Monetary policy, taxes, and the business cycle

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Abstract

This paper analyzes the interaction of inflation with the tax code and its contribution to aggregate fluctuations. We find significant effects operating through the tax on realized nominal capital gains. A tax on nominal bond income magnifies these effects. Our innovation is to combine monetary policy shocks with non-indexed taxes in a model where the central bank implements policy using an interest rate rule. Monetary policy had important effects on the behavior of the business cycle before 1980 because policymakers did not exert effective control over inflation. Monetary policy reform around 1980 led to better control, and with more stable inflation, the effect of the interaction between monetary policy and the nominal capital gains tax has become negligible.

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1. Introduction

Does the interaction of inflation and the tax code contribute considerably to aggregate fluctuations? There is a large body of work showing that the steady-state welfare effects of moderate inflation are large when nominal capital gains are taxed. These include the partial equilibrium analyses of Fischer (1981), Feldstein (1997), and Cohen et al. (1999). The literature also includes the steady-state analysis of general equilibrium models in Abel (1997), Leung and Zhang (2000), and Bullard and Russell (2004). In general equilibrium, the welfare costs arise because, for any given capital income tax rate, an increase in the inflation rate raises the real pre-tax rate of return to capital and lowers the after-tax return. The lower after-tax return causes a decline in the capital stock and a reduction in labor productivity. These analyses are about steady states and only suggestive about the cyclical impacts. This paper examines the dynamic implications for the interaction between inflation and the capital gains tax.

We specify a dynamic, stochastic, general equilibrium model that combines monetary policy shocks with taxes on nominal capital gains in a setting where the central bank implements policy using an interest rate rule. The use of an interest rate rule makes inflation highly persistent, leading to persistent changes in the expected marginal tax rate on real capital gains. We find that monetary policy had important effects on the behavior of the business cycle before 1980 because the Fed did not respond aggressively to inflation shocks that were highly persistent. Monetary policy reform around 1980 led to lower and more stable inflation. A more credible commitment to price stability and a more aggressive response to inflation shocks has led to less persistent inflation dynamics and effectively eliminated the cyclical effects of the interaction between monetary policy and the nominal capital gains tax.

Inflation persistence induces changes in expected tax rates. Dittmar et al. (2005) show that inflation persistence is common in models where the central bank uses an interest rate rule. When the central bank is using an interest rate rule, a persistent shock to the inflation trend appears as a shock to the inflation target. It leads to a persistent deviation of inflation from the steady state and, in the presence of a nominal tax on capital gains, causes a persistent change in the effective marginal tax rate on capital. Thus, a positive shock to the inflation objective distorts the consumption/saving decision and may have a long-lasting effect on capital accumulation.

The next section describes the model with taxes on realized nominal capital gains as well as on income from labor, capital, and bonds. We then consider the model dynamics, showing how inflation affects the business cycle through the tax on

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1For empirical estimates of the burden of capital gain tax using panel data, see Poterba (1987) and Auerbach (1988). For survey of the tax policy issues and recent evidence, see Auerbach (2004).

2Altig and Carlstrom (1991) use an overlapping-generations model with nominal prices (but without money explicitly included) to show that the lack of perfect indexation for inflation in the tax code could have a large cyclical effect in principle. They find, however, that their model could not account for the magnitude of cyclical variation in hours worked and that it predicts a large decline in the capital stock in the 1980s that never materialized. We find that one crucial assumption in Altig and Carlstrom—the relatively low value assumed for inflation persistence—is likely to be important for these findings.
nominal capital gains. As it turns out, only the taxes on capital gains and bond income are important for business cycle dynamics. The bond tax only matters if there is also a tax on capital gains. Note that the current U.S. tax code continues to tax nominal income from bonds and realized capital gains. Finally, the model is used with the history of inflation shocks and estimates of inflation persistence to show how the interaction of monetary policy with the tax code has affected capital, hours, and productivity in the U.S. economy.

2. A monetary model with nominal taxes

2.1. Technology

Output is produced with a constant-returns-to-scale (CRTS) production technology

\[ Y_t = z_t F(K_t, x_t N_t) = z_t K_t^{\gamma_x} (x_t N_t)^{1-\gamma_x}, \]

where \( z_t \) is a stationary technology shock and \( x_t \) is an index of labor-augmenting technical progress that increases at a deterministic (gross) growth rate \( \gamma_x = \frac{1}{(1-\gamma)} \). The implied growth rate for output, capital, and consumption, \( \gamma_x \), defines a steady-state growth path for the real economy.

The firm sells output at price \( P_t \), and purchases labor and capital services from the household at nominal wage \( W_t \) and rental price of capital \( V_t \). Along with the CRTS assumption, profit-maximization under perfect competition implies that the real wage rate, \( w_t = \frac{W_t}{P_t} \), and rental price, \( v_t = \frac{V_t}{P_t} \), will be equated with the marginal products of labor and capital.

Capital—owned by the household—follows the law of motion

\[ K_{t+1} = (1 - \delta)K_t + I_t, \]

where \( I_t \) is gross investment and \( \delta \) is the depreciation rate.

2.2. Government with a nominal tax code

A government issues money and collects revenues by imposing proportional taxes on nominal income from labor, bond interest, and capital ownership (with possibly differing tax rates). Government revenues, \( T_t \), from income taxes are

\[ T_t = \tau^N_t W_t N_t + \tau^B_t R_t B_t + \tau^K_t (v_t - \delta) P_t K_t + \tau^G_t G_t, \]

where \( R_t \) is the nominal interest rate on bonds from the previous period. The third term in Eq. (3) represents the revenue from taxes assessed on capital returns net of depreciation charges. The fourth term represents the tax on nominal capital gains. We consider two alternative versions of the capital gains tax. In the first, simple case, the capital gains tax is treated as an accrual tax

\[ G_t = (P_t - P_{t-1}) K_t. \]

In the second version, the capital gains tax applies only to realized gains, with the representative household allowed to manage the timing of realization. Specifically, the household is assumed to manage a stock of unrealized capital gains, \( U_t \), subject to an
adjustment-cost function that represents portfolio management costs

\[ U_{t+1} = U_t + (P_t - P_{t-1})K_t - \phi \left( \frac{G_t}{U_t} \right) U_t \]  

(4b)

with \( \phi > 0, \phi' > 0, \) and \( \phi'' < 0 \). We assume that the steady-state value of \( \phi(G/U) = G/U \).

Further details regarding the cost function are discussed in the calibration section.

Revenues from the income taxes are returned to the household via a lump-sum rebate. This allows us to consider the pure distortionary effects of taxation, abstracting from wealth effects associated with reallocations between the public and private sectors. The government transfers money to the public directly.

