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Endogenous money, inflation, and welfare ☆

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ABSTRACT

This paper addresses the classic question: what are the welfare costs of inflation. We employ a model in which the ratios of currency to deposits and currency to reserves are endogenously determined. The model distinguishes quantitatively between three sources of welfare cost of inflation, and provides further estimates for potential welfare gains from improvements in transaction technologies. Estimates of the marginal cost of public funds associated with the inflation tax are compared both with that of labor taxation within the model and with those reported in the public finance and macro literature. We conclude that not only is inflation an inefficient source of government revenue, but also that, in the absence of lump-sum taxation, deflationary policies may be highly inefficient.

What are the welfare gains from adopting monetary policies that reduce the inflation rate? This is among the classic questions in monetary economics, and a matter of great interest for researchers and policymakers alike. The traditional way to provide an estimate of the welfare cost has been to quantify the opportunity cost of holding non-interest-bearing money and the indirect cost resulting from the lower-than-optimal holdings of real balances.

In this paper we ask the same question, but we address it within a model economy with endogenous money supply, i.e. where the ratio of $M1$ (the sum of currency holdings and demand deposits) to $M0$ (the monetary base) is the outcome of households' optimization. Using this economy, in which currency, reserves, and deposits play distinct roles, this paper makes four contributions.

First, it provides a welfare measure of inflation that has some new features. The welfare cost reflects three distinct sources. The first is the opportunity cost of holding currency and demand deposits, the second stems from the fact that inflation distorts labor input and capital accumulation and, therefore, total output, and the third, and most important, are the costs incurred in order to avoid the inflation tax on currency and deposits. Among our findings is that, for most inflation rates, the opportunity cost of holding currency and demand deposits accounts for less than half of the estimated welfare costs of inflation, and that most of the costs are accounted for by actions taken to avoid the inflation tax.

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The second contribution is to endogenize the size of the banking sector as a function of the inflation rate. Proxies for its size are holdings of real deposits and the quantity of real deposits used for purchases. These measures of the banking sector are also closely related to the endogenously determined velocity of the monetary base.

The third contribution is to provide a framework within which we can analyze the welfare gains of potential improvements in transaction technology. As the costs of transactions are reduced, so is the cost of inflation.

And fourth, this paper not only makes a strong case for the inefficiency of inflation as a means of government revenue, but also, in the absence of lump-sum taxation, for the inefficiency of deflation. In most other models that estimate the cost of inflation, there is a close link between holdings of real balances, the welfare cost of inflation, and the object of government taxation. Within the model economy where households can take other actions and incur other costs to avoid inflation, this close link is broken. Estimates for the marginal cost of public funds from seigniorage income are compared to within-model labor-tax estimates as well as those of other empirical studies. For both cases, we find that it is inefficient not only to use inflation as a means of government revenue, but also to reduce the rate of inflation more than a tiny bit below 0%.

Prices and output are fully flexible. Building on Freeman and Kydland (2000), whose model includes inside money in the spirit of Freeman and Huffman (1991), households can make different-sized purchases of consumption goods using either currency or interest-bearing deposits. Two transaction costs affect these decisions. One is the time cost of replenishing money balances, and the other is the fixed cost of using deposits for purchases. Faced with these two costs and factors that vary over time in equilibrium, such as over the business cycle, households make decisions that, in the aggregate, determine the velocity of money and the money multiplier.

Among related literature, albeit modelling the use of money quite differently, Aiyagari et al. (1998) and Gillman (1993) estimate the welfare cost of inflation using cash-in-advance models where credit services cost resources. Chari et al. (1995) analyze how different monetary aggregates co-exist in order to account for the cyclical behavior of short-term interest rates. Other related papers include Dotsey and Ireland (1996), who address the question with a cash-in-advance model with shopping time, and Jones et al. (2004) and Simonsen and Cysne (2001), who generalize money-in-the-utility-function models to include interest-bearing assets. Natural benchmarks for estimated welfare cost of inflations are Cooley and Hansen (1989) and Lucas (2000), who report the gain from reducing inflation from 10% to 0% to be consumption equivalent of 0.38% and around 1%, respectively.

1. Model economy

1.1. Production

Output is given by a constant-returns-to-scale production function with two inputs: capital (k_t) and labor (l_t);

$$y_t = z_t f(k_t, l_t) = z_t k_t^{\alpha} l_t^{1-\alpha}.$$

The law of motion for the technology level z_t is

$$z_t = \rho z_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(\mu, \sigma^2),\ \mu > 0.$$

The depreciation rate is denoted δ, so the law of motion for the capital stock is

$$k_{t+1} = (1-\delta)k_t + i_t,$$

where i_t is gross investment.

1.2. Government

The government controls the supply of intrinsically worthless fiat money. The law of motion for the money stock is

$$M_t = \xi M_{t-1}.$$

Net revenues from printing money are transferred to households in a lump-sum fashion in the aggregate amount of

$$X_t = (\xi - 1)M_{t-1}.$$

1.3. Financial intermediation

Banks accept deposits (h), hold a fraction (θ) of the deposits as reserves, and lend the remainder to firms who invest the proceeds in capital. Free entry ensures zero profit, and the rate of return on deposits ($\tilde{r}_{t+1}$) is therefore a linear combination of the real return on capital (r_{t+1}) and the return on holding currency (p_t/p_{t+1}):

$$\tilde{r}_{t+1} = (1-\theta)r_{t+1} + \theta\frac{p_t}{p_{t+1}}.$$

Total capital stock equals the sum of non-intermediated capital (a) and the portion of deposits that banks do not hold as reserves:

$$k_{t+1} = a_t + (1-\theta)\frac{h_t}{p_t}. \tag{1}$$

The total stock of fiat money (the monetary base) is by definition equal to the combined stocks of currency (m) and reserves:

$$M_t = m_t + \theta h_t, \tag{2}$$

whereas the total money stock ($M1$) is the sum of nominal deposits and currency. It can be rewritten as the product of the monetary base and the money multiplier:

$$M1_t = m_t + h_t = M_t\left[1 + \frac{h_t(1-\theta)}{m_t + \theta h_t}\right].$$

1.4. Household's problem

There is a continuum of good types of measure c_t^*, ordered by size and indexed by j over $[0,1]$. The utility of the representative household is given by the following function

$$\mathrm{E}\sum_{t=0}^{\infty}\beta^t u\left[\min\left(\frac{c_t(j)}{(1-\omega)j^{-\omega}}\right), d_t\right], \tag{3}$$

where d_t denotes leisure. For a given level c_t^* of total consumption, the utility function implies that the household will distribute consumption over the continuum of goods according to the following optimizing rule for $c_t(j)$ over $[0,1]$:

$$c_t(j) = (1-\omega)j^{-\omega}c_t^*. \tag{4}$$

Three vehicles of savings are available to households: non-intermediated capital (a_t), nominal bank deposits (h_t), and currency (m_t). Both bank deposits and currency can be used to purchase consumption goods, but use of deposits incurs a fixed cost, denoted γ.

At the beginning of each period, households choose their real money holdings and ratio of deposits to currency. In order to purchase a given amount of consumption goods in a period, each household replenishes its money balances n times, so that n multiplied by the money balances held equals the amount of consumption. Each time a household replenishes its money balances, it spends φ units of time. Total time spent on those transactions in a period then equals φn_t.

Because of this fixed cost of using deposits for purchases, the deposit rate of return net of transaction costs goes to negative infinity as the purchase size j goes to zero. Therefore, some j^* exists below which currency is a preferred means of payment, and above which deposits are preferred.

