $\Psi$  is set equal to  $2/3$ . The parameter  $\varsigma$  corresponds to the share of capital income in output and is set equal to 0.283, an estimate obtained by Gomme and Rupert (2007) from National Income and Product Accounts (NIPA) for aggregate output close to our measure of output (see Appendix C). The share of residential land in new housing  $\varphi$  is set equal to 0.1, an estimate reported by Davis and Heathcote (2005). The depreciation rates  $\delta_K$  and  $\delta_H$  are set equal to 0.02225 and 0.01021, respectively, to be consistent with the average flow-stock ratios for capital and housing, respectively. The level of TFP,  $A$ , is set equal to 1.5321, so that steady-state output is equal to one. The stochastic process for TFP is assigned  $\rho_A = 0.9641$  and  $\sigma_A = 0.0082$ , estimates obtained by Gomme and Rupert (2007) for the Solow residual of a production function with the same  $\varsigma$  and measurements of capital and labor inputs used here (see Appendix C). The labor income tax rate is derived from NIPA using a procedure of Mendoza, Razin, and Tesar (1994), yielding  $\tau_N = 23.5\%$ . The parameter  $G$  is set equal to 0.138, in order to correspond to our measure of government expenditures (see Appendix C). The loan-to-value ratio  $\theta$  is set equal to 0.76, the average (1973-2006) of the cross-sectional mean of the loan-to-value ratio for single family newly-built home mortgages (Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10). As in Kydland et al. (forthcoming), the amortization parameters are  $\kappa = 0.00162$  and  $\alpha = 0.9946$ . These values approximate a 30-year mortgage. The weight on inflation in the monetary policy rule  $\nu_\pi$  is set equal to 1.35, which falls in the middle of the range of estimates reported by Woodford (2003), Chapter 1. The steady-state inflation rate  $\bar{\pi}$  is set equal to 0.0113, the average (1972-2006) quarterly inflation rate. In steady state, the first-order condition for  $l_t^*$  constrains  $i^F$  to equal to  $i$ . The first-order condition for  $b_t^*$  then relates  $i$  and  $\bar{\pi}$  to  $\beta$ . The above value of  $\bar{\pi}$  and  $i^F = 9.31\%$  per annum (the 1972-2006 average for 30-year FRM rate) imply  $\beta = 0.9883$ . For the participation cost function  $\Upsilon(\cdot)$ , the choice of  $\vartheta$  is guided by available studies on prices of unsecured consumer credit. Setting  $\vartheta$  equal to 0.035

gives similar interest premium schedule as in Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), Figure 6, white-collar workers.

Given the above parameter values, the second set of parameters  $(\omega, \xi, \tau_K, \tau)$  is calibrated by forcing the model to replicate, in steady state, the observed average  $K/Y$  ratio,  $H/Y$  ratio, debt-servicing costs, and  $N$ . The relationship between the four parameters and the targets is given by the steady-state versions of the first-order conditions for  $x_{Kt}$ ,  $x_{Ht}$ , and  $n_t$ , and the expression for steady-state debt-servicing costs noted above (see Appendix A for the first-order conditions). These restrictions yield  $\omega = 0.2478$ ,  $\xi = 0.6009$ ,  $\tau_K = 0.3362$ , and  $\tau = 0.4503$ .<sup>28</sup>

Finally, given the values of the first two sets of parameters,  $\zeta$ ,  $\rho_\pi$ , and  $\sigma_\pi$  are calibrated by matching certain second moments of the data. The parameters  $\rho_\pi$  and  $\sigma_\pi$  are obtained by matching the standard deviation (2.4%) and the first-order autocorrelation (0.97) of the 30-year FRM rate (annualized rate, unfiltered data).<sup>29</sup> This results in  $\rho_\pi = 0.994$  and  $\sigma_\pi = 0.0015$ . The PPF parameter  $\zeta$  controls the volatility of the expenditure components of output and is used to match the volatility of aggregate consumption, relative to the volatility of output. This has the advantage, compared to matching the volatility of one of the two investment series, that approximately the same parameter value is obtained regardless of whether the model is simulated under FRM or ARM. The resulting value is  $\zeta = 0.35$ .

Table 2 lists the steady-state values of the model's endogenous variables implied by the above calibration and, where possible, the long-run averages of their data counterparts. As can be seen, despite the highly stylized nature of the model, the steady state is broadly consistent with a number of moments not targeted in calibration.<sup>30</sup>

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<sup>28</sup>In principle,  $\tau_K$  can be measured from NIPA in the same way as  $\tau_N$ . Such alternative parameterization, however, makes the model inconsistent with the observed capital-output ratio. This is because  $\beta$  is already pinned down by the first-order condition for bonds and thus cannot be used to match the capital-output ratio. Nevertheless,  $\tau_K$  implied by the model is not far from the NIPA tax rate obtained by Gomme, Ravikumar, and Rupert (2011): 33.62% in the model v.s. 40.39% in NIPA.

<sup>29</sup>The 10-year government bond yield is actually used as a proxy for the 30-year mortgage rate. The two rates co-move closely for the period for which both series are available (from 1972), but the data for the 10-year yield are longer (1958-2007), thus providing a more accurate estimate of the parameters of the inflation target shock.

<sup>30</sup>The model has a well-defined steady state even in the presence of one unit of new land being made available each period as each new unit of land is combined with the steady-state amount of new structures

## 6 Findings

This section presents the main findings. The results of various sensitivity analysis are reported in Appendix E of the supplemental material.

### 6.1 Responses to $\bar{\pi}_t$ under FRM and ARM

The first set of findings is presented in Figures 3 and 4. The figures show responses of key variables to a 1 percentage point (annualized) increase in the level factor shock  $\bar{\pi}_t$ . Figure 3 is for the economy in which the homeowner has no access to the one-period bond market; Figure 4 is for the economy in which the homeowner has access to the bond market. In each chart, the solid line is for the ARM economy and the dash line is for the FRM economy. The charts show responses for the first 40 periods (10 years). The responses of interest and inflation rates are expressed in (annualized) percentage point deviations from steady state; the responses of other variables are expressed as percentage deviations from steady state. Even though, for some variables, convergence back to the steady state may not be apparent from the figures, eventually all variables converge back to the steady state. This, however, takes longer than 40 periods. The immediate message from Figures 3 and 4 is that the responses of real variables are stronger under ARM than under FRM.

The first two upper-left charts in Figure 3 demonstrate the level factor nature of the shock: the short-term nominal interest rate and the inflation rate, and in the case of the FRM economy also the FRM rate, all increase more or less in parallel by approximately 1 percentage point. Due to the positive inflation shock, on impact (period 1), real mortgage payments decline under both contracts. But this decline is dwarfed by the magnitudes in subsequent periods. In accordance with our discussion in Section 3, real mortgage payments on outstanding debt display a persistent gradual decline under FRM, while under ARM the payments increase sharply one period after the shock. Under FRM, housing investment (here  $X_H$  is plotted but qualitatively the same applies to  $X_S$ ) declines for the first few periods

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to form a steady-state number of new homes.



after the shock.<sup>31</sup> This reflects the price effect. Over time, however, due to the wealth effects generated by the gradual decline in real mortgage payments, housing investment increases above the steady-state level (before converging back to steady state). Under ARM, there is a quantitatively similar decline of housing investment in the first period as under FRM. This is because with the level factor shock, the price effect under the two contracts is similar, as discussed in relation to Figure 2. The major decline occurs in the second period, once the wealth effect kicks in. Over time, as the accumulated inflation sufficiently erodes real outstanding debt, negative wealth effects turn into positive wealth effects and housing investment increases above the steady-state level (before converging back to steady state). The capital owner compensates the decline in the demand for new mortgage loans by increasing investment in productive capital. Capital investment thus follows an approximately opposite path to that of housing investment, increasing at first before falling below the steady-state level later on. The next chart shows the response of the price of new homes. As Appendix A of the supplemental material shows, this price is proportional (in logs) to housing investment and thus declines as housing investment declines.

The dynamics of consumption reflect the redistribution of real income through mortgage payments. Thus, under FRM, consumption of homeowners gradually increases, while consumption of capital owners gradually declines. In contrast, under ARM, consumption of homeowners drops sharply in the second period, while consumption of capital owners increases. The increase is, however, smooth as capital owners can use capital to smooth anticipated changes in income. The behavior of output reflects predominantly the behavior of labor. In particular, output increases in the second period in the case of ARM as homeowners compensate the decline in their disposable income by working more.<sup>32</sup>

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<sup>31</sup>In equilibrium,  $X_{Ht}$  and  $X_{St}$  are related through the production function of homebuilders as  $\hat{X}_{Ht} = (1 - \varphi)\hat{X}_{St}$ , where a hat denotes percentage deviations from steady state.

<sup>32</sup>This is one possible channel how the shock may affect output (especially in countries in which households are personally liable for mortgage debt). But no doubt there are other channels, abstracted from in this model, that would likely work in the opposite direction (e.g., potentially adverse effects on financial institutions from the decline in housing market activity or input-output linkages between the construction sector and other sectors). The response of labor to the shock due to the wealth effects would also be mitigated under the Greenwood-Hercowitz-Huffman preferences.

Finally, recall from equations (21) and (22) that, in principle, the effects of the  $\bar{\pi}_t$  shock can be offset by sufficiently large adjustments in the path of the real interest rate  $r_t$ . Under FRM, the real rate persistently declines, thus working in the opposite direction of the  $\bar{\pi}_t$  shock. But this decline is clearly not sufficient to offset the increase in  $\bar{\pi}_t$  as both the nominal interest rate and inflation increase in response to the shock.<sup>33</sup>

Figure 4 shows the same plots for the economy with the access of homeowners to the bond market. The responses of real variables are essentially smooth versions of the responses in Figure 3. The magnitudes are also smaller but not insignificant. For instance, under ARM, the maximum decline of housing investment is 3.5%, compared with 6.3% in the previous case. The smoothness and smaller magnitudes of the responses reflect the fact that homeowners can now partially undo the real effects of monetary policy on their disposable income by borrowing and saving in the bond market.

## 6.2 Relating the model to empirical literature

Empirical support for some of these findings can be found in a household-level study by Di Maggio et al. (2014). The authors find that homeowners' purchases of durable and non-durable goods respond more to nominal interest rate changes in U.S. counties in which ARM debt dominates than in counties where FRM debt dominates. While the changes in nominal interest rates identified by their study may not necessarily occur due to inflation, recall from Section 3.2.3 that the immediate response of real mortgage payments under ARM to nominal interest rate shocks are approximately the same regardless of the source of the shock. Furthermore, the authors find that households that are more financially constraint respond more. Even though our model does not have heterogenous households, the above two experiments show that the responses of the representative homeowner are larger when it has no access to the one-period bond market than when it does (Appendix E also shows that

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<sup>33</sup>In the case of ARM, the response of the real interest rate even strengthens the effect of the  $\bar{\pi}_t$  shock a bit, at least initially, producing the more than one-for-one increase, relative to the shock, of inflation and the nominal interest rate in the first few periods.

increasing mortgage payments as a fraction of income increases the responses, which can be interpreted as being in line with the empirical finding that more indebted households respond more). Relevant macro-level studies are harder to find, mainly because the focus of the literature is on VAR responses to temporary monetary policy shocks of the type considered in New-Keynesian models. The identification restrictions used in this literature are not applicable here.