2.3. Households

A representative household maximizes a discounted stream of utility from consumption and leisure

\[
\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t)
\]

with \( u(C_t, L_t) = \left( \frac{C_t^{\gamma} L_1^{1-\gamma}}{C_t^{\gamma}} \right)^{-\sigma} / (1 - \sigma) \)

subject to a nominal budget constraint and a constraint on the allocation of time. The household’s nominal budget constraint can be written

\[
(1 - \tau^N)W_tN_t + (1 - \tau^K)(v_t - \delta)P_tK_t - \tau^G G_t + \tilde{T}_t + \left[ 1 + (1 - \tau^B)R_t \right] B_t + M_t + \Delta_t^M = P_tC_t + P_t[K_{t+1} - K_t] + B_{t+1} + M_{t+1},
\]

(5)

where \( \Delta_t^M \) is the lump-sum monetary transfer and \( \tilde{T}_t, \) the tax refund, is taken as exogenous by the households.

The household endowment of time (normalized to 1) can be allocated to leisure, labor input to the production process, or transaction-related activities such as shopping and trips to the bank

\[ L_t + N_t + S_t = 1. \]

(6)

The shopping-time function is increasing in the nominal value of consumption purchases and decreasing in the quantity of money held for facilitating transactions

\[ S_t = \xi \left( \frac{P_tC_t}{M_t} \right)^\eta \]

(7)

with \( \xi, \eta > 0 \). Note that the shopping-time function depends on pre-transfer money—a timing assumption used by Kydland (1989) that is also consistent with cash-in-advance timing. If we included the transfer, then it would be equivalent to end-of-period balances and more comparable with the analysis of models in which money enters the utility function directly. Both variants of shopping-time technology are discussed in Goodfriend.
and McCallum (1987). The only important result that depends on this timing is the real
determinacy of the equilibrium with a contemporaneous policy rule: Carlstrom and Fuerst
(2001) show that the determinacy conditions may depend importantly on these somewhat
arbitrary timing conventions.

2.4. Growth trends and stationarity

The price level grows at the trend inflation rate, \( \gamma_p \), so the nominal values also share
a common trend. In the computational experiments, we treat \( \gamma_p \) as stochastic, allowing
for shocks to the inflation trend. To ensure that the government’s intertemporal
budget constraint is satisfied, we impose the condition that the growth rate of bonds
and money are cointegrated with the nominal growth trend, \( \gamma_x \gamma_p \). Stationarity requires that
the \( G/U \) ratio be constant over time, with each variable growing at the nominal growth
trend.

To solve for the model’s approximate dynamics, we require a stationary representation,
which can be derived by deflating all real variables by \( (\gamma_x)^t \) and deflating all nominal
variables by a similar index of the trend rate of inflation, \( (\gamma_p)^t \).\(^4\) The timing convention is
such that \( R_{t+1} \) represents the return on a one-period bond from \( t \) to \( t+1 \), and \( \gamma_{pt+1} \)
represents the nominal trend growth rate from \( t \) to \( t+1 \).

2.5. Stochastic general equilibrium

The first-order conditions from the household’s problem, along with optimality
conditions from the firm’s problem and equilibrium conditions for clearing the markets
for goods and labor, determine the endogenous responses of the model to stochastic
shocks. All that remains is to specify the behavior of government-controlled variables and
other exogenous processes.

Without loss of generality, we will assume that government borrowing is zero in each
period, so that the household’s first-order condition with respect to bonds defines the
nominal interest rate. Tax rates are assumed to be constant and the central bank uses an
interest rate rule to achieve an inflation target. Under the interest rate rule, the money
stock is determined endogenously from the money demand relationship. In preliminary
results for this study, we found that none of our main qualitative results depended on
having output in the policy rule. Therefore, we focus on policy in which the central bank
responds only to deviations of inflation from a stochastic target. We do not attempt to
explain why the central bank allows the inflation objective to be random; rather, we show
one consequence of its doing so.\(^5\)

Writing the policy rule in terms of log-deviations from a constant steady state
\[ \hat{R}_{t+1} = (1 + \phi_x)\hat{R}_t - \phi_x\hat{\gamma}_{pt}, \]

\(^4\) This transformation is now standard in this literature as described in King et al. (1988). See the appendix for
solution details.

\(^5\) There is a large literature spawned by DeLong (1997) and Sargent (1999) that attempts to explain why the Fed
allowed inflation to follow a stochastic trend. See Nelson (2005) for a recent summary and a monetarist
interpretation.
where $\varphi_\pi$ is positive and large enough to guarantee a unique equilibrium.\(^\text{6}\) The deviation of the inflation target from the steady-state inflation rate, $\hat{\gamma}_{pt}$, follows an exogenous autoregressive process, $\hat{\gamma}_{pt} = \rho_{\pi} \hat{\gamma}_{pt-1} + \xi_{\pi}$, where the shock is assumed to be independent and identically distributed as $N(0, \sigma^2_{\pi})$.

The remaining exogenous variable, $z_t$, is similarly assumed to follow a first-order autoregressive process calibrated from the data: $z_t = \rho_{z} z_{t-1} + \xi_{z}$, where $\xi_{z}$ is assumed to be independent and identically distributed as $N(0, \sigma^2_z)$.

The model we consider here is devoid of any of the nominal frictions that are often assumed to account for real effects of monetary policy. Nevertheless, there are three interrelated features that generate non-superneutralities in our model.

From the household’s first-order conditions, we have

$$
(1 + \tilde{R}_{t+1}) = 1 + (1 - \tau^B)R_{t+1},
$$

(9)

where $\tilde{R}_{t+1}$ denotes the after tax nominal rate, defined by

$$
(1 + \tilde{R}_{t+1}) = E_t[(1 + \tilde{r}_{t+1})\pi_{t+1}],
$$

where $\pi_{t+1}$ is the gross inflation rate from period $t$ to $t + 1$, $\gamma_{pt}p_{t+1}/p_t$, and the corresponding after-tax real rate is defined by

$$
(1 + \tilde{r}_{t+1}) = E_t \left[ \frac{\gamma_{\lambda} \tilde{\lambda}_t}{\beta \tilde{z}_{t+1}} \right].
$$

where $\lambda_t$ is the marginal utility of consumption in period $t$. The implied coefficient on changes in the inflation target in the monetary policy rule, (7), is unity. From (8), it is clear that the tax rate on bonds introduces a wedge between the nominal bond rate and the after-tax rate relevant to households. Hence, a rise in inflation will cause after tax real interest rates to decline.