For the representative household, the per-period demand for real deposits is

$$n_t\frac{h_t}{p_t} = \int_{j^*}^{1} c_t(j)\,\mathrm{d}j = \int_{j^*}^{1}(1-\omega)j^{-\omega}c_t^*\,\mathrm{d}j = \bigl(1-(j^*)^{1-\omega}\bigr)c_t^* \tag{5}$$

and the per-period demand for real fiat-money balances is:

$$n_t\frac{m_t}{p_t} = \int_{0}^{j^*} c_t(j)\,\mathrm{d}j = \int_{0}^{j^*}(1-\omega)j^{-\omega}c_t^*\,\mathrm{d}j = (j^*)^{1-\omega}c_t^*. \tag{6}$$

Substituting from the optimal rule (4), the household's instantaneous utility can be written as

$$u\bigl(c_t^*, d_t\bigr) = \frac{1}{1-\nu}\bigl[\bigl(c_t^*\bigr)^{\zeta}(d_t)^{1-\zeta}\bigr]^{1-\nu}, \tag{7}$$

and the budget constraint is given by

$$c_t^* + a_t + \frac{h_t}{p_t} + \frac{m_t}{p_t} + \gamma\bigl(1-j_t^*\bigr) = w_t l_t + r_t a_{t-1} + \tilde{r}_t\frac{h_{t-1}}{p_{t-1}} + \frac{m_{t-1}}{p_t} + \frac{X_t}{p_t}. \tag{8}$$

The time available is spent on leisure, labor, and replenishment of money balances. Normalizing available time to 1, the time constraint is

$$1 = d_t + l_t + n_t\varphi. \tag{9}$$

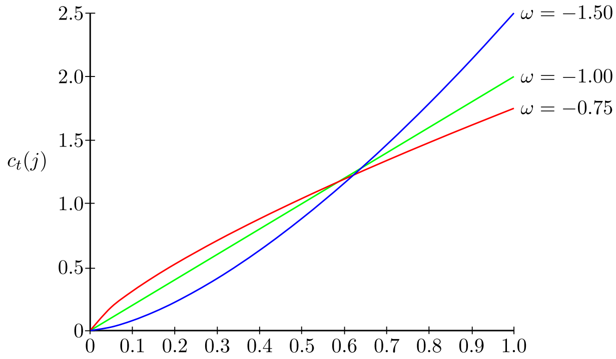


Fig. 1. Distribution of consumption across goods of different sizes.

1.5. Equilibrium

At any point in time, the economy is characterized by the technology level (z), the growth of money stock (ξ), lagged price level (p_{-1}), lagged holdings of non-intermediated capital (a_{-1}), lagged deposits (h_{-1}) and lagged currency holdings (m_{-1}). The vector $s \in S \subset \mathbb{R}^6$ is the state of the economy.

An equilibrium is a sequence of allocations $\{l(s), a(s), h(s), m(s)\}$, and a sequence of prices $\{r(s), w(s), p(s)\}$ such that

1. Each household solves its optimization problem (3) subject to its liquidity constraints (5) and (6), budget constraint (8) and time constraint (9).
2. The goods market, the asset market for capital (1), and the market for fiat money (2) clear.

2. Calibrating the model

In steady state, investment is one-quarter of output and the annual capital-output ratio 2.5. The depreciation rate is then calibrated to 0.025. The parameter α in the production function is calibrated such that labor share of national income is 0.65.

Setting the average allocation of households' time (net of sleep and personal care) to market activity equal to 0.30 restricts the value of the utility parameter ζ. The risk-aversion parameter ν equals 2, and the reserve requirement ratio θ is set equal to 0.10. The discount factor β is consistent with a steady-state net real rate of return of 4%.

The benchmark economy is calibrated such that for gross annual inflation rate equal to 1.03, the deposit-to-currency ratio is equal to 9, and the deposit portion of $M1$, net of reserves and divided by the total capital stock, is 0.05. These targets give us calibrated values for $\gamma = 0.0059$ and $\varphi = 0.00076$. They imply that, at this inflation rate, the cost associated with deposit purchases, $\gamma(1 - j^*)$, is 0.36 percent of GDP, and $n\varphi$ corresponds to approximately 75 minutes per quarter.

In order to calibrate the parameter ω of the utility function, the model business cycle properties resulting from different parameter values are compared (see Section 2.2). Consistent with Prescott (1986), the autocorrelation coefficient ρ in the technology process is set equal to 0.95, and the shocks have a standard deviation of 0.0076.

2.1. Utility function

In Fig. 1, the decision rule $c_t(j)$ in Eq. (4) is plotted for three different values of ω for the case of $c_t^* = 1$. For $\omega > -1$, the amount consumed of different goods is a concave function of the size of the goods, whereas for $\omega < -1$, this amount is a convex function of the size of the goods.

Combining Eqs. (5) and (6) gives us the cut-off size for purchases, above which deposits are preferred to currency:

$$j^* = \left(1 + \frac{h_t}{m_t}\right)^{\frac{1}{\omega - 1}}. \tag{10}$$

The derivative of j^* is negative, implying that, loosely speaking, the more convex $c_t(j)$ is, the higher is j^*, or conversely, the more concave $c_t(j)$ is, the lower is j^*.

Note that (5) and (6) combined with (10) imply

$$\int_{j^*}^{1} c_t(j)\,\mathrm{d}j = \left(1 + \frac{h_t}{m_t}\right)^{-1} c_t^*$$

Table 1
Contemporaneous correlations with output.

		$M1$	p	R_{nom}	c	i	l
Policy A:	$\omega = -0.75$	1	−0.08	−0.73	0.96	0.99	0.99
	$\omega = -1.00$	1	−0.48	−0.29	0.96	0.99	0.99
	$\omega = -1.50$	1	−0.69	0.12	0.96	0.99	0.99
Policy B:	$\omega = -0.75$	0.91	−0.06	−0.73	0.96	0.99	0.99
	$\omega = -1.00$	0.87	−0.15	−0.29	0.96	0.99	0.99
	$\omega = -1.50$	0.80	−0.26	0.12	0.96	0.99	0.99
Policy C:	$\omega = -0.75$	0.85	−0.02	−0.36	0.96	0.99	0.99
	$\omega = -1.00$	0.81	−0.09	−0.09	0.96	0.99	0.99
	$\omega = -1.50$	0.75	−0.18	0.02	0.96	0.99	0.99

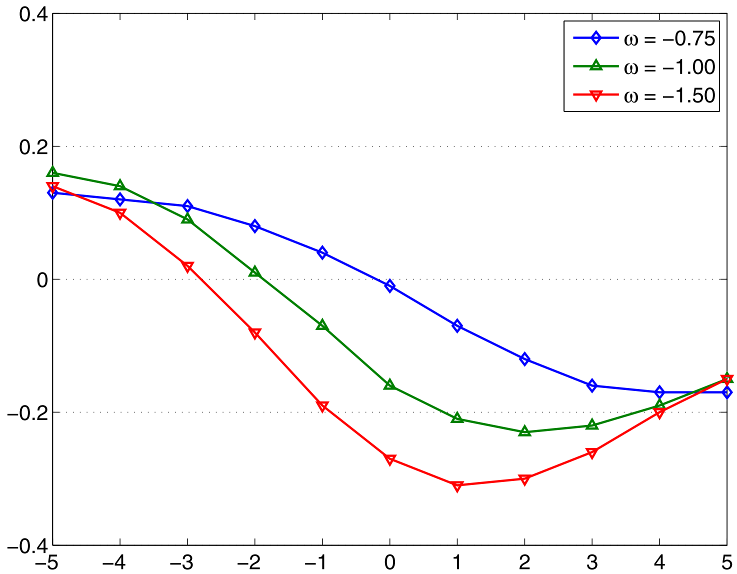


Fig. 2. Cross-correlations: Output and price level.

and

$$\int_0^{j^*} c_t(j)\,\mathrm{d}j = \left(1 + \frac{m_t}{h_t}\right)^{-1} c_t^*.$$

In words, whereas the cut-off size of purchases for which deposits are preferred to currency is a function of ω, the share of total consumption c_t^* for which deposits are preferred to currency (and vice versa) only depends on the deposit-to-currency ratio.

2.2. Business cycle properties and the parameter ω

In order to quantify the parameter ω, we examine the model's business cycle behavior under different values of ω, and for the three policy regimes considered in Freeman and Kydland (2000). In all three cases, the average growth rate of fiat money is 3 percent annually. Under the first, Policy A, the growth rate of fiat money is fixed at 3 percent. Under the second, Policy B, serially uncorrelated shocks have been added to the supply of fiat money, with a standard deviation of 0.5 percent. And under the third, Policy C, the growth rate of the monetary base is serially correlated with an autoregressive parameter of 0.7, and the shocks have a standard deviation of 0.2. For these three policies, we examine the business cycle properties for $\omega = \{-0.75, -1.0, -1.5\}$. Table 1 presents the contemporaneous correlations with output.