### 6.3 Persistence of the shock

Figure 5 demonstrates the effects on housing investment of reducing the persistence of the  $\bar{\pi}_t$  shock, using the economy in which homeowners can access the bond market. Specifically, it compares the responses of housing investment for  $\rho_\pi = 0.994$ , the baseline value, with the responses for  $\rho_\pi = 0.95$  and  $\rho_\pi = 0.5$ , the values considered in the partial equilibrium experiments in Figure 2. In the cases of  $\rho_\pi$  not being close to one, equation (21) has the form

$$i_t \approx \sum_{j=0}^{\infty} \left(\frac{1}{\nu_\pi}\right)^j E_t r_{t+1+j} - \sum_{j=1}^{\infty} \left(\frac{1}{\nu_\pi}\right)^j \rho_\pi^j \xi_t,$$

where  $\xi_t \equiv -(\nu_\pi - 1)(\bar{\pi}_t - \bar{\pi})$  in the re-arranged policy rule  $i_t = i + \nu_\pi(\pi_t - \bar{\pi}) + \xi_t$ . In addition to housing investment, Figure 5 plots the responses of the long-short spread; i.e.,  $i_t^F - i_t$ . The responses of the long-short spread show that, as the persistence of the shock declines, the shock starts to manifest itself as movements in the slope factor rather than in the level factor. The responses of housing investment becomes weaker and less persistent, at least in the case of ARM (in the case of FRM, the price effect becomes dominated by the wealth effects and the response of housing investment actually changes sign as the persistence of the shock declines).

## 6.4 Term of the loan

Figure 6 demonstrates—using again the economy in which homeowners can access the bond market—the effects of shortening the term of the loan from approximately 30 years to about 15 years ( $\kappa = 0.0094$ ,  $\alpha = 0.9912$  and steady-state  $\gamma = 0.0313$ ) and then to one period ( $\kappa = 1$ ,  $\alpha = 0$  and steady-state  $\gamma = 1$ ). The effects on housing investment are similar to the effects of reducing the shock persistence. When the term of the loan is just one period, the response becomes hardly visible (the response is positive as the only channel of transmission is a positive wealth effect for homeowners due to unexpectedly higher inflation in period 1).

## 6.5 Refinancing

In the model so far, the homeowner holds on to FRM mortgages taken out in the past even when interest rates fall. In actual economies, homeowners will likely refinance—repay the existing mortgage and take out a new mortgage at the lower interest rate. Refinancing involves costs, both monetary and in terms of time. When interest rates rise, homeowners have no incentive to refinance. Refinancing thus works like an option. Mortgage investors understand this behavior and price FRMs accordingly. Besides such ‘strategic’ refinancing, there are always homeowners who refinance for exogenous reasons, such as selling the house due to divorce or moving for a new job.

To make refinancing tractable in our framework, we consider only ‘symmetric’ refinancing. Each period, the homeowner chooses a fraction  $\varrho_t$  of the outstanding debt that he wishes to refinance, subject to a quadratic cost function in terms of time. Leisure in the utility function becomes  $1 - n_t - n_{Ft}$ , with the time spent in refinancing given by  $n_{Ft} = \varpi(\varrho_t - \varrho)^2$ , where  $\varpi > 0$  is a parameter and  $\varrho$  is the fraction of debt that is refinanced in steady state (due to the exogenous reasons). The refinancing cost is specified only in terms of time so that the definition of output is unaffected by this extension.

The laws of motion for the mortgage variables are as follows. First, debt evolves as

$$d_{t+1} = (1 - \varrho_t)(1 - \gamma_t)d_t + l_t,$$

where the implicit assumption is that refinancing occurs after the current-period mortgage payments (and thus amortization payments) have been made. New loans then consist of mortgages used for new house purchases and loans that are being refinanced

$$l_t = \theta p_t p_{Ht} x_{Ht} + \varrho_t(1 - \gamma_t)d_t.$$

Combining the above two equations gives back the original law of motion for debt (15). Second, the law of motion for the amortization rate stays the same, given by equation (16). This implicitly assumes that debt that is being refinanced has the initial amortization rate the same as what would be applied to it if it wasn't refinanced (this captures the notion that, for instance, a loan that is refinanced 10 years before maturity is replaced with a 10-year loan, rather than a loan of the full length of 30 years). This leads to a very stark characterization of refinancing, explained below. And third, the law of motion for the interest rate becomes

$$R_{t+1} = (1 - \Theta_t)R_t + \Theta_t i_t^F,$$

where  $\Theta_t \equiv l_t/d_{t+1}$ . Mortgage payments are as before,  $m_t = (R_t + \gamma_t)d_t$ . Thus, what refinancing does is to change the weights on the old effective interest rate and the current market interest rate in calculating the effective interest rate on the new stock of debt, without tying this change to new housing investment. The capital owner's laws of motion are changed analogously and the first-order condition for  $i_t^F$  now prices in the refinancing behavior of the homeowner. Notice that when  $\varrho_t = 1$ , i.e., the whole existing stock is refinanced,  $\Theta_t = 1$  and the evolution of  $R_{t+1}$  becomes the same as under ARM. When  $\varrho_t = 0$ ,  $\Theta_t = \theta_t$  (defined in Section 4.1.2) and the evolution of  $R_{t+1}$  becomes the same as in the original FRM case.

Thus, whether the model with refinancing resembles the original FRM version or the ARM version depends on parameter values.

The parameter  $\varrho$  is set so that the fraction of new loans in steady state due to refinancing,  $\varrho(1 - \gamma)d/l$ , is equal to 0.39, a long-run average (1987-2006, Freddie Mac's Weekly Primary Mortgage Market Survey). This implies  $\varrho = 0.02$ , i.e., 2% of the outstanding debt is refinanced quarterly in steady state.  $\varpi$  is set so as to match the elasticity of the fraction of new loans due to refinancing to the mortgage rate, which restricts  $\varpi$  to equal to 1.4.

Figure 7 shows the effect of refinancing, comparing the responses of housing investment with those in Figure 3. In response to a higher mortgage rate (up by 1 percentage point per annum), the fraction of outstanding debt that is refinanced declines on impact by about 1.6 percentage points (implying the fraction of new loans due to refinancing dropping from 0.39 to about 0.2). Refinancing then increases as the mortgage rate declines. By rebalancing the weights on the new and old interest rate in the direction of the lower rate, refinancing reduces the cost of mortgage finance for the homeowner. As a result, the wealth effects—due to higher inflation—dominate the price effect in the response of housing investment in Figure 7.

## 7 Concluding remarks

Mortgage payments and mortgage debt constitute a substantial part of household mandatory expenses and financial liabilities, respectively. In combination with the fact that mortgages are long-term loans set in nominal terms, it is natural to ask what role does mortgage finance play in the transmission of monetary policy? This paper attempts to establish these connections, currently missing from the literature. Like goods market imperfections provide a breeding ground for nominal price rigidities to play a role in the transmission of monetary policy in New-Keynesian models, financial market imperfections (incomplete asset markets) facilitate a transmission of monetary policy through mortgage contracts in our framework. Two channels of transmission are identified: the effective price of new housing

and current and expected future wealth effects of outstanding mortgage debt. These channels are embedded in a dynamic general equilibrium model populated by two household types, homeowners and capital owners, with the key characteristics of these groups observed in the data. General equilibrium considerations are, at least a priori, important as endogenous price adjustments (especially of the real interest rate) may potentially eliminate any significant real effects of monetary policy suggested by partial equilibrium reasoning.

Three key properties of the mortgage transmission mechanism emerge. First, the entire paths of nominal interest rates and inflation matter; monetary policy shocks that work like a level factor in the nominal yield curve have larger effects than transitory shocks, manifesting themselves as movements in the long-short spread. Second, the real effects are larger under ARM than FRM. And third, higher inflation—associated with shocks to the level factor—redistributes real income from mortgage investors to homeowners under FRM, but from homeowners to mortgage investors under ARM. While the redistribution under FRM is back-loaded, under ARM it is front-loaded and is akin to a real interest rate shock, even if monetary policy has no direct effect on the short-term real interest rate as in traditional models. We have further demonstrated that the effects are stronger the more costly it is for homeowners to smooth consumption over time through other financial assets and that shortening the term of the loan reduces the effectiveness of monetary policy. Under one-period loans, the real effects essentially disappear.

For the purpose of clarity, we have abstracted from the usual frictions in goods and labor markets as well as other channels through which housing finance affects the macroeconomy. The number of shocks was also limited. A natural extension, bringing the model closer to the data, should incorporate these additional features and study their interaction with the channels proposed here.

Another interesting extension would be to study the price and wealth effects in an overlapping generations model with realistic life-cycle dynamics. It is possible that the agents who face the price effects are different from those who face the wealth effects. Price effects

thus may affect housing investment while wealth effects may only feed into current and future consumption. The added complexity at the level of the household is likely to require compromises at the general equilibrium level. We have found that the general equilibrium price adjustments do not overturn in our model the key results obtain from partial equilibrium reasoning. Extrapolating from this, keeping the real interest rate exogenous (by considering a small open economy) may be a reasonable modeling choice in such endeavor.

A third possible extension involves studying the effects of uncertainty in monetary policy on housing investment and the economy. One can imagine that long-run inflation and interest rate risk should matter in the presence of long-term nominal loans. We have experimented in our model with recursive preferences on both the homeowner's and the capital owner's side. But like in other macro models with representative agents, the effect on the dynamics of aggregate quantities turned out to be small. Further analysis in this direction may require a richer model with individual-level costs of adjusting housing and mortgage debt, perhaps in combination with individual-level income risk.

Finally, an interesting normative question regards the design of optimal monetary policy in an environment like ours. Optimal monetary policy is likely to depend on the prevalent mortgage type, ARM or FRM, in the economy. Different countries may thus follow different policies depending on their institutional environment. This aspect of the optimal policy is likely to complicate matters further if a common monetary policy is to be conducted for an area with different mortgage markets, such as the Eurozone. Extending the model to allow for default and a banking sector may also generate interesting interactions between optimal monetary and macroprudential policies. All these extension are left for future research.



## References

- Alm, J., Follain, J. R., 1984. Alternative mortgage instruments, the tilt problem, and consumer welfare. *Journal of Financial and Quantitative Analysis* 19, 113–26.
- Aruoba, S. B., Davis, M. A., Wright, R., 2012. Homework in monetary economics: Inflation, home production, and the production of homes. NBER Working Paper 18276.
- Atkeson, A., Kehoe, P. J., 2009. On the need for a new approach to analyzing monetary policy. In: *NBER Macroeconomics Annual, Volume 23*. National Bureau of Economic Research, Inc.
- Auclert, A., 2014. Monetary policy and the redistribution channel. Mimeo.
- Benigno, P., Woodford, M., 2006. Optimal taxation in an RBC model: A linear-quadratic approach. *Journal of Economic Dynamics and Control* 30, 1445–89.
- Bernanke, B. S., Gertler, M., 1995. Inside the black box: The credit channel of monetary transmission. *Journal of Economic Perspectives* 9, 27–48.
- Brunnermeier, M. K., Julliard, C., 2008. Money illusion and housing frenzies. *Review of Financial Studies* 21, 135–180.
- Calza, A., Monacelli, T., Stracca, L., 2013. Housing finance and monetary policy. *Journal of the European Economic Association* 11, 101–122.
- Campbell, J. Y., 2012. Mortgage market design. NBER Working Paper 18339.
- Campbell, J. Y., Cocco, J. F., 2003. Household risk management and optimal mortgage choice. *Quarterly Journal of Economics* 118, 1449–94.
- Campbell, J. Y., Cocco, J. F., 2015. A model of mortgage default. *Journal of Finance* Forthcoming.
- Chambers, M., Garriga, C., Schlagenhauf, D., 2009a. Accounting for changes in the homeownership rate. *International Economic Review* 50, 677–726.
- Chambers, M., Garriga, C., Schlagenhauf, D., 2009b. The loan structure and housing tenure decisions in an equilibrium model of mortgage choice. *Review of Economic Dynamics* 12, 444–68.
- Chatterjee, S., Corbae, D., Nakajima, M., Rios-Rull, J.-V., 2007. A quantitative theory of unsecured consumer credit with risk of default. *Econometrica* 75, 1525–89.
- Chatterjee, S., Eyigungor, B., 2011. A quantitative analysis of the US housing and mortgage markets and the foreclosure crisis. Working Paper 11-26, Federal Reserve Bank of Philadelphia.
- Corbae, D., Quintin, E., 2011. Mortgage innovation and the foreclosure boom. Mimeo.