This feature of the model interacts closely with the direct effect of the capital gains tax

$$
(1 + \tilde{r}_{t+1}) = E_t \left\{ 1 + \left[ (1 - \tau^K)(v_{t+1} - \delta) \right] - \left[ \frac{\tau^G}{\phi'(g_{t+1}/u_{t+1})} \right] (1 - 1/\pi_{t+1}) \right\}.
$$

(10)

The last term in Eq. (10) reflects the taxation of nominal capital gains. The expression for the accrual-based capital gains tax is obtained by setting $\phi'(g_{t+1}/u_{t+1}) = 1$. A higher inflation rate lowers after-tax returns to capital through this channel, lowering investment and capital accumulation. This is the primary mechanism driving the model’s response to our policy shocks. The presence of a tax on nominal bond interest amplifies the transitory response of inflation to an inflation target shock, which amplifies the real effects of the capital gains tax.

Inflation also matters because it lowers real returns on money and bonds. For a given baseline real return, an increase in inflation requires a higher nominal bond rate and a higher nominal return to money holdings in equilibrium. In the case of money, higher nominal returns are associated with lower real money balances and higher shopping-time costs.

\(^6\text{Edge and Rudd (2002) show that adding taxes to the model restricts the size of the parameter space for which the model has a unique equilibrium. In our model with the baseline calibration, }\varphi_\pi \text{ must be greater than 0.3.}\)
After some substitution from the household’s first-order conditions, the condition for optimal money holdings can be written in a form that can be interpreted as a money demand relationship:

$$m_{t+1} + 1 \over p_{t+1} = \left[ \eta \tilde{\zeta} (1 - \tau^N)^{1/2} (1 - z) (y_{t+1}/N_{t+1}) c_{t+1}^{\gamma} \right]^{1/(1+\eta)} \frac{(1 - \tau^B) R_{t+1}}{(1 - \tau^B) R_{t+1}}.$$  \hspace{1cm} (11)

The expression looks complicated, but it actually has a familiar double-log form. Calibrating the shopping-time function with $\eta = 1$ implies an interest elasticity of $-\frac{1}{2}$.\(^7\) Note also that because consumption and productivity are cointegrated, the scale variable in the numerator of (11), $[(y/N)c^\gamma]$, implies a long-run income elasticity equal to unity.\(^8\) Note that, in addition to the inflation tax, both the labor and bond tax rates affect real money demand.

2.6. Steady-state and model calibration

The model’s dynamics will be approximated as proportional deviations from a baseline steady state, defined by the model parameters (including the baseline growth rates of technology and prices, $\gamma_s$ and $\gamma_p$). Our calibrations are based on long-run characteristics of the data and/or are common to the literature using computable general equilibrium models. The baseline calibration for the model is shown in Table 1.

In our baseline calibration, the coefficient on the deviation of inflation from target $(1 + \phi_p)$ is set equal to 1.375. This is less than the 1.5 that Taylor (1993) suggested for the post-1980 data, but larger than many estimates using data from the earlier period. Clarida et al. (2000) estimate a value around 0.8 for $(1 + \phi_p)$ using U.S. data from the period before October 1979. Lower values of $\phi_p$ result in equilibria with more price variability and larger interactions between inflation shocks and the tax code. As noted in Edge and Rudd (2002), the inclusion of a bond tax increases the area of indeterminacy associated with interest rate rules—the region rises from unity to about 1.34 for our baseline calibration of the model.

In principle, we could calibrate the time-series process for the inflation trend using data on either money growth or inflation. Because the U.S. data were generated in an economy in which the Fed usually followed an interest rate rule, the model suggests that we should calibrate the model to persistence in the inflation data, not the money growth data.\(^9\) Gavín and Kydland (2000), Kim et al. (2004), and many others, show that the autocorrelation of inflation dropped significantly after the policy change in October 1979. Therefore, we estimate the persistence in the inflation rate separately for pre- and post-October 1979 periods. Using an augmented Dickey-Fuller method, we estimate the persistence to be 0.97 before October 1979 and 0.84 afterwards. The standard deviation of the residual is approximately 0.4 percent at a quarterly rate in both periods. Under this specification, the lower unconditional variance of inflation after 1979 is all due to lower persistence. Our baseline case is set to the pre-1979 estimate.

\(^7\)Pakko (1998) shows that a specification of this type is associated with real welfare effects of inflation that are consistent with the typical welfare-triangle analysis of the money demand literature.

\(^8\)Because both consumption and labor productivity tend to be procyclical—but with smaller amplitude than output—the short-run income elasticity of the money demand relationship will be less than 1.

\(^9\)See Balke and Wynne (2004) for an identification procedure which uses M2 and disaggregated price data to identify the monetary policy shock to the trend in M2 growth.
Steady-state tax rates for labor, interest, capital income, and the capital gains tax are set to equal the average marginal tax rates for 1960–2002, calculated using the NBER TAXSIM model and reported in Table 9 of Feenberg and Poterba (2003). They report 24 percent for labor, 26 percent for interest income, 34 percent for capital income, and 20 percent for realized capital gains.

Calibration of the parameters of the capital gains accrual Eq. (4b) requires quantitative restrictions on the adjustment cost function, \( \phi(G/U) \). As mentioned earlier, \( \phi(G/U) = G/U \) is assumed so that the adjustment costs apply only to deviations from the steady state. On average, for this period, realized capital gains were about 40 percent of changes in the nominal capital stock measured as the net stock of private nonresidential assets. Accordingly, we calibrate the steady-state ratio of capital gains realized to capital gains accrued, \( G_t/(P_t - P_{t-1})K_t \), to equal 0.4. From Eq. (4b), this calibration results in a steady-state \( G/U \) ratio of 0.0094 (the ratio of capital gains realized to accumulated unrealized gains). Note these ratios are so low because some capital gains are never realized. Some are held by tax exempt institutions such as pension funds and some are bequeathed to heirs, in which case the basis for the capital gains is reset to the current market value and no capital gain tax is paid (the estate may be taxed, however).