Notice first that the correlations of the real variables c, i and l are hardly affected by changes in monetary policy or the curvature of the utility function. We see also that $M1$ is strongly correlated with real output. Under Policy A, in which there is no randomness, the correlation is 1. Under the other two policy regimes, $M1$ is less tightly correlated, but still highly correlated.

An interesting pattern is the countercyclical behavior of the price level (Fig. 2). For all policies, the price level is more countercyclical the more negative ω is. Further, from the figures we see that, for $\omega = -1.5$, prices are lagging the cycle by

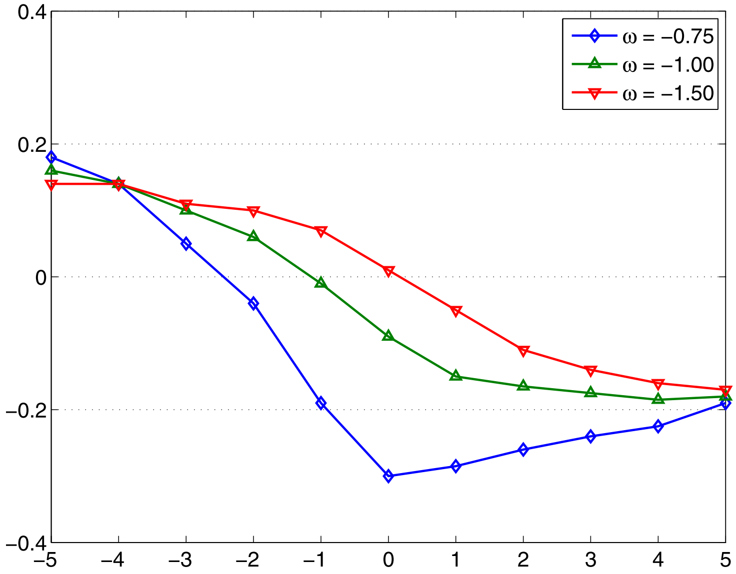


Fig. 3. Cross-correlations: Output and nominal interest rates.

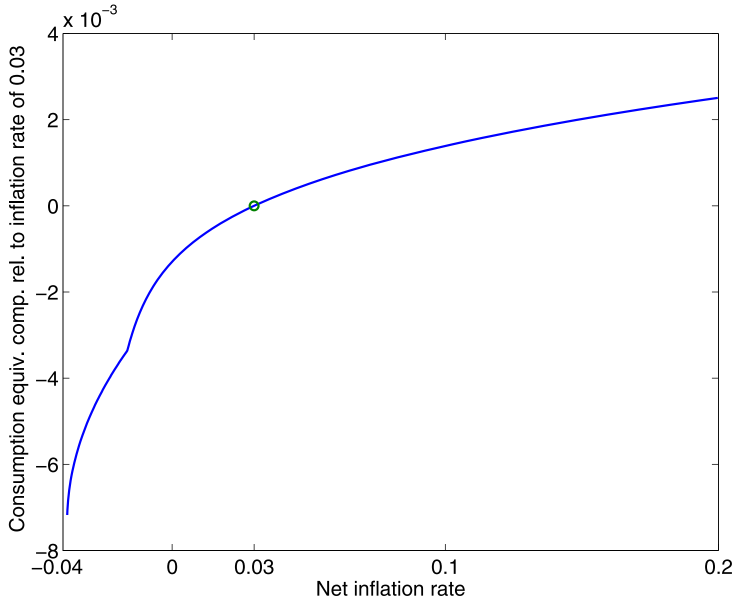


Fig. 4. Welfare cost of inflation.

one quarter. Of the three values of ω, the value of $\omega = -1.5$ yields price movements closest to those reported for U.S. data by Gavin and Kydland (1999).

We notice also that for $\omega = -1.5$ the cyclical behavior of the nominal interest rate is closer to what is observed in the data (Fig. 3). Consistent with reported business cycle statistics, for $\omega = -1.5$ the nominal interest rate is weakly procyclical. In contrast, for the other two values of ω, the nominal rate of return (R_{nom}) is countercyclical.

Until we have data from which we can map to ω, we choose $\omega = -1.5$ as our benchmark value, as this value gives business cycle statistics closest to those observed along well-defined dimensions.

3. Quantitative findings

Fig. 4 plots the benchmark welfare cost function λ, defined such that

$$u\big(\lambda c(\pi), d(\pi)\big) = u\big(c(\tilde{\pi}), d(\tilde{\pi})\big), \tag{11}$$

Table 2
Steady-state welfare costs, benchmark calibration.

	−3.85%	−2%	−1.64%	0%	3%	10%	25%	50%
n	0.0000	1.4635	1.5981	1.1837	1.2088	1.4310	1.8213	2.2558
l	0.2998	0.2988	0.2988	0.2997	0.3000	0.3000	0.2999	0.2996
a	9.9944	9.9607	9.9548	9.5526	9.5000	9.5535	9.6359	9.6943
h	0.0000	0.0000	0.0000	2.5439	4.7368	6.7335	8.0260	8.6803
m	1.0000	1.0000	1.0000	0.7456	0.5263	0.3267	0.1974	0.1320
h/m	0.0000	0.0000	0.0000	3.4118	9.0000	20.6138	40.6597	65.7757
j^*	1.0000	1.0000	1.0000	0.5523	0.3981	0.2925	0.2250	0.1863
p	0.0000	1.9591	2.1405	5.2160	8.5263	13.5505	20.1115	26.7238
c	0.7496	0.7470	0.7466	0.7465	0.7462	0.7456	0.7447	0.7438
d	0.7002	0.7001	0.7000	0.6993	0.6990	0.6988	0.6986	0.6985
u	−1.3964	−1.3981	−1.3983	−1.3991	−1.3997	−1.4004	−1.4011	−1.4018
K	9.9944	9.9607	9.9598	9.9915	10.0000	10.0008	9.9951	9.9867
N	0.2998	0.2988	0.2988	0.2997	0.3000	0.3000	0.2999	0.2996
Y	0.9994	0.9960	0.9955	0.9992	1.0000	1.0001	0.9995	0.9987
$\lambda_{1.03}$	−0.74%	−0.37%	−0.32%	−0.12%	0.0	0.14%	0.30%	0.43%

where $\tilde{\pi}$ is set equal to the average inflation rate since 1960 (about 3% annually). When comparing steady states, we use the parameter values of our model economy (α, β, δ, φ, γ, ν, ζ, θ, ω, and $\bar{z}$) as they are calibrated for an annual inflation rate equal to 3%, then vary the inflation rate and solve for combinations of a, c, d, h, l, m, n, p, and w. The values of these endogenous variables for inflation rates between −4% and 50% are given in Table 2.

Bailey (1956) and Friedman (1969) show that, under given assumptions, the optimal rate of inflation is the inverse of the real rate of return on capital, and such that the net nominal interest rate is 0. The intuition for this result is that the opportunity cost of holding currency is the nominal interest rate. As long as the net nominal interest rate differs from 0, people will spend time and resources to economize on their nominal currency holdings.

In our model economy, there turns out to be a uniquely defined lower bound of inflation, weakly greater than the inverse of the real rate of return on capital, below which no one holds deposits. In other words, inside money ceases to exist, all capital held is non-intermediated, and the total money stock equals the monetary base ($M1 = M$). If we also consider inflation rates below the lower bound of the inside-money economy, we find that the Bailey–Friedman level of inflation is also optimal in our model economy.

Fig. 4 plots the steady-state welfare cost for annual inflation rates ranging from the inverse of the steady-state net real return on non-intermediated capital to 20%. We see that as the inflation rate approaches an annual rate of 20%, the welfare cost compared with the benchmark is slightly less than 0.25% of consumption.

The most striking feature of the graph is the predicted welfare gain from reducing inflation below 3% annually. The welfare gain from reducing annual inflation towards the lower bound of the inside-money economy, −1.64%, is about the same magnitude as the welfare cost of increasing inflation from 3% to 33%. Further reducing the inflation rate beyond the level at which inside money has ceased to exist also results in a sizeable welfare gain. Attaining the Bailey–Friedman rule will give a consumption equivalent welfare gain of about 0.7% compared with the benchmark.