- Davis, M. A., Heathcote, J., 2005. Housing and the business cycle. *International Economic Review* 46, 751–84.
- Di Maggio, M., Kermani, A., Ramcharan, R., 2014. Monetary policy pass-through: Household consumption and voluntary deleveraging. Mimeo.
- Doepke, M., Schneider, M., 2006. Inflation and the redistribution of nominal wealth. *Journal of Political Economy* 114, 1069–97.
- Doepke, M., Schneider, M., Selezneva, V., 2015. Distributional effects of monetary policy. Working Paper 14, Hutchins Center on Fiscal and Monetary Policy at Brookings.
- Dressler, S. J., Li, V. E., 2009. Inside money, credit, and investment. *Journal of Economic Dynamics and Control* 33, 970–84.
- Edge, R. M., 2000. The effect of monetary policy on residential and structures investment under differential project planning and completion times. *International Finance Discussion Paper 671*, Board of Governors of the Federal Reserve System.
- European Mortgage Federation, 2009. EMF Factsheet 2009—France. Brussels.
- European Mortgage Federation, 2012a. Study on mortgage interest rates in the EU. Brussels.
- European Mortgage Federation, 2012b. EMF Factsheet 2012—Germany. Brussels.
- European Mortgage Federation, 2012c. EMF Factsheet 2012—Denmark. Brussels.
- Favilukis, J., Ludvigson, S. C., Van Nieuwerburgh, S., 2011. The macroeconomic effects of housing wealth, housing finance and limited risk sharing in general equilibrium. Mimeo, New York University.
- Gallmeyer, M. F., Hollifield, B., Palomino, F., Zin, S. E., 2007. Arbitrage-free bond pricing with dynamic macroeconomic models. *Federal Reserve Bank of St. Louis Review* July/August, 305–326.
- Garriga, C., Schlagenhaut, D. E., 2009. Home equity, foreclosures, and bail-outs. Mimeo.
- Gavin, W. T., Keen, B. D., Pakko, M. R., 2005. The monetary instrument matters. *Reserve Bank of St. Louis Quarterly Review* September/October, 633–58.
- Ghent, A., 2012. Infrequent housing adjustment, limited participation, and monetary policy. *Journal of Money, Credit, and Banking* 44, 931–55.
- Gomes, J., Jermann, U., Schmid, L., 2013. Sticky leverage. Mimeo.
- Gomme, P., Ravikumar, B., Rupert, P., 2011. The return to capital and the business cycle. *Review of Economic Dynamics* 14, 262–78.
- Gomme, P., Rupert, P., 2007. Theory, measurement and calibration of macroeconomic models. *Journal of Monetary Economics* 54, 460–97.

- Graham, L., Wright, S., 2007. Nominal debt dynamics, credit constraints and monetary policy. *The B.E. Journal of Macroeconomics* 7, Article 9.
- Green, R. K., Wachter, S. M., 2005. The American mortgage in historical and international context. *Journal of Economic Perspectives* 19, 93–114.
- Hancock, M., Wood, R., 2004. Household secured debt. *Bank of England Quarterly Bulletin* Autumn, 291–301.
- Hansen, G. D., Prescott, E. C., 1995. Recursive methods for computing equilibria of business cycle models. In: Cooley, T. F. (Ed.), *Frontiers of Business Cycle Research*. Princeton University Press.
- Hilscher, J., Raviv, A., Reis, R., 2014. Inflating away the public debt? An empirical assessment. *Mimeo*.
- Huffman, G. W., Wynne, M. A., 1999. The role of intratemporal adjustment costs in a multisector economy. *Journal of Monetary Economics* 43, 317–50.
- Iacoviello, M., 2005. House prices, borrowing constraints, and monetary policy in the business cycle. *American Economic Review* 95, 739–64.
- Iacoviello, M., 2010. Housing in DSGE models: Findings and new directions. In: de Bandt, O., Knetsch, T., Peñalosa, J., Zollino, F. (Eds.), *Housing Markets in Europe: A Macroeconomic Perspective*. Springer-Verlag Berlin.
- Iacoviello, M., Minetti, R., 2008. The credit channel of monetary policy: Evidence from the housing market. *Journal of Macroeconomics* 30, 69–96.
- International Monetary Fund, 2011. Housing finance and financial stability—back to basics? In: *Global Financial Stability Report, April 2011: Durable Financial Stability—Getting There from Here*. International Monetary Fund, Washington, DC.
- Ireland, P. N., 2007. Changes in the Federal Reserve’s inflation target: Causes and consequences. *Journal of Money, Credit, and Banking* 39, 1851–82.
- Kearl, J. R., 1979. Inflation, mortgage, and housing. *Journal of Political Economy* 87, 1115–38.
- Kearl, J. R., Rosen, K., Swan, C., 1975. Relationships between the mortgage instruments, the demand for housing, and mortgage credit. In: *New Mortgage Designs for Stable Housing in an Inflationary Environment*. Federal Reserve Bank of Boston, Boston.
- Koijen, R. S. J., Van Hemert, O., Van Nieuwerburgh, S., 2009. Mortgage timing. *Journal of Financial Economics* 93, 292–324.
- Kydland, F. E., Rupert, P., Sustek, R., 2014. Housing dynamics over the business cycle. *Discussion Paper 2014-23*, Centre for Macroeconomics.
- Kydland, F. E., Rupert, P., Sustek, R., forthcoming. Housing dynamics over the business cycle. *International Economic Review*.

- Lessard, D. R., Modigliani, F., 1975. Inflation and the housing market: Problems and potential solutions. In: *New Mortgage Designs for Stable Housing in an Inflationary Environment*. Federal Reserve Bank of Boston, Boston.
- Li, V. E., Chang, C. Y., 2004. The cyclical behavior of household and business investment in a cash-in-advance economy. *Journal of Economic Dynamics and Control* 28, 691–706.
- Meh, C. A., Rios-Rull, J. V., Terajima, Y., 2010. Aggregate and welfare effects of redistribution of wealth under inflation and price-level targeting. *Journal of Monetary Economics* 57, 637–52.
- Mendoza, E. G., Razin, A., Tesar, L. L., 1994. Effective tax rates in macroeconomics: Cross-country estimates of tax rates on factor incomes and consumption. *Journal of Monetary Economics* 34, 297–323.
- Miles, D., 2004. The UK mortgage market: Taking a longer-term view. Special report, HM Treasury.
- Piazzesi, M., 2006. Affine term structure models. In: Ait-Sahalia, Y., Hansen, L. P. (Eds.), *Handbook of Financial Econometrics*. Elsevier, Amsterdam.
- Poterba, J. M., 1984. Tax subsidies to owner-occupied housing: An asset-market approach. *Quarterly Journal of Economics* 99, 729–52.
- Rajan, R. G., Zingales, L., 1995. What do we know about capital structure? Some evidence from international data. *Journal of Finance* 50, 1421–60.
- Rubio, M., 2011. Fixed- and variable-rate mortgages, business cycles, and monetary policy. *Journal of Money, Credit, and Banking* 43, 657–88.
- Scanlon, K., Whitehead, C., 2004. International trends in housing tenure and mortgage finance. Special report for the Council of Mortgage Lenders, London School of Economics.
- Schwab, R. M., 1982. Inflation expectations and the demand for housing. *American Economic Review* 72, 143–53.
- Sheedy, K., 2013. Debt and incomplete financial markets: A case for nominal GDP targeting. CEP Discussion Paper 1209.
- Sterk, V., Tenreyro, S., 2013. The transmission of monetary policy operations through redistributions and durable purchases. Discussion Paper 2013-05, Centre for Macroeconomics.
- Woodford, M., 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.

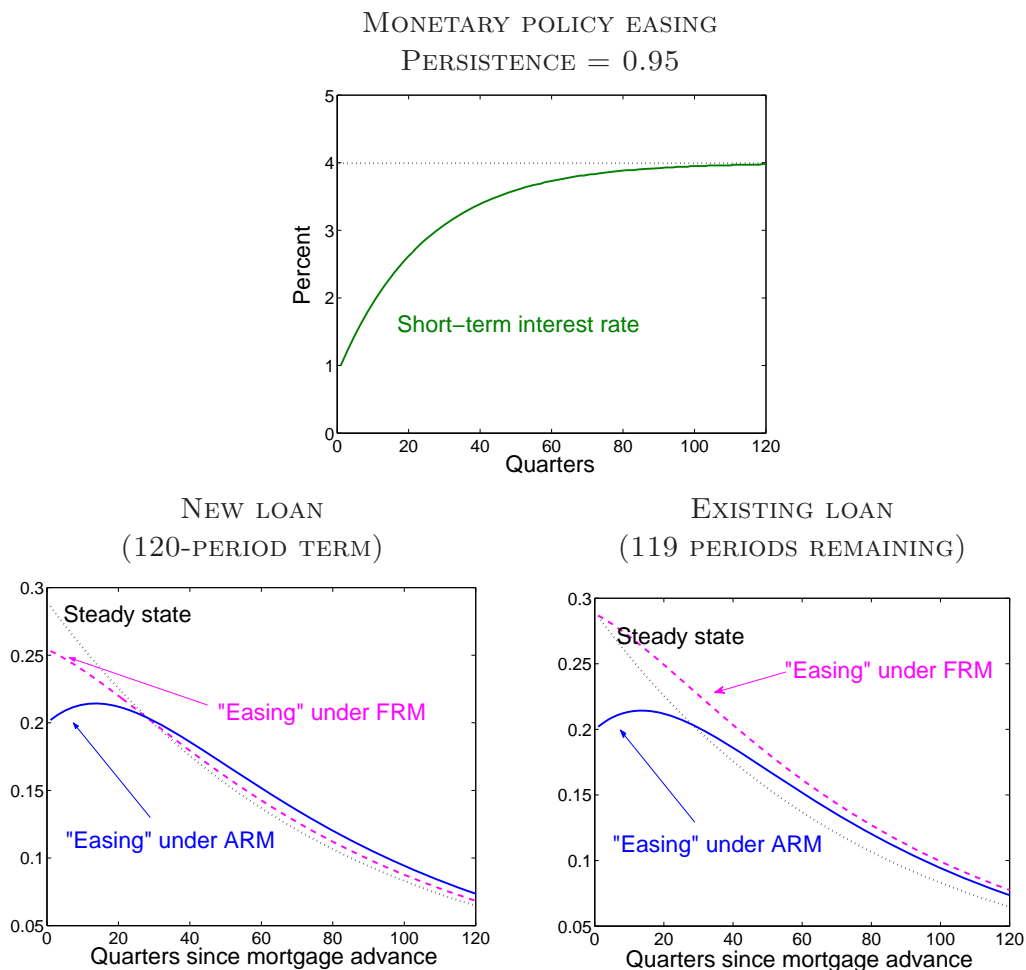
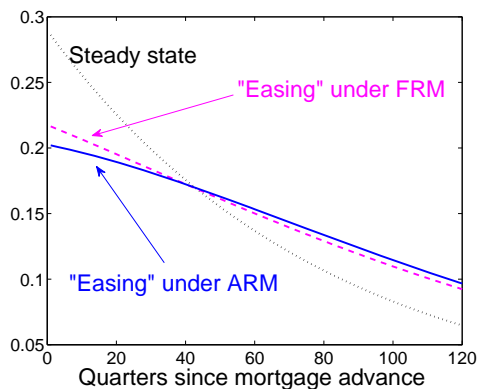