In order to calculate the linearly approximated version of the model, the first two derivatives of \( \phi(G/U) \) also require calibration. From the first-order condition determining the optimal accumulation of unrealized capital gains, the first derivative is equal to the steady-state nominal interest rate (see Appendix). The elasticity of marginal adjustment costs with respect to the \( G/U \) ratio, \( \zeta = (G/U)\phi''(G/U)/\phi'(G/U) \), is calibrated to be consistent with Auerbach’s (1988) regression results showing that a one-percent increase in the capital gains tax rate is associated with a 0.56 decline in realized capital gains.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depreciation rate</td>
<td>( \delta )</td>
<td>0.02</td>
</tr>
<tr>
<td>Discount factor</td>
<td>( \beta )</td>
<td>0.99</td>
</tr>
<tr>
<td>Relative risk aversion</td>
<td>( \sigma )</td>
<td>2</td>
</tr>
<tr>
<td>Capital share in production</td>
<td>( \alpha )</td>
<td>0.38</td>
</tr>
<tr>
<td>Shopping-time parameter</td>
<td>( \eta )</td>
<td>1</td>
</tr>
<tr>
<td>Steady-state share of time labor time</td>
<td>( N )</td>
<td>0.3</td>
</tr>
<tr>
<td>Steady-state share of shopping time</td>
<td>( S )</td>
<td>0.003</td>
</tr>
<tr>
<td>Labor tax rate</td>
<td>( \tau^N )</td>
<td>0.24</td>
</tr>
<tr>
<td>Capital tax rate</td>
<td>( \tau^K )</td>
<td>0.34</td>
</tr>
<tr>
<td>Bond tax rate</td>
<td>( \tau^B )</td>
<td>0.26</td>
</tr>
<tr>
<td>Capital-gains tax rate</td>
<td>( \tau^G )</td>
<td>0.20</td>
</tr>
<tr>
<td>Steady-state ratio of realized to accumulated capital gains</td>
<td>( G/U )</td>
<td>0.0094</td>
</tr>
<tr>
<td>Elasticity of marginal adj costs w.r.t ( G/U )</td>
<td>( \zeta )</td>
<td>-1.1</td>
</tr>
<tr>
<td>Steady-state output growth</td>
<td>( \gamma_x )</td>
<td>1.004</td>
</tr>
<tr>
<td>Steady-state inflation</td>
<td>( \gamma_p )</td>
<td>1.01</td>
</tr>
<tr>
<td>Fed’s reaction to inflation</td>
<td>( \Phi_z )</td>
<td>0.5</td>
</tr>
<tr>
<td>S.D. of the technology shock</td>
<td>( \sigma_z )</td>
<td>0.0075</td>
</tr>
<tr>
<td>Persistence in the technology shock</td>
<td>( \rho_z )</td>
<td>0.95</td>
</tr>
<tr>
<td>S.D. of the inflation shock</td>
<td>( \sigma_x )</td>
<td>0.0040</td>
</tr>
<tr>
<td>Persistence in the inflation shock</td>
<td>( \rho_x )</td>
<td>0.97</td>
</tr>
</tbody>
</table>
A simulation experiment using the time series property of Auerbach’s data on capital gains realizations generates approximately this result with an elasticity measure $\zeta$ equal to $-1.1$.

Balcer and Judd (1987) model the complexity of the tax code in a life cycle model and argue that, in a frictionless world with complete markets, the effective marginal tax rate would be negligible. Nevertheless, in a study of a panel of 13,000 individual tax returns collected throughout the period from 1985 to 1994, Auerbach et al. (2000) measure the marginal effect tax rates by income class. They find that all but highest income class paid the statutory rate on realized capital gains. Those in the highest class, the most sophisticated taxpayers, faced effective marginal tax rates that were about 90 percent of the statutory rates.

In the final section, we include sensitivity analysis for particular parameters that are important for the results. These include parameters in the monetary policy rule, $\varphi_\pi$ and $\rho_\pi$. The coefficient of relative risk aversion, $\sigma$, and the steady-state ratio of capital gains realized to those accrued, $G_t/(P_t - P_{t-1})K_t$.

2.7. Steady-state welfare costs

The main operative mechanism of the model—the interaction of inflation with the nominal tax code—is illustrated in the steady-state welfare calculations presented in Table 2.

The small welfare costs of inflation attributable to non-neutrality from the shopping-time function are shown in the first row. These losses are associated with typical “welfare triangle” type calculations: Higher rates of inflation induce households to economize on real money holdings, requiring greater shopping time (at the expense of leisure and work effort). For an inflation rate of 10 percent, output and consumption are only 0.44 percent lower than they would be in a zero-inflation steady state. Leisure is only marginally lower than in the zero-inflation environment. The final two columns of the table show the combined effects of lower consumption and leisure on household utility, using a measure of compensating variation calculated as the $\kappa$ that solves

$$U(c^0_t, L^0_t) = U((1 - \kappa)c^0_t, L^0_t),$$

where superscripts denote the steady-state inflation rate. For the first row, this value represents a cost of only 0.49 percent of steady-state consumption in the zero-inflation environment.

Table 2

<table>
<thead>
<tr>
<th>Welfare effects of a steady-state 10 percent inflation rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effects on steady-state values (percent)</td>
</tr>
<tr>
<td>$Y$</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>No taxes</td>
</tr>
<tr>
<td>Taxes w/o capital gains</td>
</tr>
<tr>
<td>Taxes incl. capital gains</td>
</tr>
<tr>
<td>Accrual based</td>
</tr>
<tr>
<td>Realization based</td>
</tr>
</tbody>
</table>
environment. The second row shows that—with the exception of the capital gains tax—the addition of taxes to the model has almost no effect on the welfare costs of inflation.

The third and fourth rows show the dramatic effect that nominal taxation of capital gains has on the steady state. The third row shows the results using the accrual-based specification. In the high-inflation environment, output is 12.13 percent lower than it would be at zero inflation, while consumption is lowered by 8.49 percent. The main effect of inflation is revealed in the capital/output ratio, which is 14.69 percent lower in the 10 percent inflation regime. As a result, wages and employment are lower (so that leisure is actually higher for this case). In terms of the compensating variations, 10 percent inflation represents a cost of 7.03 percent of steady-state consumption, or 5.59 percent of output.

As shown in the fourth row of Table 2, the ability to shift tax capital gains taxation into the infinite future mitigates the steady-state welfare effects of inflation. The 10 percent inflation environment is associated with a 2.98 percent decline in output and a 2.09 percent decline in consumption. The associated compensating variations are 1.79 percent and 1.41 percent. Although these measures are considerably smaller than found using the accrual-based capital gains tax, they are an order of magnitude larger than the welfare costs implied by the model without any nominal capital gains taxation.\(^{10}\) In the model, the 10 percent increase in steady-state inflation results in a permanent decline in the \(G/U\)-ratio of about 0.66 percent.

These calculations confirm that our model framework captures the effects highlighted by Feldstein, Fisher, and others—namely, that the nominal taxation of capital gains interacts with inflation to suppress capital accumulation. The model dynamics presented below show how this mechanism can generate real fluctuations in response to stochastic inflation.