As Fig. 5 shows, as the steady-state inflation rate increases further, the associated welfare gain flattens out. If the inflation rate is increased to 100%, the cost of inflation in terms of consumption compensation is still less than 0.6% — and still of a lesser magnitude than the welfare gain associated with a nominal interest rate of 0%.

The welfare gain of inflation can be decomposed into reductions of direct and indirect costs. The latter are costs associated with changes in labor and capital input, and hence total output. Fig. 6 plots the *direct* transaction and opportunity costs of inflation: "Use of deposits for purchases" is equal to the fixed cost of purchasing goods with deposits, $\gamma(1-j^*) = \gamma(1-(1+\frac{h}{m})^{\frac{1}{\omega-1}})$; "Replenish money balances" is equal to the wage rate times time spent, $wn\varphi$; "Opportunity cost, currency" is equal to the difference in return between non-intermediated capital and currency times real currency holdings, $(r-\frac{1}{\xi})\frac{m}{p}$; and "Opportunity cost, deposits" is equal to the difference in return between non-intermediated capital and that on real deposit holdings, $(r-\tilde{r})\frac{h}{p} = (r-\frac{1}{\xi})\frac{\theta h}{p}$. Household optimization implies that the cost of replenishing money balances is equal to the discounted opportunity cost of holding currency and deposits:

$$wn\varphi = \frac{1}{r}\left(r - \frac{1}{\xi}\right)\left(\frac{m+\theta h}{p}\right). \tag{12}$$

For positive inflation rates below about 100% annually, "use of deposits for purchases" is the largest direct cost. As inflation increases, and so also the opportunity cost of holding deposits, households will replenish their money balances more frequently. Almost paradoxically, as inflation increases, the total opportunity cost of holding currency decreases. This is because households in the economy in part substitute real money balances from currency to deposits, and in part optimally hold smaller real balances and instead replenish their money balances more frequently. It should also be noted that, for all inflation rates, the sum of resources spent on using deposits for purchases and on replenishing money balances is greater than the sum of opportunity costs of holding deposits and currency.

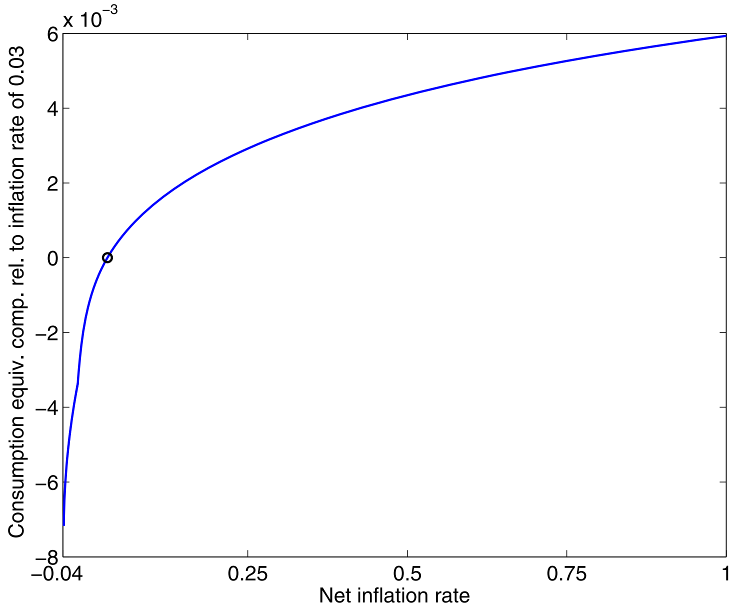


Fig. 5. Welfare cost of inflation.

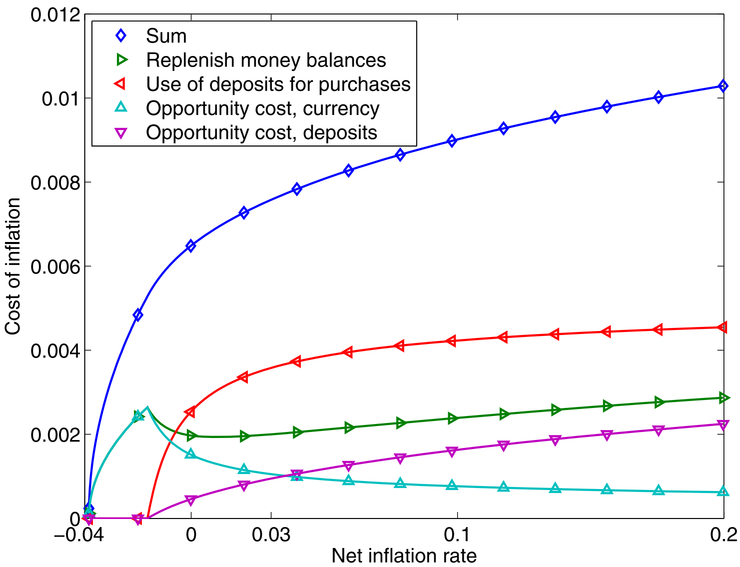


Fig. 6. *Direct* transaction and opportunity costs of inflation.

Below the lower inflation bound of the inside-money economy, we have $h = 0$ and $m = M$. Between the Bailey–Friedman rule and this level of inflation, the price level is governed by the liquidity constraint

$$n\left(\frac{m+h}{p}\right) = n\frac{M}{p} = c^*,$$

and the marginal condition from Eq. (12)

$$wn\varphi = \frac{1}{r}\left(r - \frac{1}{\xi}\right)\left(\frac{M}{p}\right).$$

As the rate of inflation decreases, households choose to replenish money balances less frequently and rather hold higher real money balances. Hence, as the return difference between non-intermediated capital and currency goes to zero, the price level converges towards zero and real balances, $\frac{\bar{M}}{p}$, to infinity.

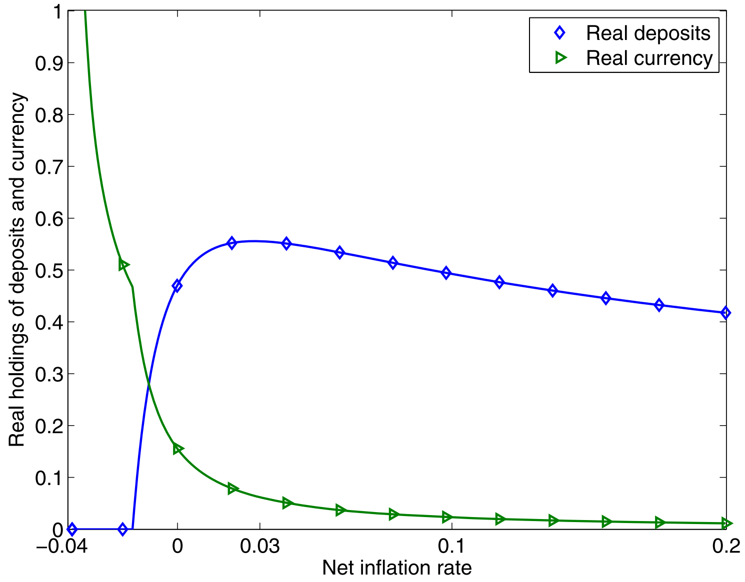


Fig. 7. Real deposit and currency holdings.

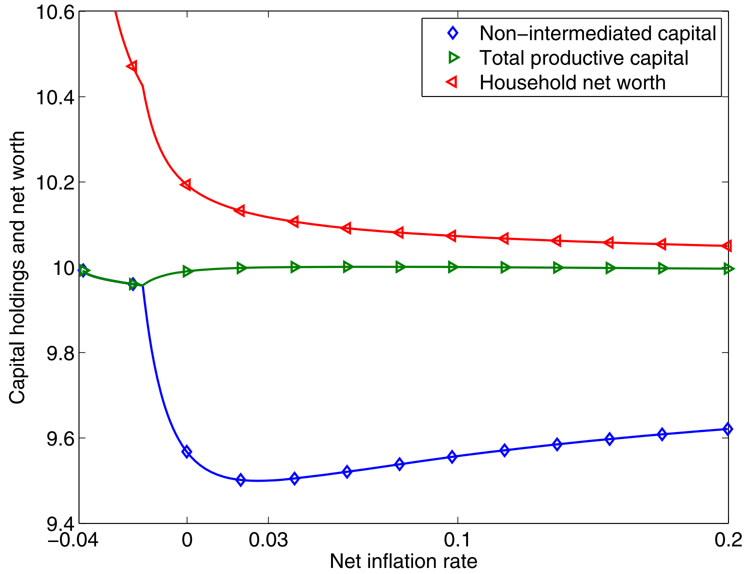


Fig. 8. Total productive capital and household net worth.