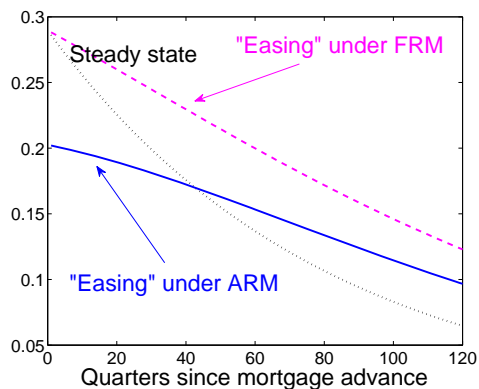
Figure 1: Illustration of price and expected future wealth effects. Debt-servicing costs over a term of a new and an existing 30-year mortgage under alternative paths of the short-term nominal interest rate. The label ‘steady-state’ refers to the case when the short rate is at its steady-state level of 4%. The mortgage is equal to four times the household’s income; the real interest rate is held constant at 1% per annum.

**A. High persistence (0.99) of the short rate decline**

NEW LOAN  
(120-PERIOD TERM)

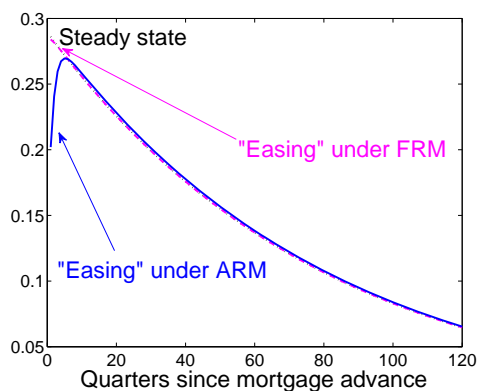


EXISTING LOAN  
(119 PERIODS REMAINING)



**B. Low persistence (0.5) of the short rate decline**

NEW LOAN  
(120-PERIOD TERM)



EXISTING LOAN  
(119 PERIODS REMAINING)

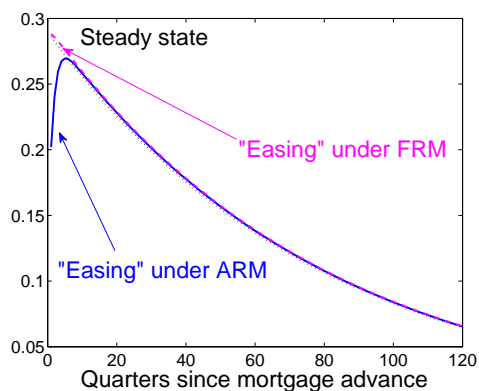


Figure 2: Illustration of price and expected future wealth effects for high and low persistence of the mean-reverting short rate decline by 3 percentage points.

Table 1: Calibration

Symbol	Value	Description
Population		
$\Psi$	2/3	Share of homeowners
Technology		
$A$	1.5321	Steady-state level of TFP
$\varsigma$	0.283	Capital share of output
$\delta_K$	0.02225	Depreciation rate of capital
$\delta_H$	0.01021	Depreciation rate of housing
$\zeta$	0.35	Curvature of PPF
$\varphi$	0.1	Land share of new housing
Fiscal		
$G$	0.138	Government expenditures
$\tau_N$	0.235	Labor income tax rate
$\tau_K$	0.3362	Capital income tax rate
$\tau$	0.4503	Transfer
Preferences		
$\beta$	0.9883	Discount factor
$\omega$	0.2478	Cons. composite's share in utility
$\xi$	0.6009	Share of market cons. in composite
Mortgages		
$\theta$	0.76	Loan-to-value ratio
$\kappa$	0.00162	Initial amortization rate
$\alpha$	0.9946	Amortization adjustment factor
Bond market		
$\vartheta$	0.035	Participation cost function
Monetary policy rule		
$\nu_\pi$	1.35	Weight on inflation
$\bar{\pi}$	0.0113	Steady-state inflation rate
Exogenous processes		
$\rho_A$	0.9641	Persistence of TFP shocks
$\sigma_A$	0.0082	Std. of TFP innovations
$\rho_\pi$	0.994	Persistence of infl. target shocks
$\sigma_\pi$	0.0015	Std. of infl. target innovations

Table 2: Nonstochastic steady state and long-run averages of data

Symbol	Model	Data	Description
Normalized:			
$Y$	1.0	N/A	Output
Targeted in calibration:			
$K$	7.06	7.06	Capital stock
$H$	5.28	5.28	Housing stock
$X_K$	0.156	0.156	Capital investment
$X_S$	0.054	0.054	New housing structures
$N$	0.255	0.255	Hours worked
$\widetilde{M}/(wN - \Psi\tau)$	0.185	0.185	Debt-servicing costs (pre-tax)
$i^M$	0.0233	0.0233	Mortgage rate
Not targeted:			
Aggregate mortgage variables			
$\widetilde{D}$	1.61	2.35 <sup>†</sup>	Mortgage debt
$\gamma$	0.0144	0.0118 <sup>‡</sup>	Amortization rate
Capital owner's variables			
$(1 - \tau_K)(r - \delta_K)$	0.012	0.013 <sup>§</sup>	Net (post-tax) rate of return on capital
$[(r - \delta)k + \widetilde{m}^*]/[(r - \delta)k + \widetilde{m}^* + \tau^*]$	0.33	0.39 <sup>¶,§§</sup>	Income from assets to total income
Homeowner's variables			
$\tau_H$	0	N/A	Housing wedge
$\widetilde{m}/[(1 - \tau_N)(wn - \tau)]$	0.24	N/A	Debt-servicing costs (post-tax)
$(wn - \tau)/(wn - \tau)$	1.00	0.81 <sup>¶</sup>	Income from labor to total income
Distribution of wealth			
$(K + \widetilde{D})/(K + H)$	0.71	0.82 <sup>¶</sup>	Capital owners
$(H - \widetilde{D})/(K + H)$	0.29	0.18 <sup>¶</sup>	Homeowners

Note: Rates of return and interest and amortization rates are expressed at quarterly rates; capital owners = the 5th quintile of the SCF wealth distribution; homeowners = the 3rd and 4th quintiles of the SCF wealth distribution.

<sup>†</sup> Upper bound for the mortgage debt in the model due to the presence in the data of equity loans, second mortgages, and mortgages for purchases of existing homes.

<sup>‡</sup> For a standard 30-year mortgage.

<sup>§</sup> NIPA-based estimate of Gomme et al. (2011).

<sup>¶</sup> 1998 SCF; the model counterpart is defined so as to be consistent with the definition in SCF.

<sup>§§</sup> The sum of capital and business income.



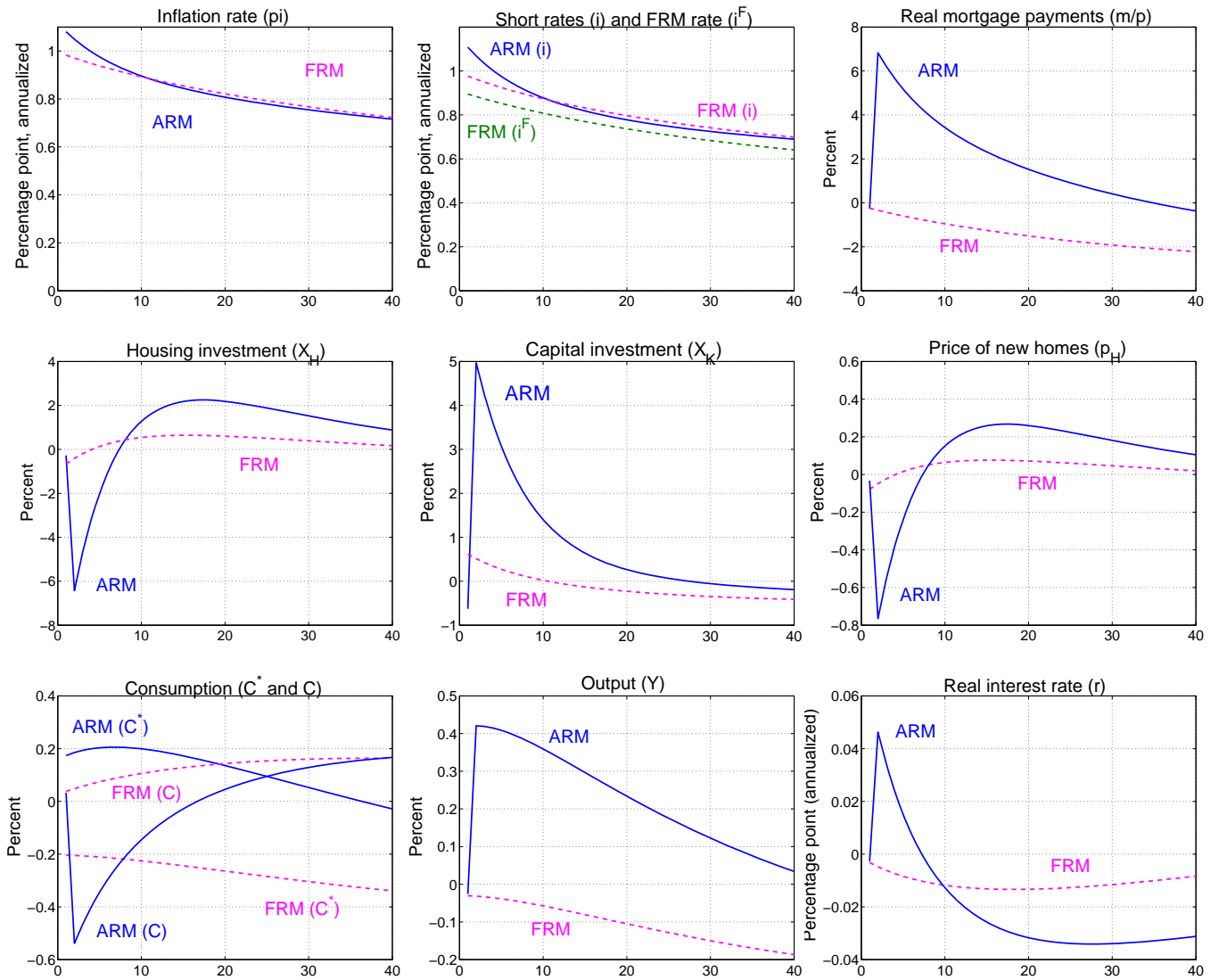


Figure 3: General equilibrium responses to 1 percentage point (annualized) increase in  $\bar{\pi}_t$  in period 1; version with no access of homeowners to the 1-period bond market.