3. Model dynamics

This section reports computational experiments that show how the model economy responds to monetary policy shocks under alternative assumptions about tax and monetary policies. Before turning to the issue of how inflation and taxes interact to generate model dynamics, it is instructive to consider the response of the model to changes in the capital gains tax. Fig. 1 shows how reported capital gains and the portfolio of unreported gains respond to a persistent 10 percent increase in the capital gains tax rate. We calibrate the persistence in the average marginal tax rate on capital gains to be equal to the largest root in the time series process using U.S. data from 1954 to 2001.\(^{11}\) The tax change is known one period before it goes into effect. In our model, a persistent 10 percent increase in the tax rate, beginning in the next period, will cause investors to increase realizations by 4 percent in the period before the higher tax becomes effective. In the first period following the shock, realizations fall 5 percent below the steady state, but return to the trend within a few years. Unrealized gains rise gradually to about 1.45 percent above trend after nine years and then return very gradually to the steady state.

\(^{10}\)Welfare effects for the accrual-based capital gains tax can be made approximately equal to the realization-based measures by using an accrual-equivalent tax rate of approximately 4 percent. This is close to the rate calculated by Bailey (1969) and assumed by Feldstein and Summers (1979).

\(^{11}\)The time series on capital gains tax rates are from Feenberg and Poterba (2003) and Auerbach (1988)—for the years before 1960. The time series has an AR(1) coefficient equal to 0.83 in annual data.
In our baseline calibration, inflation reacts more than one for one with a persistent shock to the inflation target. Fig. 2 shows the response of inflation to a persistent 1 percent shock to the nominal growth trend, $g_p$, with and without a tax on bond income. Without the bond tax, a 1 percent shock to the inflation trend causes the inflation rate to jump to 0.8 percent before gradually returning to the steady state. With the 26 percent tax on interest income, the inflation rate jumps to almost 3 percent and decays gradually. Here, the relatively weak response of policy to the inflation shock causes a significant magnification of the inflation target shock on the current inflation rate.

The effect on the real economic dynamics of our model is best seen by comparing the response of the capital stock under these alternative regimes. The impulse responses of the capital stock to a monetary policy shock under four tax regimes are shown in Fig. 3. The tax regime with the smallest impact is the one with the seigniorage tax only. Here, a persistent 1 percent shock to the inflation target causes capital to decline only a tiny fraction of a percent. When we include all taxes except capital gains taxes, the maximum decline is about 0.29 percent after a decade. The decline is entirely due to the bond tax because it drives a larger wedge between the before- and after-tax interest rate. The interesting cases are those with a capital gains tax, with and without a bond tax. Braun (1994) and McGrattan (1994) show that both the labor tax and the capital income tax have large welfare effects. These effects, however, do not change with inflation and do not interact with fluctuations in the inflation rate as does the bond tax. In the third tax regime, we reinstate the capital gains tax but eliminate the tax on bond income. Here the peak effect is a 0.57 percentage decline in the capital stock that persists for several decades. When we include all taxes, the total effect is almost an order of magnitude larger. A 1 percent shock raising the inflation target reduces the capital stock by 3.0 percent by the

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12Chang (1995) considered the capital income tax, but also did not investigate the interaction with inflation.
12th year. The bond tax is important because it magnifies the impact on inflation (see Fig. 1) and therefore magnifies the increase in the effective tax on capital gains.

Fig. 4 shows the impulse responses of some key macroeconomic variables following a 1 percent inflation shock. Both output and hours worked decline sharply upon impact with the decline in investment. Output follows capital stock along a protracted path of below the steady state. Hours converge back to the steady state with a convergence rate that has a half-life of about 6 years. The model produces a counterfactual increase in consumption because there is no cost of adjusting capital and it is freely consumed if the stock is too high. The lower left-hand panel of Fig. 4 shows that this effect is quite short-lived compared with the long period of depressed consumption that follows an inflationary shock. Labor productivity also displays a short-lived increase upon impact, followed by a long period of convergence back to the trend.
Fig. 4. Responses to a 1% inflation target shock.
4. Business cycle effects

Next, we turn to the question posed at the beginning, “Does the interaction of inflation and the tax code contribute considerably to aggregate fluctuations?” We calculate the business cycle effects under two separate regimes for inflation: In the first regime, the policy parameters are those used as our baseline. They were chosen to reproduce the volatile and persistent inflation observed in the period before October 1979. The autoregressive parameter on the inflation target shock, $\rho_\pi$, is set to 0.97 and the central bank’s reaction to the deviation of inflation from target, $(1 + \varphi_\pi)$, is set to 1.375. For the period after October 1979, we choose parameters that reproduce the relative price stability observed since. The persistence parameter is lowered to 0.84 and the Fed’s reaction to inflation is raised to 1.5, the value suggested by Taylor (1993) looking at post 1980 data. In each of the computational experiments, the technology shock is assumed to have a first-order AR parameter of 0.95 and a shock variance of 0.0075.

Table 3 shows standard deviations and correlations with output for some key macroeconomic variables, comparing versions of the model with alternative capital gains tax assumptions. In both the top and bottom panels, the first block of results report the second moments calculated using U.S. data. The second block reports the model results when there is no capital gains tax, but there are taxes on income from labor, capital and interest as well as an inflation tax. In the third block, we report the results of an experiment in which nominal capital gains are taxed on accrual, but where the tax rate is lowered from 20 percent to 4 percent. This is the rate that replicates the steady-state welfare consequences of our model when taxes are paid on realization. The last block reports the results when we give households the opportunity to choose when to realize capital gains and allow 60 percent of the gains to avoid taxation altogether.

The top panel reports results for the early period. The model without capital gains taxes accounts for 75 percent of the variability in the cyclical standard deviation in output. In this simple model without the capital gains tax, the variability of hours is low and the comovement between output and other variables far too high relative to the data—particularly for productivity. These moments are nearly identical to those that would obtain in a model without either taxes or inflation. Persistent shocks to the inflation objective have no measurable impact on output in the model without a capital gains tax.

The inclusion of the accrual-based tax raises the standard deviation of output fluctuations by 7 percent. The low correlation of consumption with output reflects the initial rise in consumption that occurs as households adjust to changes in the desired capital stock. The resulting increase in the expected future effective capital gains tax causes households to consume capital on impact, generating a relatively low contemporaneous correlation with output (0.52). Adding the capital gains tax increases the standard deviation of each of the variables considered. The standard deviation of hours is approximately 46 percent larger in the model with a capital gains tax. In addition, the inclusion of capital gains taxes introduces a propagation channel for inflation shocks that lowers the high correlation between output and other macroeconomic variables that is typical the baseline RBC model.

13It is interesting to note that Protopapadakis (1983) argues that accrual-equivalent marginal tax rates were perhaps as low as 5 percent.
The right block includes the results with the realization-based capital gains tax. The statistics are very similar to case with the 4 percent accrual tax. The cyclical effect of the inflation-tax interaction is only slightly lower than in the case of the accrual-based tax. There is slightly larger reduction in the standard deviations of hours. The biggest difference with the capital gains tax, either the accrual-equivalent or the realization-based tax, is that investment becomes more volatile than observed in the data. Other variables—hours, in particular—become more volatile, but the variability remains below that of output.