Fig. 7 plots the holdings of real currency (m/p) and real deposits (h/p) as functions of the inflation rate. Real currency holdings is a monotonously decreasing function for all inflation rates. In contrast, real deposit holdings is an increasing function of the inflation rate for very low inflation rates, and then converges towards zero as inflation goes to infinity, but at a slower rate than real currency holdings.

As a corollary to Figs. 6 and 7, Fig. 8 shows how total net worth (the sum of real currency, real deposits, and non-intermediated capital holdings) is a monotonously decreasing function of the inflation rate. Except for low levels of inflation, where households choose to hold relatively large real deposits, non-intermediated capital as share of total productive capital increases with inflation.

In Fig. 8 we notice that total productive capital (K) attains its minimum value at the lower bound of inflation at which the economy still has a positive quantity of inside money. This result is also reflected in total output as plotted in Fig. 9. Total output reaches its highest level at net annual inflation rate equal to 6.4%. At this inflation rate, output is about 0.44% larger than at the lower bound.

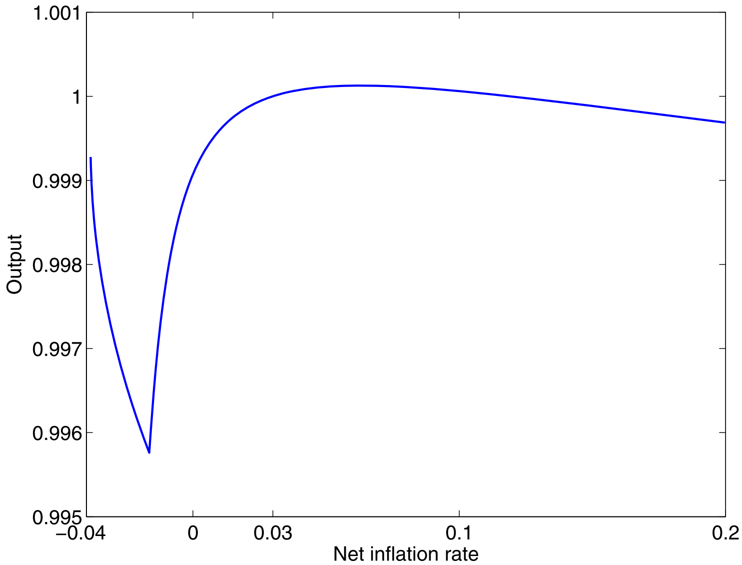


Fig. 9. Output.

Figs. 6 and 7 also graph our two proxies for the financial sector of our model economy. As we saw from Fig. 6, the cost spent on facilitating transactions using deposits is an increasing, concave function of the inflation rate. On the other hand, and as we have discussed, real deposit holdings reaches its global maximum at an inflation rate slightly above 3%.

3.1. Fiscal considerations

Government steady-state real seigniorage income is equal to

$$\frac{X}{p} = (\xi - 1)\frac{M}{p}. \tag{13}$$

The basis of government revenue is $M = (m + \theta h)$.

In contrast to cash-in-advance models, households in our model face more realistic trade-offs, and have other margins to avoid inflation tax than holding less real balances. As we described in previous sections, as the rate of inflation increases, households optimally allocate more resources to the use of deposits for purchases and to replenishing their money balances more frequently. The flip side of this observation is that, as inflation rises, they optimally hold less real balances and thereby reduce the basis of government revenue.

Fig. 10 plots the government seigniorage income (x), household cost of holding currency $(\frac{1}{r}(r - \frac{1}{\xi})(\frac{m+\theta h}{p}))$, and total direct cost of inflation $(\gamma(1 - j^*) + wn\varphi + \frac{1}{r}(r - \frac{1}{\xi})(\frac{m+\theta h}{p}))$ for inflation rates between 0% and 100% annually. As expected, government seigniorage income and the household cost of holding currency are closely related, and are equal for $\xi = 1 + \sqrt{1 - \frac{1}{r}}$. As the inflation rate approaches 100%, the direct cost of inflation, ignoring general equilibrium effects, is more than 2.7 times larger than government seigniorage revenue.

3.2. The inefficiency of deflation

Suppose that government expenditures are fixed and that lost revenue from a decrease in inflation must be made up by an increase in other sources. Then, whether such a decrease is welfare improving or not depends on the marginal cost of public funds. If the monetary authority has access to lump-sum taxation, the marginal cost of public funds is zero. As we have shown, the welfare-maximizing rate of inflation is then the inverse of the real interest rate.

In the absence of lump-sum taxation, it is illuminating to determine the maximum marginal cost of public funds that makes a reduction in the inflation rate a welfare improvement. This measure, $\chi(\pi)$, satisfies the following equation

$$\chi(\pi)\frac{\partial(X/p)}{\partial\pi} = \frac{\partial(\lambda c(\pi))}{\partial\pi}. \tag{14}$$

The right-hand side is the marginal change in the computed compensating variation by a marginal change in the inflation rate. The left-hand side is the product of the compensating variation from a marginal change in distortionary taxation and the marginal change in government revenue from a marginal change in inflation.

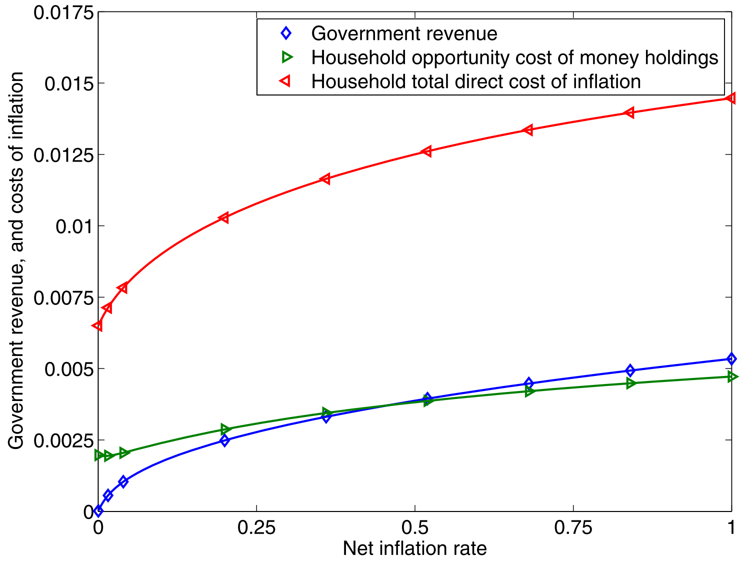


Fig. 10. Cost of inflation divided by government inflation tax income.

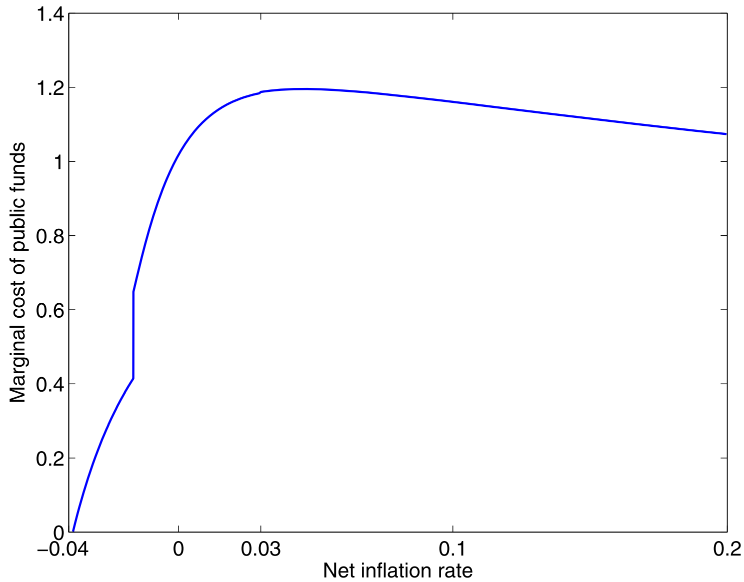


Fig. 11. Maximum marginal cost of public funds for a given inflation rate to be a welfare improvement.