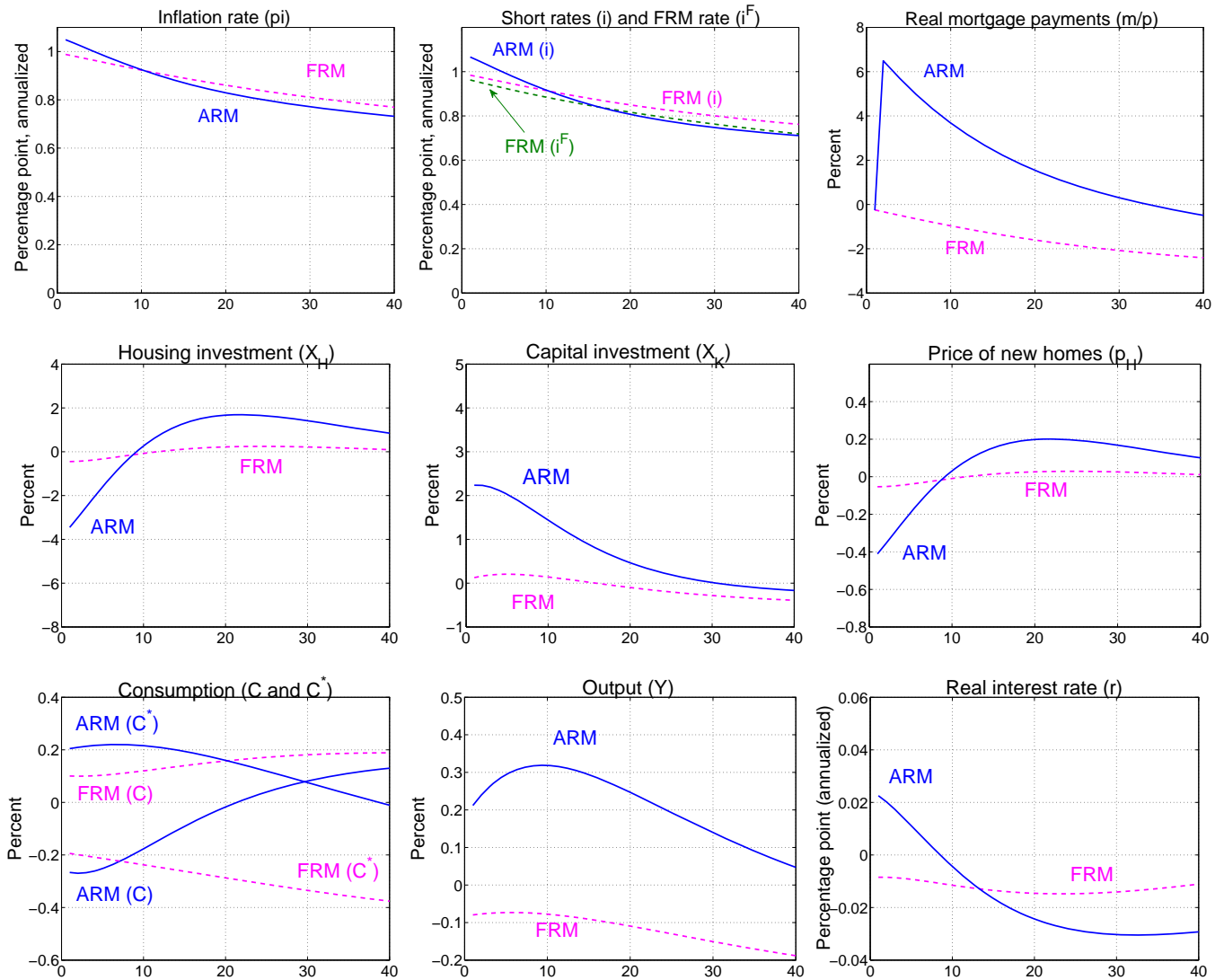


Figure 4: General equilibrium responses to 1 percentage point (annualized) increase in  $\bar{\pi}_t$  in period 1; version with access of homeowners to the 1-period bond market.

A.  $\rho_\pi = 0.994$

B.  $\rho_\pi = 0.95$

C.  $\rho_\pi = 0.50$

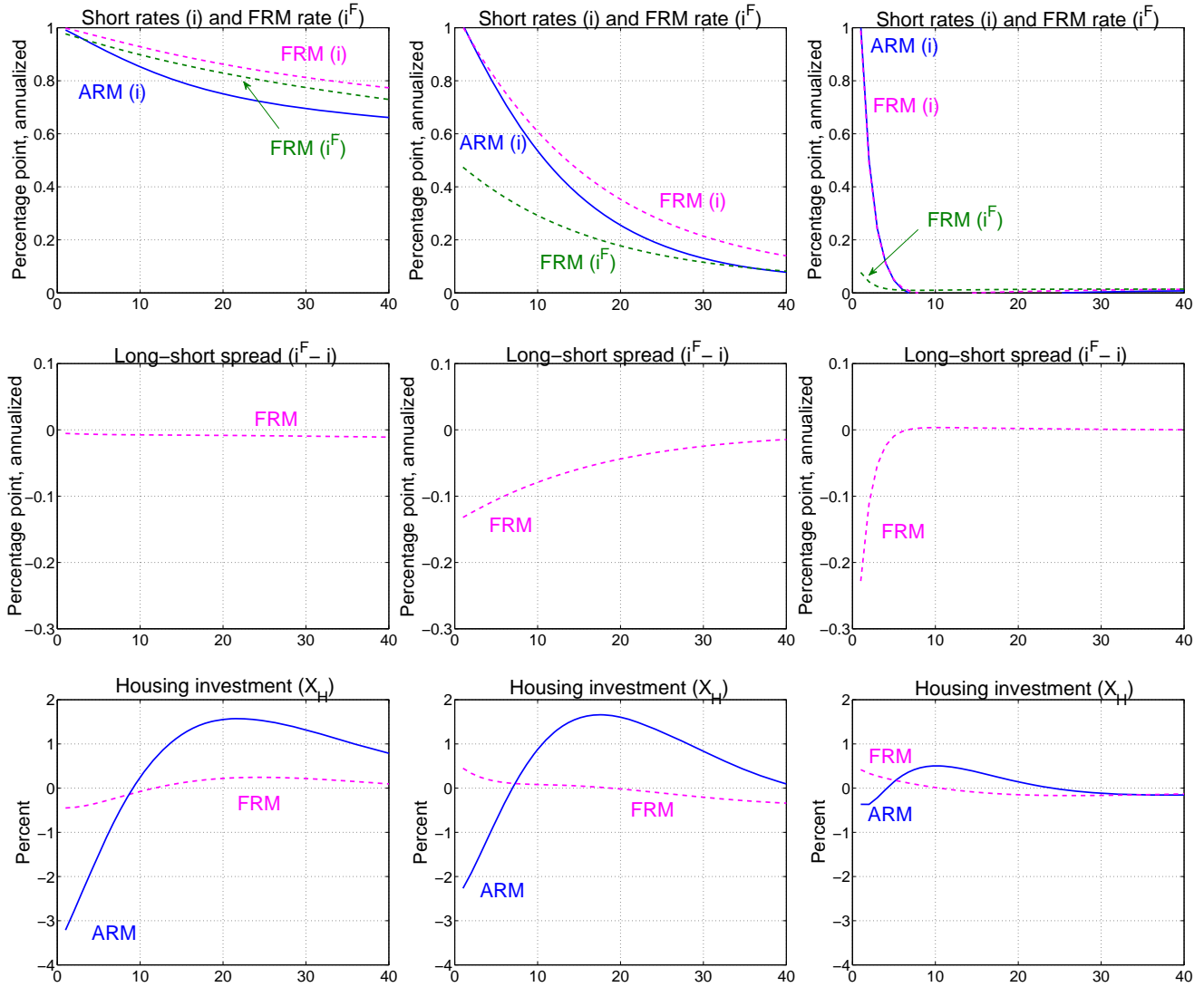


Figure 5: The effect of the persistence of the short-term nominal interest rate. General equilibrium responses to an increase in  $\bar{\pi}_t$  in period 1 scaled so as to generate 1 percentage point increase in the short rate in period 1; version with access of homeowners to the 1-period bond market.

A. approx. 120 periods (30yrs)

B. approx. 60 periods (15yrs)

C. 1 period

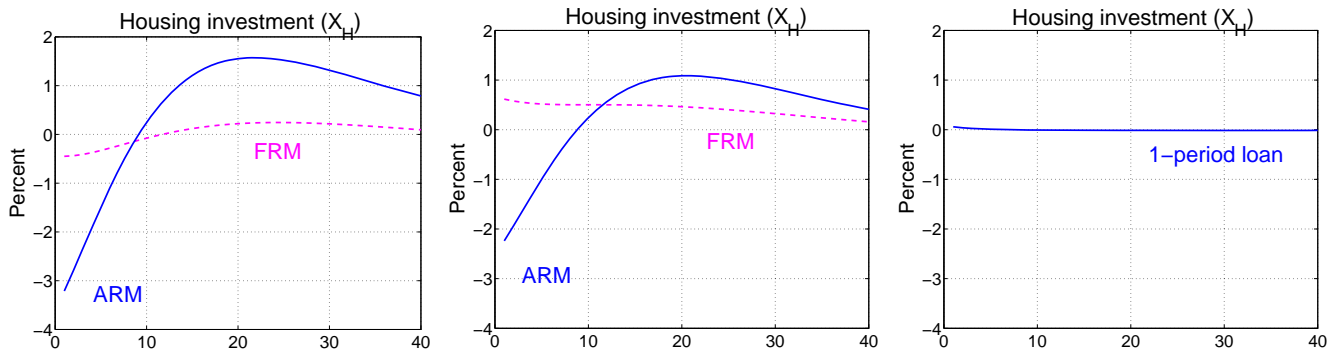


Figure 6: The effect of the term of the loan. General equilibrium responses to 1 percentage point (annualized) increase in  $\bar{\pi}_t$  in period 1; version with access of homeowners to the 1-period bond market. Shock persistence is  $\rho_\pi = 0.994$ .

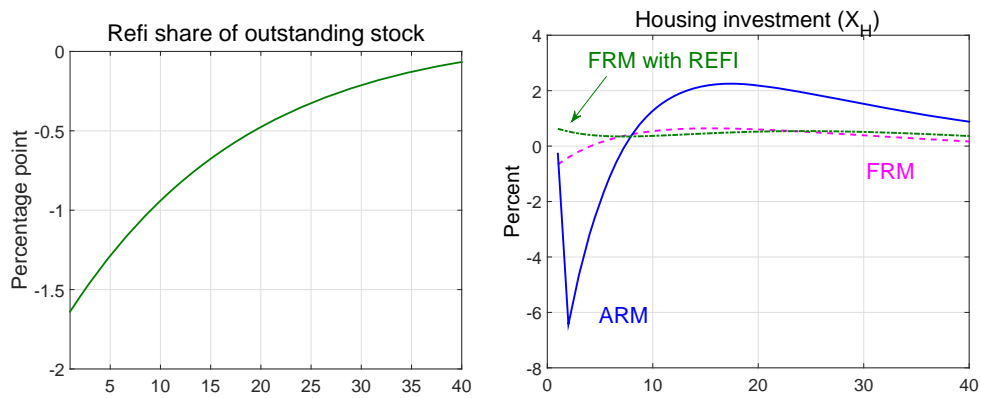


Figure 7: The effect of refinancing. General equilibrium responses to 1 percentage point (annualized) increase in  $\bar{\pi}_t$  in period 1; version with no access of homeowners to the 1-period bond market. Shock persistence is  $\rho_\pi = 0.994$ .