Inflation target shocks have real effects because they change expected future tax rates. That the accrual-equivalent is close to the realization-based tax was suggested by Viard (2000), who found that the asset pricing implications of expected future tax changes were similar in accrual and realization-based tax systems.

In the later period, with $\rho_x = 0.84$ and $\phi_x = 0.5$, the qualitative results are similar but much smaller. The standard deviation of output deviations is no higher than it is without the capital gains tax. Overall, except for a slight rise in the volatility of investment, there appear to be no measurable cyclical effects of adding the capital gains tax when persistence in the inflation target shock is as low as 0.84.

The statistics for U.S. data reported in Table 3 illustrate the widely documented decline in the volatility of real macroeconomic variables during the 1980s. The analysis of the model suggests that the lower persistence of inflation since 1979 might have played a partial role in this volatility decrease. With high persistence in the inflation process, inflation shocks interact with the capital gains tax to have large effects on real variables.

### Table 3
Second moments (HP filtered)

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<td>Corr((\bullet, y))</td>
<td>SD((\bullet))</td>
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<td>1.77</td>
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<td>1.33</td>
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<tr>
<td>Consumption</td>
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<td>0.97</td>
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<tr>
<td>Investment</td>
<td>5.21</td>
<td>0.79</td>
<td>4.04</td>
<td>0.99</td>
</tr>
<tr>
<td>Hours</td>
<td>1.91</td>
<td>0.87</td>
<td>0.67</td>
<td>0.89</td>
</tr>
<tr>
<td>Productivity</td>
<td>1.23</td>
<td>0.61</td>
<td>0.80</td>
<td>0.92</td>
</tr>
</tbody>
</table>

**Panel A:** $\rho_x = 0.97$ and $\phi_x = 0.375$

<table>
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<tr>
<td></td>
<td>SD((\bullet))</td>
<td>Corr((\bullet, y))</td>
<td>SD((\bullet))</td>
<td>Corr((\bullet, y))</td>
</tr>
<tr>
<td>Output</td>
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<tr>
<td>Consumption</td>
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<td>0.55</td>
<td>0.98</td>
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<tr>
<td>Investment</td>
<td>4.47</td>
<td>0.80</td>
<td>4.01</td>
<td>0.99</td>
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<tr>
<td>Hours</td>
<td>1.65</td>
<td>0.89</td>
<td>0.53</td>
<td>0.98</td>
</tr>
<tr>
<td>Productivity</td>
<td>0.88</td>
<td>0.36</td>
<td>0.79</td>
<td>0.99</td>
</tr>
</tbody>
</table>

**Panel B:** $\rho_x = 0.84$ and $\phi_x = 0.5$

\(\text{aWe use an accrual equivalent tax rate} = 4\ \text{percent based on steady state welfare effects in Table 2.}\)
4.1. Simulations of U.S. data

The computational experiments suggest that we should see important effects from the interaction of inflation and the capital gains tax before 1980, but the effects may be too small to be measurable afterwards. To illustrate this feature of the model, we use estimated shocks to the inflation trend to see what our model implies for movements of capital, hours worked, and labor productivity in U.S. history, assuming a policy break in 1979:Q3. We use the same calibrations for the policy process that are used in Table 3. The contribution of estimated inflation shocks to the real economy is summarized in Fig. 5.

In the period leading up to 1980, the effects of the interaction between inflation and the capital gains tax are large enough to be measurable. As we saw in Fig. 2, the effects on the...
capital stock go on for such a long time that the damage from rising inflation in the 1960s and 1970s continued to have a depressing effect on the capital stock into the 1990s. The stabilization of the price level after the Korean War led to a rise in the capital stock to 4 percent above the steady state in 1965. The acceleration of inflation in the 1960s and 1970s caused the capital stock to fall 2.4 percent below the zero inflation steady state by 1980. Since then, the effects of the earlier inflation on the capital stock have gradually dissipated.

The impact on labor input works through the economy quickly. The upward drift of inflation caused hours worked to fall below the steady-state level for most of the 1970s. Corresponding to the inflationary effects of the oil price shocks of the 1970s, the model implies sharp declines in employment associated with those events. The severity of the 1975 recession was attributed to the negative effect of high relative oil prices on the efficiency of the existing capital stock. Wei (2003) calibrates a vintage capital model to the U.S. economy and shows that such a model cannot explain the negative response of hours worked and output to the sharp spike in the relative price oil. Our model suggests that the Fed’s reaction to the oil price shock that raised inflation expectations may help to explain the episode. Since 1980, the effect on hours worked is insignificant.

The impact on productivity reflects a combination of the effect on the capital stock and on hours worked. The upward drift in inflation combined with the nominal tax on capital gains to exert an increasingly negative impact on labor productivity from the late 1960s until after 1980. Since the 1980s, this effect led to a small, but steady, rise in labor productivity.

4.2. Sensitivity analysis

The goal in this paper is to analyze the business cycle consequences of interaction between inflation policy and a non-indexed tax system. There are reasons that our baseline case may over- or underestimate the effects of inflation operating through the tax code. In Fig. 6, we show how the standard deviation of output fluctuations (deviations from an HP trend) depend on the parameters that are important for our results. The dashed lines show the results when the capital gains tax is set to zero.

First, the Fed’s policy parameters are critically important for inducing these business cycle effects. In the upper left panel of Fig. 6, we show how the results depend on the Taylor rule parameter, $\phi_{\pi}$. We assume that the policy parameter is large enough to determine a unique equilibrium. The notion that policy during the 1970s was close to the region of indeterminacy (which begins around 0.34 for our baseline calibration) suggests that this may be an important source of business cycle fluctuations. A relatively small value of $\phi_{\pi}$ would interact with the capital gains tax to make output more variable. If the parameter is raised to 0.5, much of the extra variability disappears, even with the highly autocorrelated inflation shocks.