Fig. 11 plots the value of the marginal cost of public funds that equalizes the marginal gain from changing the inflation to the marginal change in seigniorage income. This curve is very steep for negative inflation rates, which means that, in the absence of lump-sum taxation, it would be highly inefficient to set the rate of inflation equal to the inverse of the real interest rate. If the marginal cost of public funds is as large as 1.0, it is even inefficient to reduce the rate of inflation below 0.5%.

In order to compare the marginal cost of public funds associated with the inflation tax to those of other forms of taxation, we add to the model a tax on labor. Government tax revenue is transferred back to the representative household in a lump-sum fashion, modifying the budget constraint to

$$c_t^* + a_t + \frac{h_t}{p_t} + \frac{m_t}{p_t} + \gamma\left(1 - j_t^*\right) = (1-\tau) w_t l_t + r_t a_{t-1} + \tilde{r}_t \frac{h_{t-1}}{p_{t-1}} + \frac{m_{t-1}}{p_t} + \frac{X_t}{p_t} + T_t,$$

where τ_t is the labor tax rate, T_t is the lump-sum transfers of labor-tax government income, and

$$T_t = \tau w_t l_t.$$

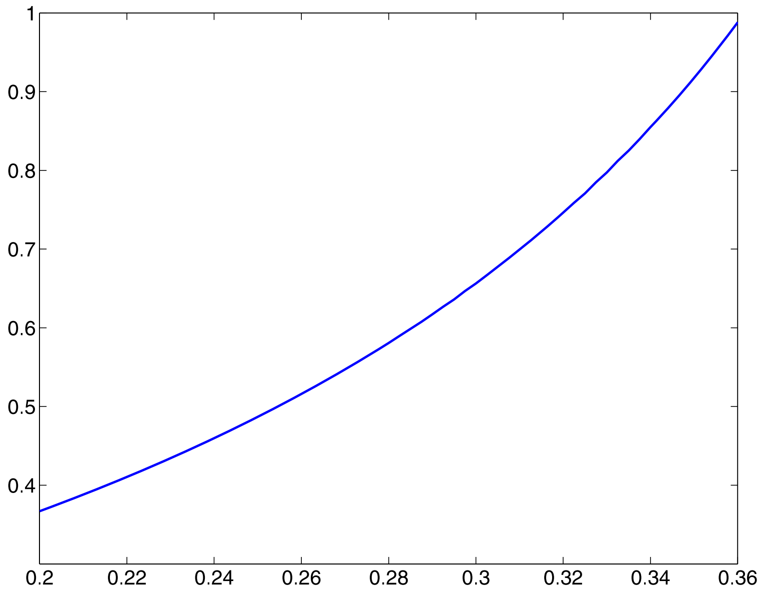


Fig. 12. Marginal cost of public funds as a function of the marginal tax rate on labor.

We compute the measure $\psi(\tau)$ that satisfies the following condition

$$\psi(\tau)\frac{\partial(T(\tau))}{\partial\tau} = \frac{\partial(\lambda(\tau)c(\tau))}{\partial\tau}. \tag{15}$$

The right-hand side is the marginal change in the computed compensating variation resulting from a marginal change in the labor tax rate. The left-hand side is the product of the compensating variation from a marginal change in distortionary taxation and the marginal change in government revenue from a marginal change in the labor tax rate.

Several careful studies have calculated the average tax rate on individual income; see, in particular, Barro and Sahasakul (1983, 1986), Seater (1985) and Stephenson (1998). While there are slight differences in the methods, the results show the tax rate between 1954 and 1994 to be (roughly) between 22% and 30%. Mendoza et al. (1994) calculate average effective tax rates and report labor income tax rates for the United States ranging from 17% to 30%.

In Fig. 12, we plot the measure $\psi(\tau)$ for marginal tax rates varying from 20% to 36%. It is a convex function for which the computed marginal cost of public funds is just below 0.4 when the tax rate is about 20%, and just below one as the labor-tax rate approaches 36%.

These model-based estimates of the marginal cost of public funds are robust and consistent with estimates reported elsewhere in the public finance and macro literature. For labor income tax being the only source, Browning (1987) estimates the marginal cost of public funds for the United States to be in the range 0.32 to 0.47. Jorgenson and Yun (1990) use all taxes, and estimate the marginal cost of public funds to be 0.47. Feldstein (1999) estimates the marginal cost of increasing the labor tax to be about one.

If the marginal cost of public funds is about the magnitude reported by Browning (1987) and Jorgenson and Yun (1990), the cost of sustaining a level of inflation below −1% is higher than the marginal gain. If the marginal cost of public funds is as large as estimated by Feldstein (1999), it is inefficient to reduce inflation below 0.5%.

In sum, given estimates of the marginal cost of public funds both from the model and from other studies, not only is positive inflation an inefficient way to raise government revenue, but negative inflation rates are also very costly.

Estimates indicate that more than half of all U.S. currency in circulation is held abroad (see e.g. Porter and Judson, 1996). The volume of U.S. dollars circulating abroad means that non-U.S. citizens bear part of the seigniorage burden. If welfare considerations are only made for domestic citizens, and the elasticity of non-citizens' currency holdings is different from that of domestic citizens, the fiscal considerations would change.

More specifically, if welfare considerations are made only for domestic citizens, the right-hand side of (14) would remain unchanged. If non-citizens' holdings of currency were weakly less elastic with respect to inflation than those of domestic citizens, real seigniorage income would be less sensitive to changes in inflation. An implication is that, for a given rate of inflation, $\chi(\pi)$ would be lower. Moreover, the level of the inflation rate below which it is inefficient to reduce it any further, would be higher. We leave it to future research to explore in greater detail the welfare implications of foreign holdings of U.S. currency.

Table 3
Sensitivity: γ.

Intermediated/ total capital	Deposit-to-currency ratio		
	6	9	12
3%	0.00464	0.00357	0.00297
5%	0.00774	0.00595	0.00495
7%	0.01083	0.00833	0.00694

Table 4
Sensitivity: φ.

Intermediated/ total capital	Deposit-to-currency ratio		
	6	9	12
3%	0.00036	0.00027	0.00023
5%	0.00101	0.00076	0.00064
7%	0.00199	0.00150	0.00127

3.3. Uncertainty

Lucas (2000, p. 258) rephrases the belief that he claims "[m]any economists [have] that a deterministic framework like Bailey's or [his] misses important costs of inflation that are thought to arise from price or inflation rate *variability*". Lucas continues by stating that "[he is] very confident that the effects of such a modification on the welfare costs estimated [in his paper] would be negligible".

Our results support Lucas' conjecture. For inflation rates above the lower bound of the inside-money economy, if we keep the exogenous technology process the same and vary the standard deviation of the monetary process, the first two moments of the consumption and leisure sequences remain almost the same. The economic intuition for this result must be that only a small fraction of household net worth is held as nominal assets, and that therefore the effect of a marginal increase in nominal uncertainty is insignificant.

3.4. Lower bound

A theoretical novelty of the model is the existence of a lower bound of inflation for the economy with endogenous money supply. At this level of inflation, the real rate of return net of transaction costs on deposits used on n purchases of the largest consumption goods must equal the rate of return on currency, that is,

$$\tilde{r}_{t+1} - \frac{\gamma n}{(1-\omega)c_t^*} r_{t+1} = \frac{p_t}{p_{t+1}},$$

where the second term is the ratio of the cost of n purchases of the largest consumption goods over the size of these purchases multiplied by the alternative rate of return.

Three important technology and policy parameters determine the rate-of-return difference between non-intermediated capital and currency: the time cost of replenishing money balances (φ), the cost of using deposits for purchases (γ), and the reserve requirement ratio (θ). The rate-of-return difference at the lower bound is smaller the higher is φ, the lower is γ, and the lower is θ.