# Supplemental material—appendices

## Appendix A: Equilibrium conditions

This appendix lists the conditions characterizing the equilibrium defined in Section 4.2. Throughout, the notation is that, for instance,  $u_{ct}$  denotes the first derivative of the function  $u$  with respect to  $c$ , evaluated in period  $t$ . Alternatively,  $v_{2t}$ , for instance, denotes the first derivative of the function  $v$  with respect to the second argument, evaluated in period  $t$ .

### Capital owner's optimality

The first-order conditions with respect to, respectively,  $x_{Kt}$ ,  $b_{t+1}^*$ , and  $l_t^*$ :

$$\begin{aligned} 1 &= E_t \left\{ \beta \frac{u_{c,t+1}}{u_{ct}} [1 + (1 - \tau_K)(r_{t+1} - \delta_K)] \right\}, \\ 1 &= E_t \left[ \beta \frac{u_{c,t+1}}{u_{ct}} \left( \frac{1 + i_t}{1 + \pi_{t+1}} \right) \right], \\ 1 &= E_t \left\{ \beta \frac{\widehat{U}_{d,t+1}}{u_{ct}} + \beta \frac{U_{\gamma,t+1}}{u_{ct}} \zeta_{Dt}^* [\kappa - (\gamma_t^*)^\alpha] + \beta \frac{U_{R,t+1}}{u_{ct}} \zeta_{Dt}^* (i_t^F - R_t^*) \right\}. \end{aligned}$$

In the first-order condition for  $l_t^*$ , which—as discussed in the text—applies only in the FRM case,  $\widehat{U}_{dt} \equiv p_{t-1} U_{dt}$  is a normalization to ensure stationarity in the presence of positive steady-state inflation and  $U_{dt}$ ,  $U_{\gamma t}$ , and  $U_{Rt}$  are the derivatives of the capital owner's value function with respect to  $d_t^*$ ,  $\gamma_t^*$ , and  $R_t^*$ , respectively. These derivatives are given by the Benveniste-Scheinkman (BS) conditions:

$$\widetilde{U}_{dt} = u_{ct} \frac{R_t^* + \gamma_t^*}{1 + \pi_t} + \beta \frac{1 - \gamma_t^*}{1 + \pi_t} E_t \left\{ \widetilde{U}_{d,t+1} + \zeta_{lt}^* [(\gamma_t^*)^\alpha - \kappa] U_{\gamma,t+1} + \zeta_{lt}^* (R_t^* - i_t^F) U_{R,t+1} \right\},$$

$$\begin{aligned} U_{\gamma t} &= u_{ct} \left( \frac{\widetilde{d}_t^*}{1 + \pi_t} \right) - \beta \left( \frac{\widetilde{d}_t^*}{1 + \pi_t} \right) E_t \widetilde{U}_{d,t+1} \\ &+ \beta \left( \frac{\widetilde{d}_t^*}{1 + \pi_t} \right) \left\{ \zeta_{lt}^* [\kappa - (\gamma_t^*)^\alpha] + \frac{(1 - \gamma_t^*) \alpha (\gamma_t^*)^{\alpha-1}}{\frac{1 - \gamma_t^*}{1 + \pi_t} \widetilde{d}_t^* + \widetilde{l}_t^*} \right\} E_t U_{\gamma,t+1} \\ &+ \beta \left( \frac{\widetilde{d}_t^*}{1 + \pi_t} \right) \zeta_{lt}^* (i_t^F - R_t^*) E_t U_{R,t+1}, \end{aligned}$$

$$U_{Rt} = u_{ct} \left( \frac{\widetilde{d}_t^*}{1 + \pi_t} \right) + \beta \left( \frac{\frac{1 - \gamma_t^*}{1 + \pi_t} \widetilde{d}_t^*}{\frac{1 - \gamma_t^*}{1 + \pi_t} \widetilde{d}_t^* + \widetilde{l}_t^*} \right) E_t U_{R,t+1}.$$

In these expressions,  $\tilde{d}_t^* \equiv d_t^*/p_{t-1}$ ,  $\tilde{l}_t^* \equiv l_t^*/p_t$ ,

$$\zeta_{lt}^* \equiv \frac{\tilde{l}_t^*}{\left(\frac{1-\gamma_t^*}{1+\pi_t} \tilde{d}_t^* + \tilde{l}_t^*\right)^2} \in (0, 1),$$

and

$$\zeta_{Dt}^* \equiv \frac{\frac{1-\gamma_t^*}{1+\pi_t} \tilde{d}_t^*}{\left(\frac{1-\gamma_t^*}{1+\pi_t} \tilde{d}_t^* + \tilde{l}_t^*\right)^2} \in (0, 1).$$

Notice that for a once-and-for-all mortgage loan ( $l_t^* = l^*$  in period  $t$  and  $l_t^* = 0$  thereafter) and no outstanding mortgage debt ( $d_t^* = 0$  in period  $t$ ), we have  $\zeta_{Dt}^* = 0$  and  $\zeta_{l,t+j}^* = 0$ , for  $j = 1, 2, \dots$ . In this case, the first-order condition for  $l_t^*$  and the BS condition for  $\tilde{U}_{dt}$  simplify, as the terms related to  $U_{\gamma t}$  and  $U_{Rt}$  drop out. Once combined, the two optimality conditions result in an equation that is a straightforward infinite-horizon extension of the mortgage-pricing equation (1) in the two-period mortgage example of Section 3:

$$1 = E_t [Q_{1t}^* (i_t^F + \gamma_{t+1}^*) + Q_{2t}^* (i_t^F + \gamma_{t+2}^*) (1 - \gamma_{t+1}^*) + \dots],$$

where

$$Q_{Jt}^* \equiv \prod_{j=1}^J \beta \frac{u_{c,t+j}}{u_{c,t+j-1}} \frac{1}{1 + \pi_{t+j}} \quad J = 1, 2, \dots$$

The terms related to  $U_{\gamma t}$  and  $U_{Rt}$  in the general form of the optimality conditions arise because the mortgage payment  $m_t^*$  entering the budget constraint of the capital owner pertains to payments on the entire outstanding mortgage debt, not just the new loan. In this case, the terms related to  $U_{\gamma t}$  and  $U_{Rt}$  capture the marginal effect of  $l_t^*$  on the average interest and amortization rates of the outstanding debt, and thus the marginal effect of  $l_t^*$  on the mortgage payments on the outstanding debt.

The capital owner's constraints:

$$c_t^* + k_{t+1} + \tilde{b}_{t+1}^* + \tilde{l}_t^* = [1 + (1 - \tau_K)(r_t - \delta_K)] k_t + (1 + i_{t-1}) \frac{\tilde{b}_t^*}{1 + \pi_t} + \tilde{m}_t^* + \tau_t^* + \frac{pL_t}{1 - \Psi},$$

$$\tilde{m}_t^* = (R_t^* + \gamma_t^*) \frac{\tilde{d}_t^*}{1 + \pi_t},$$

$$\tilde{d}_{t+1}^* = \frac{1 - \gamma_t^*}{1 + \pi_t} \tilde{d}_t^* + \tilde{l}_t^*,$$

$$\gamma_{t+1}^* = (1 - \phi_t^*) (\gamma_t^*)^\alpha + \phi_t^* \kappa,$$

$$R_{t+1}^* = \begin{cases} (1 - \phi_t^*) R_t^* + \phi_t^* i_t^F, & \text{if FRM,} \\ i_t, & \text{if ARM,} \end{cases}$$

where  $\phi_t^* \equiv \tilde{l}_t^*/\tilde{d}_{t+1}^*$  and  $\tilde{b}_t^* \equiv b_t^*/p_{t-1}$ .

## Homeowner's optimality

The first-order conditions with respect to, respectively,  $n_t$ ,  $x_{Ht}$ , and  $b_{t+1}$ :

$$v_{ct}(1 - \tau_N)w_t = v_{2t},$$

$$v_{ct}(1 - \theta)p_{Ht} = \beta E_t \left\{ V_{h,t+1} + p_{Ht}\theta \left[ \tilde{V}_{d,t+1} + \zeta_{Dt}(\kappa - \gamma_t^\alpha)V_{\gamma,t+1} + \zeta_{Dt}(i_{t+1}^M - R_t)V_{R,t+1} \right] \right\},$$

$$1 = E_t \left[ \beta \frac{v_{c,t+1}}{v_{ct}} \left( \frac{1 + i_t + \Upsilon_t}{1 + \pi_{t+1}} \right) \right],$$

where  $\tilde{V}_{dt} \equiv p_{t-1}V_{dt}$  and  $V_{ht}$ ,  $V_{dt}$ ,  $V_{\gamma t}$ , and  $V_{Rt}$  are the derivatives of the homeowner's value function. Further,  $i_{t+1}^M = i_t^F$  in the FRM case and  $i_{t+1}^M = i_t$  in the ARM case. Analogously to the case of the capital owner,

$$\zeta_{Dt} \equiv \frac{\frac{1-\gamma_t}{1+\pi_t}\tilde{d}_t}{\left(\frac{1-\gamma_t}{1+\pi_t}\tilde{d}_t + \tilde{l}_t\right)^2} \in (0, 1).$$

The derivatives of the value function with respect to  $d_t$ ,  $\gamma_t$ , and  $R_t$  are given by BS conditions, which take similar forms as those of the capital owner:

$$\tilde{V}_{dt} = -v_{ct}\frac{R_t + \gamma_t}{1 + \pi_t} + \beta\frac{1 - \gamma_t}{1 + \pi_t}E_t \left[ \tilde{V}_{d,t+1} + \zeta_{lt}(\gamma_t^\alpha - \kappa)V_{\gamma,t+1} + \zeta_{lt}(R_t - i_{t+1}^M)V_{R,t+1} \right],$$

$$V_{\gamma t} = -v_{ct} \left( \frac{\tilde{d}_t}{1 + \pi_t} \right) - \beta \left( \frac{\tilde{d}_t}{1 + \pi_t} \right) E_t \tilde{V}_{d,t+1}$$

$$+ \beta \left( \frac{\tilde{d}_t}{1 + \pi_t} \right) \left[ \zeta_{lt}(\kappa - \gamma_t^\alpha) + \frac{(1 - \gamma_t)\alpha\gamma_t^{\alpha-1}}{\frac{1-\gamma_t}{1+\pi_t}\tilde{d}_t + \tilde{l}_t} \right] E_t V_{\gamma,t+1}$$

$$+ \beta \left( \frac{\tilde{d}_t}{1 + \pi_t} \right) \zeta_{lt}(i_{t+1}^M - R_t)E_t V_{R,t+1},$$

$$V_{Rt} = -v_{ct} \left( \frac{\tilde{d}_t}{1 + \pi_t} \right) + \beta \left( \frac{\frac{1-\gamma_t}{1+\pi_t}\tilde{d}_t}{\frac{1-\gamma_t}{1+\pi_t}\tilde{d}_t + \tilde{l}_t} \right) E_t V_{R,t+1},$$

where

$$\zeta_{lt} \equiv \frac{\tilde{l}_t}{\left(\frac{1-\gamma_t}{1+\pi_t}\tilde{d}_t + \tilde{l}_t\right)^2} \in (0, 1).$$