The upper right panel shows that the persistence of inflation target shocks is also important. Stock (1991) shows that when using the augmented Dickey-Fuller procedure, estimated values of the largest root close to unity have very wide confidence intervals and are biased downward. Stock’s bias adjustment for our case makes the value very close to one. In a recent study of the Fed’s implied inflation targets, Ireland (2005) finds a random walk in the inflation target throughout the post-war period. Many studies in the macroeconomic time-series literature finds a unit root in the inflation process for this early period. In recent research on inflation expectations embedded in the yield curve, Kozicki
Fig. 6. Sensitivity of cyclical variability to model assumptions. *Note:* Boxes mark the pre October 1979 policy calibration.
and Tinsley (2005), Ellingsen and Söderström (2004), and Dewachter and Lyrio (2006) all present evidence shocks to long-run inflation expectations are the major source of volatility in long-term bond yields. Using data from the bond market on comparable indexed and non-indexed bonds for a recent period, Gürkaynak et al. (2003) find that the one-year forward rate, 10 years ahead, responds significantly to macroeconomic news because expected inflation at that horizon responds to news. They attribute this effect to the Federal Reserve’s decision not to choose an explicit numerical objective for the long-run inflation trend. Nevertheless, many others have estimated lower values for the largest root in the inflation series. For example, using Bayesian methods, Kim et al. (2004) estimate the posterior mean of the persistence parameter to be 0.94 before 1979:Q2. The upper right panel in Fig. 6 shows that the variability of output is very sensitive to the values of this parameter, $\rho_n$, for values above 0.97. We calibrate the post-1979 policy parameter to be 0.84, where the business-cycle effects of the inflation–capital gains tax interaction are very small. In this model, changing either one of the policy parameters would effectively eliminate these effects.

In the case of both parameters, $\phi_n$ and $\rho_n$, there are reasonable values at which monetary policy can explain some of the output variation, even with no capital gains tax. This is due to the magnification of inflation shocks by the tax on nominal bond income. Still the effects are small relative to those that occur with the tax on capital gains.

In the bottom left panel, we show how the results depend upon our assumption about the steady-state ratio of capital gains realized to those accrued. Obviously, there is no effect of this ratio if there is no capital gains tax. In the bottom right hand panel, we consider the coefficient on relative risk aversion, a utility parameter that is particularly important for our results. Balcer and Judd (1987) report that the results should be sensitive to the curvature of the utility function. We find that is true. As the degree of risk aversion rises, the effect of the capital gains tax declines. Even without the capital gains tax, the cyclical variability depends on this parameter, but clearly the presence of the capital gains tax and certain monetary policies can exacerbate the effects. The cyclical effects of persistent nominal shocks operating through the capital gains tax appear to be measurable in specifications where the coefficient of relative risk aversion is less than 4.

5. Conclusion

When the central bank operates with an interest rate, persistent shocks to the inflation target can have large real effects on the business cycle if the tax system is not indexed for inflation. In our model, there is a tax on realized nominal capital gains. The business cycle effects of inflation interacting with the tax code were large before 1980 both because the shocks to the inflation target were highly persistent and because the Fed responded weakly to deviations of inflation from target. Monetary policy reform around 1980 led to better control of inflation, and with more stable inflation, the effect of the interaction between monetary policy and the nominal capital gains tax has become negligible.

We present a model of the capital gains realization problem in a representative agent setting. Using a common calibration for all parameters except for those in the monetary policy function, we find that bad monetary policy may partially explain the slowdown in productivity growth before 1980. The upward trend in the average inflation rate interacted with the tax on nominal capital gains to reduce productivity growth in the 1960s and
1970s. Better policy after 1980 may partially explain the revival of productivity and the lower variability of real variables since then.

We find that accrual equivalent capital gains tax rate of 4 percent results in the same welfare costs of a 10 percent inflation as we get with the realization based tax and a 20 percent tax rate. This estimate of an accrual-equivalent rate is in line with earlier estimates by Bailey (1969) and Protopapadakis (1983). We also find that the business cycle effects of the 4 percent accrual-equivalent tax are about the same as the effects using a 20 percent tax on realized gains.

Our study is aimed at understanding business cycle effects, not welfare effects. The welfare effects of these taxes may be quite large even if the cyclical effects are negligible. The results in this paper suggest that taking account of them would be important for understanding the nature of the U.S. economy, especially before 1980. One explanation given for the relative stability of the post-1980 economy is that monetary policy was much improved. This article demonstrates one channel for real effects of monetary policy that is consistent with that explanation.

Appendix. First-order conditions and steady-state calibration

This appendix details the equations for the model with a realization-based capital gains tax. To solve for the model’s approximate dynamics, we require a stationary representation, which can be derived by deflating all real variables by \((g_x)\) and deflating all nominal variables by a similar index of the trend rate of inflation, \((g_p)\). The resulting transformed household optimization problem, in which all nominal and real variables are stationary, can be written

\[
\max E_0 \sum_{t=0}^{\infty} \beta (c_t^0 L_t^{1-\theta})^{1-\sigma}/(1-\sigma)
\]

subject to

\[
(1 - \tau^N)w_t N_t + (1 - \tau^K)(v_t - \delta)k_t - \tau^G g_t + \tilde{t}_t/p_t
\]

\[
+ [1 + (1 - \tau^B_R)] b_t/p_t + m_t/p_t = c_t + \left[\gamma_x k_{t+1} - k_t\right] + \gamma_{p_t+1}^\gamma_x b_{t+1}/p_t + \gamma_{p_t+1}^\gamma_x m_{t+1}/p_t
\]

(A.1)

and

\[
L_t + N_t + \zeta \left(\frac{p_t c_t}{m_t}\right)^{\eta} = 1,
\]

(A.2)

\[
\gamma_{p_t+1}^\gamma_x \frac{u_{t+1}}{p_t} = \frac{u_t}{p_t} + \left(1 - \frac{p_{t-1}}{\gamma_{p_t} p_t}\right) k_t - \phi \left(\frac{g_t}{u_t}\right) \frac{u_t}{p_t}.
\]

(A.3)

In the transformed problem, lower-case variables represent inflation-adjusted, growth-adjusted stationary variables.

The first-order conditions to the household’s optimization problem can be expressed as

\[
U_c(\cdot) = \lambda_t + \omega_t \eta (S_t/c_t),
\]

(A.4)
\[ U_L(t) = \omega_t, \tag{A.5} \]
\[ \lambda_t(1 - \tau^N_t)w_t = \omega_t, \tag{A.6} \]
\[ \tau^G_t \lambda_t = \phi'(g_t/u_t) \phi_t, \tag{A.7} \]
\[ \beta E_t \left\{ \frac{\lambda_{t+1} + \omega_{t+1}p_{t+1} \eta(S_{t+1}/m_{t+1})}{\pi_{t+1}} \right\} = \gamma_x \lambda_t, \tag{A.8} \]
\[ \beta E_t \left\{ \lambda_{t+1} [1 + (1 - \tau^B_t)R_{t+1}] / \pi_{t+1} \right\} = \gamma_x \lambda_t, \tag{A.9} \]
\[ \beta E_t \lambda_{t+1} \left\{ 1 + [(1 - \tau^K_{t+1})(v_{t+1} - \delta)] - \frac{p_{t+1}}{\lambda_{t+1}} \left( 1 - \frac{p_t}{\gamma_{\pi t+1}^{p t+1}p_{t+1}} \right) \right\} = \gamma_x \lambda_t, \tag{A.10} \]
\[ \beta E_t \frac{p_{t+1}}{\pi_{t+1}} \left\{ 1 - \phi' \left( \frac{g_{t+1}}{u_{t+1}} \right) + \frac{g_{t+1}}{u_{t+1}} \phi' \left( \frac{g_{t+1}}{u_{t+1}} \right) \right\} = \gamma_x \phi_t, \tag{A.11} \]

where \( \lambda_t, \omega_t, \) and \( \phi_t \) are utility-denominated, present-valued shadow prices associated with constraints (A.1), (A.2) and (A.3), respectively, and \( \pi_{t+1} = \gamma_{\pi t+1}^{p t+1}p_{t+1}/p_t \).