For the benchmark calibration of the model, the lower bound of inflation of the economy with endogenous money is −1.64%. Given estimates for the marginal cost of public funds, both from labor taxation within the model and those reported elsewhere, it is inefficient to reduce the inflation rate to this level.

3.5. Sensitivity

For the benchmark economy, the two transaction-cost parameters, γ and φ, are calibrated so that the steady-state deposit-to-currency ratio is 9, and intermediated/total capital is 5%. These values correspond to empirical averages for the sample period, where included in currency is an estimate of the portion held in the United States, as described in Freeman and Kydland (2000), and an estimate of intermediated capital is the deposit portion of M1 net of reserves. Both of these ratios have declined steadily, however, from 12 to 6 in the case of the former, and from 7% to 3% for the latter. In Tables 3 and 4, we list the implied values of γ and φ associated with such changes. As we see from these tables, the resulting values of both γ and φ increase in the estimate for intermediated/total capital stock, and are insensitive or slightly decreasing in the estimate of the deposit-to-currency ratio.

Over these ranges, the min and max values of γ and φ are (0.00297, 0.01083) and (0.00023, 0.00199), respectively. At an annual inflation rate of three percent, these estimates imply that the cost associated with deposit purchases is in the range between 0.18 and 0.66 percent of GDP, and time spent per quarter to replenish money balances is in the range of 16 to 143 minutes.

Table 5
Sensitivity: Welfare.

Intermediated/ total capital	Deposit-to-currency ratio		
	6	9	12
3%	0.00118	0.00078	0.00060
5%	0.00196	0.00130	0.00101
7%	0.00275	0.00182	0.00141

Table 5 displays the consumption-equivalent welfare gains from reducing inflation from 3% annually to zero for the range of values of γ and φ spanned by Tables 3 and 4. For the boundary cases, $(\gamma, \varphi) = (0.00297, 0.00023)$ and $(\gamma, \varphi) = (0.01083, 0.00199)$, the estimated welfare gain would have been 0.06% and 0.28%, respectively, compared with 0.12% for the benchmark case.

3.6. Improved transaction technologies

Over time, the economy's technology level has risen. It's reasonable to presume that this improvement is reflected in declines in transaction costs. Based on the experiments in the preceding section, the model clearly suggests that this is what in all likelihood happened. Looking at Tables 3 and 4, reality, in terms of the empirical counterparts of the variables measured along the two axes, more or less moved, over the sample period, from the bottom right-hand to the upper left-hand corners. In the case of both γ and φ, substantial declines in their values are implied. In this section, we look systematically at how welfare gains due to improvements in the transaction technologies compare, quantitatively, with those associated with reductions in the inflation rate when the transaction technologies are kept constant.

Here we do an experiment in which transaction costs are reduced by three-quarters. Figs. 13.1 and 13.2 show welfare gains as φ and γ are reduced, respectively. As we see, the welfare gain if each of the transaction-cost parameters is divided by four and the other kept fixed is about 0.3%. Fig. 13.3 shows that if both transaction-cost parameters are simultaneously reduced by three-quarters, the consumption equivalent welfare gain is almost 0.5%.

As implied by Fig. 4, in the absence of lump-sum taxation, the welfare gain associated with this substantial reduction in transaction cost is significantly less than the gain of about 0.7% when inflation is reduced to the Bailey–Friedman rule for given transaction costs. At the same time, it is larger than the welfare gain associated with reducing inflation to the lower bound of the inside-money economy.

Figs. 14 and 15 plot the *direct* transaction and opportunity costs of inflation ("Use of deposits for purchases", "Replenish money balances", "Opportunity costs, currency", and "Opportunity costs, deposits") as the γ and φ are reduced, respectively. If the time cost of each replenishment of money balances decreases while the cost of using deposits for purchases is kept constant (Fig. 14), households will substitute away from deposit holdings towards holding more currency, and replenish their money balances more frequently. The result is that both the opportunity cost of holding currency and the total cost of replenishing money balances increase, but the decrease in both the opportunity cost of deposits and the cost of using deposits for purchases more than compensates for that increased cost.

As we see from Fig. 15, if the cost of using deposits for purchases (γ) decreases and the time cost of replenishing money balances is held constant, households hold more real deposits and the opportunity cost of holding deposits increases. The substitution towards more real deposits is still sufficiently small to make the income effect from improvement of the transaction technology dominate, and the total cost of using deposits for purchases decrease.

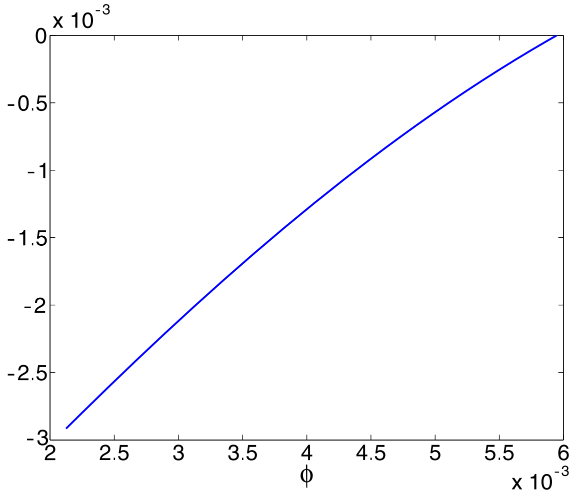
4. Concluding remarks

Compared to the existing literature on the welfare cost of inflation, the model in this paper contains several novel features. Importantly, people make purchases using both inside and outside money. The proportions of purchases made by either are determined by economic decisions in which the liquid assets' relative returns play an important role. The model allows for two transaction costs, one associated with using deposits when making purchases, and one incurred when liquid balances are replenished during the period. The additional margin through which households may avoid the inflation tax on currency lets us distinguish between costs households incur due to lower return on currency and costs due to actions taken to avoid the inflation tax.

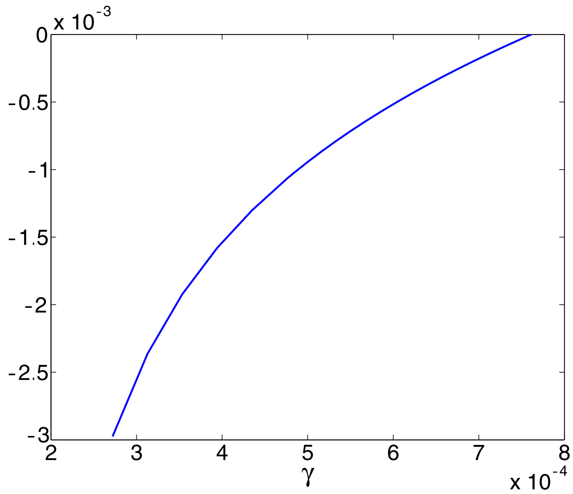
Our welfare-cost estimates are somewhat lower than what Cooley and Hansen (1989) and Lucas (2000) report. An interesting finding is that the welfare-cost curve is quite concave, meaning that the cost goes up steeply with steady-state inflation for low inflation rates (especially around 5 percent and lower) before flattening out considerably.

Another interesting finding is that only a small fraction of the total welfare cost of inflation can be accounted for by the opportunity cost of holding currency, and that most of the cost can instead be accounted for by the direct transaction costs and by the distortions to labor input and capital accumulation.