In addition, there is a BS condition for the derivative with respect to  $h_t$ :

$$V_{ht} = v_{ht} + \beta(1 - \delta_H)E_t V_{h,t+1}.$$



Rearranging the first-order condition for  $x_{Ht}$  yields

$$v_{ct}p_{Ht}(1 + \tau_{Ht}) = \beta E_t V_{h,t+1},$$

where the wedge  $\tau_{Ht}$  is given by

$$\tau_{Ht} \equiv -\theta E_t \left[ 1 + \beta \frac{\tilde{V}_{d,t+1}}{v_{ct}} + \zeta_{Dt}(\kappa - \gamma_t^\alpha) \beta \frac{V_{\gamma,t+1}}{v_{ct}} + \zeta_{Dt}(i_{t+1}^M - R_t) \beta \frac{V_{R,t+1}}{v_{ct}} \right].$$

For the same reasons as in the case of the mortgage-pricing equation of the capital owner, the wedge is more complicated than in the case of the two-period mortgage. Again, it becomes a straightforward infinite-horizon extension of either equation (2) or (3) in the main text if the housing investment decision is once-and-for-all and there is no outstanding mortgage debt ( $\Rightarrow \zeta_{Dt} = 0$  and  $\zeta_{l,t+j} = 0$ , for  $j = 1, 2, \dots$ ):

$$\tau_{Ht} \equiv -\theta E_t \left\{ 1 - [Q_{1t}(i_{t+1}^M + \gamma_{t+1}) + Q_{2t}(i_{t+2}^M + \gamma_{t+2})(1 - \gamma_{t+1}) + \dots] \right\},$$

where

$$Q_{Jt} \equiv \prod_{j=1}^J \beta \frac{v_{c,t+j}}{v_{c,t+j-1}} \frac{1}{1 + \pi_{t+j}}.$$

The constraints pertaining to the homeowner are:

$$c_t + p_{Ht}x_{Ht} - \tilde{l}_t + \tilde{b}_{t+1} = (1 - \tau_N)w_t n_t - \tau_t + (1 + i_{t-1} + \Upsilon_{t-1}) \frac{\tilde{b}_t}{1 + \pi_t} - \tilde{m}_t,$$

where

$$\tilde{m}_t = (R_t + \gamma_t) \frac{\tilde{d}_t}{1 + \pi_t},$$

$$\tilde{l}_t = \theta p_{Ht}x_{Ht},$$

$$x_{Ht} = h_{t+1} - (1 - \delta_H)h_t.$$

Due to the aggregate consistency conditions  $(1 - \Psi)\tilde{d}_t^* = \Psi\tilde{d}_t$ ,  $\gamma_t^* = \gamma_t$ , and  $R_t^* = R_t$ , it is not necessary to include the homeowners laws of motion for the mortgage variables among the equations characterizing the equilibrium.

## Production

The producer's first-order conditions:

$$r_t = A_t f_1((1 - \Psi)k_t, \Psi n_t),$$

$$w_t = A_t f_2((1 - \Psi)k_t, \Psi n_t).$$

Output:

$$Y_t = A_t f((1 - \Psi)k_t, \Psi n_t).$$

The relative price of structures (i.e., the curvature of the production possibilities frontier):

$$q_t = q(\Psi x_{St}).$$

### Homebuilding

Land market clearing:

$$X_{Lt} = 1.$$

The production function and the first-order conditions of homebuilders (for the Cobb-Douglas production function) after imposing the land market clearing condition:

$$x_{St} = \frac{1}{\Psi} (\Psi x_{Ht})^{\frac{1}{1-\varphi}},$$

$$p_{Ht} = q_t \frac{(\Psi x_{St})^\varphi}{1-\varphi},$$

$$p_{Lt} = p_{Ht} \varphi (\Psi x_{St})^{1-\varphi}.$$

For a given  $x_{Ht}$ , the first equation determines  $x_{St}$ , the second  $p_{Ht}$ , and the third  $p_{Lt}$ . Notice that when  $\varphi = 0$ ,  $x_{Ht} = x_{St}$  and  $p_{Ht} = q_t$ .

### Monetary policy and the government

The monetary policy rule:

$$i_t = (i - \bar{\pi} + \bar{\pi}_t) + \nu_\pi (\pi_t - \bar{\pi}_t).$$

The government budget constraint:

$$G + (1 - \Psi)\tau_t^* = \tau_K(r_t - \delta_K)(1 - \Psi)k_t + \tau_N(w_t\Psi n_t) + (\tau_t - \Omega_t)\Psi.$$

### Market clearing

The labor and capital market clearing conditions have already been imposed in the production sector. And the land and structures market clearing conditions have already been imposed in the homebuilding sector. The remaining market clearing conditions are for the bond market:

$$(1 - \Psi)\tilde{b}_t^* + \Psi\tilde{b}_t = 0;$$

and mortgage market:

$$(1 - \Psi)\tilde{l}_t^* = \Psi\tilde{l}_t.$$

It is straightforward to verify that the Walras' law holds (i.e., the goods market clears and national accounts hold):

$$(1 - \Psi)c_t^* + \Psi c_t + (1 - \Psi)x_{Kt} + q_t\Psi x_{St} + G = Y_t = r_t(1 - \Psi)k_t + w_t\Psi n_t.$$

## Stochastic processes

TFP:

$$\log A_{t+1} = (1 - \rho_A) \log A + \rho_A \log A_t + \epsilon_{A,t+1}, \quad \text{where } \epsilon_{A,t+1} \sim iidN(0, \sigma_A).$$

Inflation target:

$$\bar{\pi}_{t+1} = (1 - \rho_\pi) \bar{\pi} + \rho_\pi \bar{\pi}_t + \epsilon_{\pi,t+1}, \quad \text{where } \epsilon_{\pi,t+1} \sim iidN(0, \sigma_\pi).$$

## Appendix B: Computation

The recursive competitive equilibrium (RCE) is computed using a linear-quadratic (LQ) approximation method for distorted economies with exogenously heterogeneous agents (see Hansen and Prescott, 1995), adjusted along the lines of Benigno and Woodford (2006) to take into account nonlinear constraints of the agents. The centering point of the approximation is the nonstochastic steady state and the LQ approximation of the Bellman equations is computed using numerical derivatives. All variables in the approximation are either in percentage deviations or percentage point deviations (for rates) from the steady state. Before computing the equilibrium, the model is made stationary by expressing all nominal variables in real terms and replacing ratios of price levels with the inflation rate, as in Appendix A.

The nonlinearity in the constraints of the agents comes from the laws of motion for the mortgage variables. The nonlinearity means that these equations cannot be substituted out into the per-period utility function, as required by the standard LQ approximation procedure. For this reason, as noted above, the method is modified along the lines of Benigno and Woodford (2006). This involves forming a Lagrangian, consisting of the per-period utility function and the laws of motion for the mortgage variables. The Lagrangian is then used as the return function in the Bellman equation being approximated. This adjustment is necessary to ensure that second-order cross-derivatives of the utility function and the constraints are taken into account in the LQ approximation. This modification, as applied to the homeowner, is described in detail by Kydland, Rupert, and Sustek (2014). The specification for the capital owner is analogous. We therefore refer the reader to that paper for details.

An alternative procedure—implemented, for instance, by Dynare—would be to log-linearize the model’s equilibrium conditions in Appendix A and use a version of the Blanchard-Kahn method to arrive at the equilibrium decision rules and pricing functions. As is well known, this method yields the same linear equilibrium decision rules and pricing functions as the adjusted LQ approximation; i.e., the same approximation to the set of functions  $W(z, S)$ .

## Appendix C: Data counterparts to variables

This appendix describes the data used to calculate the aggregate ratios employed in calibrating the model. Adjustments to official data are made to ensure that the data correspond conceptually more closely to the variables in the model. To start, for reasons discussed by Gomme and Rupert (2007), the following expenditure categories are taken out of GDP: gross housing value added, compensation of general government employees, and net exports.

In addition, we also exclude expenditures on consumer durable goods (as our ‘home capital’ includes only housing) and multifamily structures (which are owned by corporate entities and rented out to households mainly in the 1st and 2nd quintiles of the wealth distribution). With these adjustments, the data counterparts to the expenditure components of output in the model are constructed from BEA’s NIPA tables as follows: consumption ( $C$ ) = the sum of expenditures on nondurable goods and services less gross housing value added; capital investment ( $X_K$ ) = the sum of nonresidential structures, equipment & software, and the change in private inventories; housing structures ( $X_S$ ) = residential gross fixed private investment less multifamily structures; and government expenditures ( $G$ ) = the sum of government consumption expenditures and gross investment less compensation of general government employees. Our measure of output ( $Y = C + X_K + X_S + G$ ) accounts, on average (1958-2006), for 74% of GDP.

BEA’s Fixed Assets Tables and Census Bureau’s M3 data provide stock counterparts to capital and housing investment: capital stock ( $K$ ) = the sum of private nonresidential fixed assets and business inventories; housing stock ( $H$ ) = residential assets less 5+ unit properties.<sup>34</sup> Federal Reserve’s Flow of Funds Accounts provide data on mortgages and we equalize mortgage debt in the model ( $D$ ) with the stock of home mortgages for 1-4 family properties. The Flow of Funds data, however, include mortgage debt issued for purchases of existing homes, second mortgages, and home equity loans. In contrast, the model speaks only to first mortgages on new housing. The data thus provide an upper bound for  $D$  in the model.

## Appendix D: Estimation of mortgage debt servicing costs

A key measurement for calibrating the model concerns the mortgage debt servicing costs of homeowners. Unfortunately, such information for the United States is not readily available. Four different procedures are therefore used to arrive at its estimate. The four procedures exploit the notion that the homeowners in the model correspond to the 3rd and 4th quintiles of the U.S. wealth distribution. Some of these estimates arguably overestimate the debt servicing costs, while others underestimate it. Nevertheless, all four procedures yield estimates in the ballpark of 18.5% of pre-tax income, the value used to calibrate the model.

The first procedure, for FRM (1972-2006) and ARM (1984-2006), combines data on income from the Survey of Consumer Finances (SCF) and the model’s expression for debt servicing costs. Suppose that all mortgage debt is FRM. The model’s expression for steady-state debt-servicing costs,  $(R + \gamma)[D/(pwN - p\tau\Psi)]$ , can then be used to compute the average debt-servicing costs of homeowners. The various elements of this expression are mapped into data in the following way:  $D/(pwN - p\tau\Psi)$  corresponds to the average ratio of mortgage debt (for 1-4 unit structures) to the combined personal income (annual, pre-tax) of the 3rd and 4th quintiles, equal to 1.56;  $R$  corresponds to the average FRM annual interest rate for a conventional 30-year mortgage, equal to 9.31%; and  $\gamma$  corresponds to the average amortization rate over the life of the mortgage, equal to 4.7% per annum. This yields debt-servicing costs of 22%. This estimate is likely an upper bound as some of the

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<sup>34</sup>Separate stock data on 2-4 unit properties are not available, but based on completions data from the Census Bureau’s Construction Survey, 2-4 unit properties make up only a tiny fraction of the multifamily housing stock.

outstanding mortgage debt in the data is owed by the 5th quintile (the 1st and 2nd quintiles are essentially renters) and the effective interest rate on the stock in the data is likely lower than the average FRM rate due to refinancing. When all mortgage debt is assumed to be ARM, this procedure yields 17.5% (based on the average Treasury-indexed 1-year ARM rate for a conventional 30-year mortgage).