Eq. (A.4) sets the marginal utility of consumption equal to the shadow goods price plus a factor reflecting the shopping-time cost. Eqs. (A.5) and (A.6) determine the shadow value of time and reflect the optimal condition that the marginal utility of leisure is equal to an after-tax wage rate (denominated in utility units). Eq. (A.7) relates the capital gains tax rate to the marginal portfolio adjustment cost. Eqs. (A.8) and (A.9) represent the marginal efficiency conditions for holding nominal assets (money and bonds), while (A.10) determines optimal capital accumulation. In the accrual-based version of the model, the ratio \( \frac{\lambda_{t+1}/\pi_{t+1}}{\lambda_t/\pi_t} \) is replaced by \( \frac{g_{t+1}}{u_{t+1}} \). The accrual-based version of the model also omits Eqs. (A.3), (A.7) and (A.11)—which determines the optimal accumulation of unrealized capital gains.

From the firm’s profit-maximization condition, the marginal product of labor is equal to the real wage
\[ w_t = (1 - \alpha)(y_t/N_t) \tag{A.12} \]
and the firm’s demand for capital determines that the real rental price will be equal to capital’s marginal product
\[ v_t = \alpha(y_t/k_t). \tag{A.13} \]

Eqs. (A.4) and (A.10), along with a transformed stationary representation of the capital accumulation equation
\[ \gamma_x k_{t+1} = (1 - \delta)k_t + i_t \tag{A.14} \]
imply household demand functions for consumption and real investment—and, hence, the future capital stock, \( k_{t+1} \). The presence of marginal shopping-time costs in the consumption-demand Eq. (A.4), defined by the shopping-time function
\[ S_t = \varepsilon \left( \frac{p_t c_t}{m_t} \right)^{\eta} \tag{A.15} \]
demonstrates one source of non-neutrality in the model. In addition, the presence of $\pi_t$ in Eq. (A.8) implies another source of interaction between the goods market and the nominal asset market.

Assuming equilibrium in the nominal asset markets, the condition for equilibrium in the goods market can be derived from the household’s budget constraint

$$y_t = c_t + i_t$$  \hspace{1cm} (A.16)

and the production function

$$y_t = z_t k_t^a N_t^{1-a}.$$  \hspace{1cm} (A.17)

Equilibrium in the goods market determines consumption, investment, and output—with the equilibrating price being the shadow value of capital, $\hat{\lambda}_{t+1}$; i.e., the after-tax real interest rate

$$(1 + \hat{r}_{t+1}) = \frac{\gamma_x \hat{\lambda}_t}{\beta \hat{\lambda}_{t+1}}.$$  \hspace{1cm} (A.18)

Steady-state relationships

Several key steady-state ratios are useful for deriving values for the remaining model parameters and for specifying the linear approximations used to calculate the model’s dynamics.

First, Eqs. (A.10) and (A.13) can be used to derive the steady-state capital/output ratio

$$\frac{k}{y} = \frac{\gamma_x (1 - \tau^K)}{\gamma_x - \beta [1 - (1 - \tau^K) \delta] + \beta \tau^G (\gamma_p - 1)/\gamma_p}.$$  \hspace{1cm} (A.19)

From (A.14) the share of output used for investment will be

$$\frac{i}{y} = \left[ \gamma_x - (1 - \delta) \right] \frac{k}{y}$$  \hspace{1cm} (A.20)

and from (A.16) the consumption share is

$$\frac{c}{y} = 1 - \frac{i}{y}.$$  \hspace{1cm} (A.21)

From (A.5) and (A.6), the marginal rate of substitution between consumption and leisure is related to the two shadow prices and the parameters of the shopping-time function. Substituting the values of the relative shadow prices from (A.8), we can derive the following relationship:

$$\frac{\theta}{1 - \theta} \left( \frac{L}{N} \right) = \frac{1}{(1 - \tau_N)(1 - z)} \left( \frac{c}{y} \right) + \eta \left( \frac{S}{N} \right).$$  \hspace{1cm} (A.22)

Given a calibrated allocation of time among labor, leisure, and shopping—along with a value of $\eta$ (selected to generate money demand elasticity) and the consumption/output ratio from (A.20)—Eq. (A.21) determines the value of the parameter $\theta$ to be used.
Combining Eqs. (A.6) and (A.8) yields

\[ 1 + (1 - \tau^N)(1 - z) \left( \frac{py}{m} \right) \eta \left( \frac{S}{N} \right) = \frac{\gamma \lambda \gamma_p}{\beta} \]  

(A.22)

which defines the steady-state ratio of nominal output to money (velocity). With this value in hand, we use the shopping-time definition (A.15), along with the consumption–output ratio above, to specify a value for the scale parameter, \( \xi \), consistent with the calibrated allocation of time for shopping.

The steady-state version of capital gains realization Eq. (A.3) is

\[ (\gamma \lambda \gamma_p - 1)u = (1 - 1/\gamma_p)k - g. \]  

(A.23)

Noting that the average of realized capital gains is approximately equal to 40% of total changes in the nominal value of the capital stock, Eq. (A.23) implies a value of approximately 0.01 for the \( G/U \)-ratio.

Eq. (A.11), which determines the optimal capital-gains realization, implies a steady-state relationship

\[ \left\{ 1 - \phi \left( \frac{g}{u} \right) + \frac{g}{u} \phi' \left( \frac{g}{u} \right) \right\} = \frac{\gamma \lambda \gamma_p}{\beta}. \]  

(A.24)

Given the normalizing assumption that the steady-state value of \( \phi(g/u) = g/u \), Eq. (A.24) defines the steady-state value of the first derivative of the adjustment-cost function, \( \phi'(g/u) \). The final value needed to calibrate the adjustment cost function, its second derivative, is calibrated using the dynamic experiment described in the text.

References