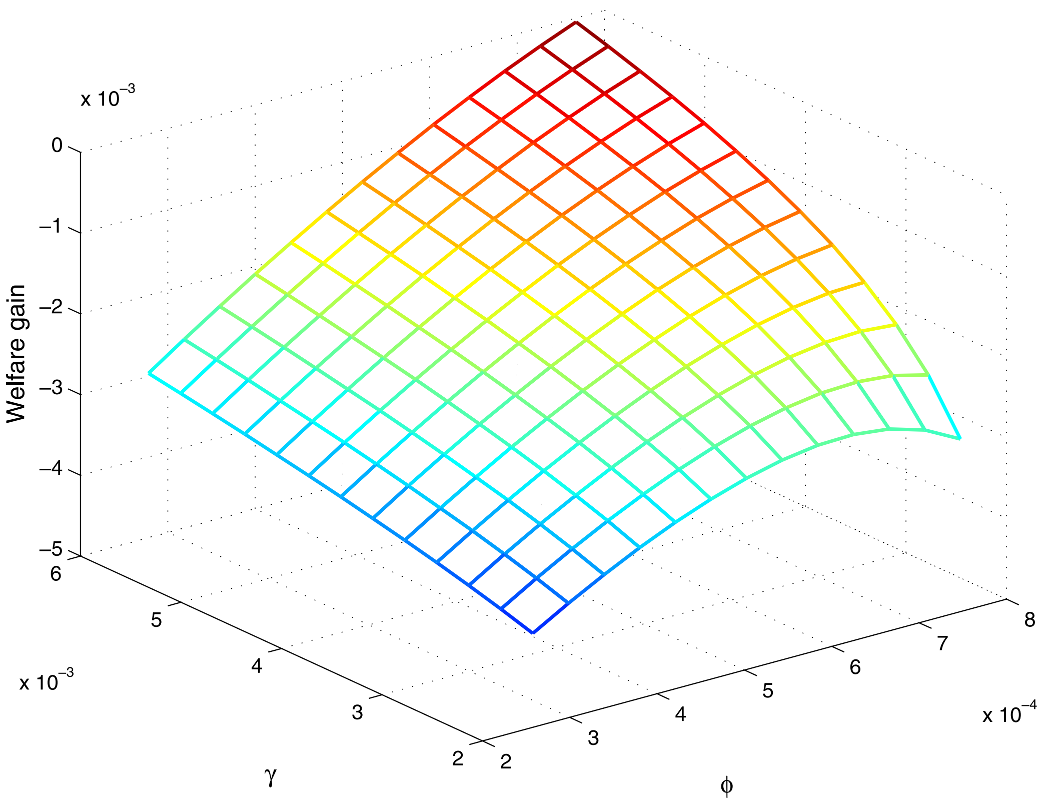
The model's two proxies for the banking sector, real deposits and resources spent on transaction services, give somewhat ambiguous answers for how the size of this sector varies with the inflation rate. Whereas resources spent on transaction costs increase monotonically in the inflation rate, holdings of real deposits reach their maximum level at an inflation rate just above 3%.



13.1: φ



13.2: γ



13.3: φ and γ

Fig. 13. Potential welfare gains from improved transaction technologies.

We analyze the sensitivity of the welfare-cost estimate to several features. In particular, it is quite sensitive to the magnitudes of the two quantities used to calibrate the transaction-cost parameters. This is an interesting discovery, as these parameters most likely have changed over the past decades and may be expected to decrease further. As our numerical experiments show, in the economy with lump-sum taxes, the welfare gain from potential technological progress leading to a decrease of both transaction parameters by 75% is larger than the welfare gain from setting the inflation rate to zero, but less than the gain from reducing inflation to the inverse of the real rate of return when the government has access to lump-sum taxation.

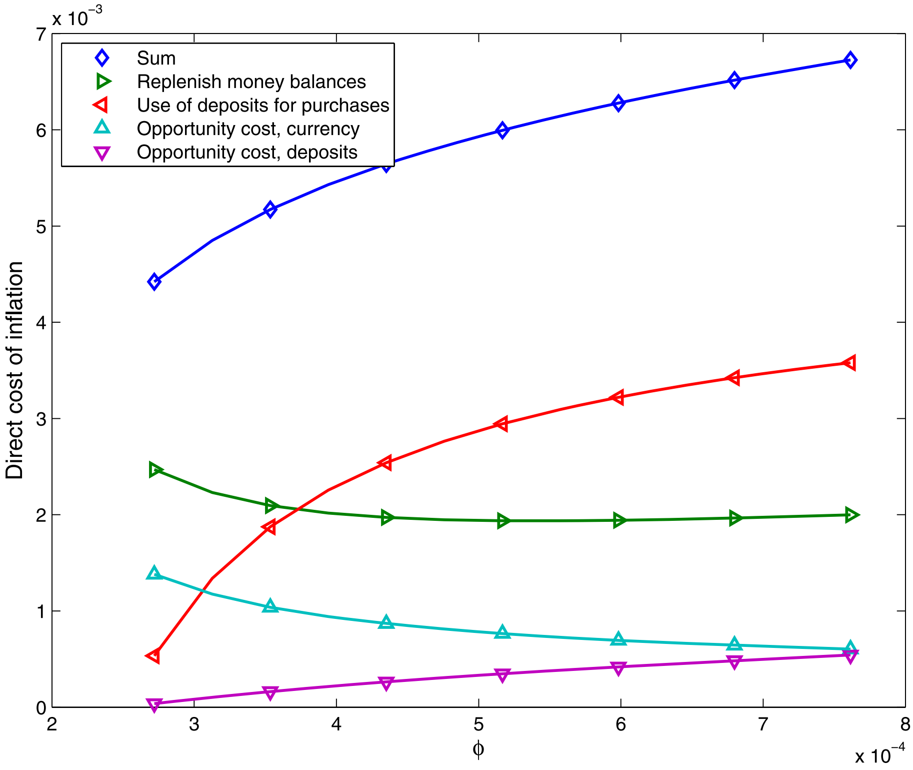


Fig. 14. Direct costs of inflation as φ is reduced.

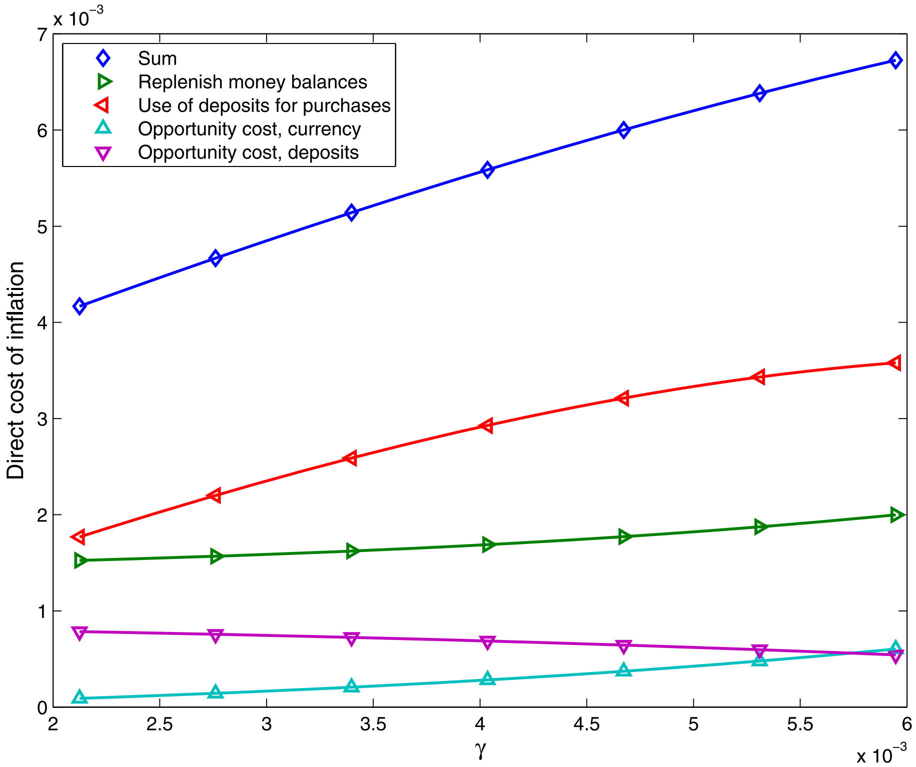


Fig. 15. Direct costs of inflation as γ is reduced.

An intriguing theoretical novelty of the model is the existence of a rate of inflation below which households do not hold any deposits, i.e. outside money ceases to exist. For strictly positive transaction cost parameters, this lower bound is strictly larger than the inverse of the real rate of return.

More importantly, in this model economy where households can choose other margins to avoid the inflation tax than holding less real balances, seigniorage is a highly inefficient way of raising public funds. Given estimates of the marginal cost of public funds, both within the model and from the labor and macro literature, we conclude not only that inflation is a suboptimal source of government revenue, but that, in the absence of lump-sum taxation, deflationary policies may be highly inefficient.

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