The second estimate is based on Federal Reserve's Financial Obligation Ratios (FOR) for mortgages (1980-2006). FOR report all payments on mortgage debt (mortgage payments, homeowner's insurance, and property taxes) as a fraction of NIPA's share of disposable income attributed to homeowners. For our purposes, the problem with these data is that members of the 5th quintile of the wealth distribution are also counted as homeowners in the data (as long as they own a home), even though they do not represent the typical homeowner in the sense of Campbell and Cocco (2003). To correct for this, we apply the shares of the aggregate SCF personal income attributed to the 3rd, 4th, and 5th quintiles of the wealth distribution to disposable income from NIPA. This gives us an estimate of NIPA disposable income attributed to these three quintiles. This aggregate is then multiplied by the financial obligation ratio to arrive at a time series for total mortgage payments. Assuming again that all mortgage payments are made by the 3rd and 4th quintiles, the total mortgage payments are divided by NIPA personal (pre-tax) income attributed to just these two quintiles (using the SCF shares). This procedure yields average debt-servicing costs of 20%.

Third, we use the ratio of all debt payments to pre-tax family income for the 50-74.9 percentile of the wealth distribution, reported in SCF for 1989-2007. The average ratio is 19%. About 80% of the payments are classified as residential by the purpose of debt, yielding an average ratio of 15.2%. A key limitation of this procedure is that the data exclude the 1970s and most of the 1980s—periods that experienced almost twice as high mortgage interest rates, on average, than the period covered by the survey. Another issue is that the information reported in the survey is not exactly for the 3rd and 4th quintiles.

The fourth procedure is based on the Consumer Expenditure Survey (CEX), 1984-2006. This survey reports the average income and mortgage payments (interest and amortization) of homeowners with a mortgage. To the extent that homeowners without a mortgage are likely to belong to the 5th quintile of the wealth distribution—they have 100% of equity in their home and thus have higher net worth than homeowners with a mortgage—the survey's homeowners with a mortgage should closely correspond to the notion of homeowners used in this paper (CEX does not contain data on wealth). The resulting average, for the available data period, for mortgage debt servicing costs of this group (pre-tax income) is 15%. Given that the data do not cover the period of high mortgage rates of the late 1970s and early 1980s, like the third estimate, this estimate probably also underestimates the debt servicing costs for the period used in calibrating the model.

Taken together, the four procedures lead us to use 18.5%, a value in the middle of the range of the estimates, as a target in calibration.

## Appendix E: Sensitivity analysis

See Figure A1. The figure is for the economy with no access of homeowners to the one-period bond market so as to prevent them from partially undoing the effects of the shocks. This gives alternative parameterization the best chance to matter.

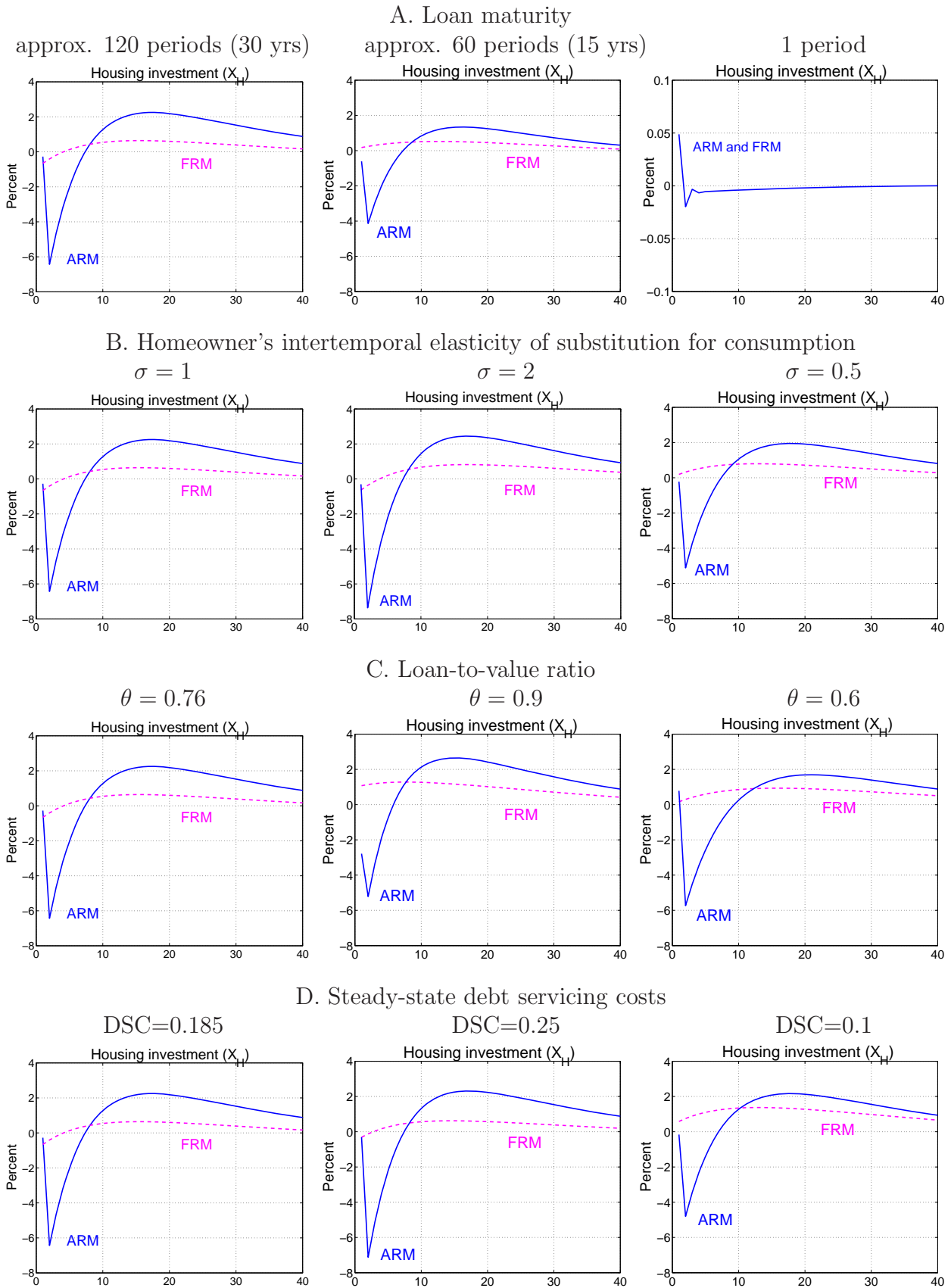


Fig A1. Sensitivity analysis; version with no access of homeowners to the 1-period bond market.

## Appendix F: Business cycle properties

From Figures 3 and 4 one could conclude that the model has counterfactual implications for the cyclical behavior of residential and nonresidential investment, as housing and capital investment in the two figures move in opposite directions. However, the business cycle in the U.S. (as in other developed economies) is characterized by positive correlations between the two types of investment and output. This appendix reports the cyclical properties of the model economy, once it is subjected to both TFP and the level factor shocks. The moments generated by the model are compared with corresponding moments of the U.S. business cycle.<sup>35</sup>

Table A.1 shows that, with these two shocks, the model accounts for about half of the volatility of U.S. output (our measure of output is close to private sector output; see Appendix C for details). As in the data, all three expenditure side components of output in the model ( $C$ ,  $X_S$ , and  $X_K$ ) are positively correlated with output. The correlations are stronger than in the data, arguably due to the presence of only two shocks in the model, of which the TFP shock has the larger impact. The ranking of volatilities is also consistent with the data: residential investment is most volatile, followed by nonresidential investment, and then consumption. The volatility of residential investment is higher under FRM than ARM. This is because, as in the data, the nominal interest rate is pro-cyclical (so is inflation). The pro-cyclical movements of the short rate then dampen, under ARM, the responses of  $X_{St}$  to TFP shocks (recall that housing investment responds more to interest rate and inflation movements under ARM than FRM).

In contrast to the short rate and inflation, the long rate is almost acyclical (the three variables are positively correlated with each other at lower than business cycle frequencies due to the level factor shock). The procyclicality of the short rate occurs due to the slope factor, which is mainly driven by TFP. Because the long rate does not respond much to (temporary) TFP shocks, the long rate is acyclical and the long-short spread is negatively correlated with output, as in the data.

The model is also consistent with a pro-cyclical behavior of the relative price of new residential structures and new homes. The volatility of new home prices in the model is about 60 – 70% as high as in the data. The shortfall, and the relatively high correlation of new home prices with output, is due to the absence in the model of ‘housing supply shocks’ (house prices in the model are driven only by housing demand). Notice that the volatility of the relative price of structures in the model is only about 30% as high as in the data and that its correlation with output is too high. Shocks to  $q_t$ , reflecting, for instance, shocks specific to the construction industry, as in Davis and Heathcote (2005), may be a source of the remaining volatility of the price of structures and new homes.

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<sup>35</sup>U.S. moments are for HP-filtered series, post-Korean war data. The model moments are averages of moments for 150 runs of the model; the artificial series of each run have the same length as the data series and are HP filtered.

Table A1. Business cycle properties

	US data	Model			US data	Model	
		FRM	ARM			FRM	ARM
<b>Std(Y)</b>	1.92	0.94	1.04				
<b>Rel. std</b>				<b>Corr with Y</b>			
$Y$	1.00	1.00	1.00	$Y$	1.00	1.00	1.00
$C$	0.42	0.42	0.35	$C$	0.79	0.88	0.94
$X_S$	6.94	9.48	8.20	$X_S$	0.60	0.99	0.85
$X_K$	2.45	1.76	3.01	$X_K$	0.73	0.92	0.83
$\pi$	0.58	0.85	0.81	$\pi$	0.14	0.23	0.41
$i$	0.58	0.85	0.85	$i$	0.36	0.32	0.48
$i^F$	0.35	0.77	N/A	$i^F$	0.01	0.09	N/A
$i^F - i$	0.42	0.21	N/A	$i^F - i$	-0.49	-0.98	N/A
$q$	0.58	0.18	0.15	$q$	0.41	0.99	0.85
$p_H$	1.57	1.13	0.97	$p_H$	0.55	0.99	0.85

Note: All U.S. moments are for HP-filtered series, post-Korean war data. Interest and inflation rates are annualized. The 10-year government bond yield is used as a proxy for  $i_t^F$  due to its longer time availability; the inflation rate of the GDP deflator is used for  $\pi_t$ ; the 3-month T-bill yield is used for  $i_t$ ; the ratio of the residential investment deflator to the GDP deflator is used for  $q_t$ ; the ratio of the average price of new homes sold (Census Bureau) and the GDP deflator is used for  $p_{Ht}$  (1975-2006). The model moments are averages of moments for 150 runs of the model; the artificial series of each run have the same length as the data series and are HP filtered.