Chapter 17

Dynamic Flow-of-Funds Networks

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When the notion of a credit network was first suggested in the literature [12], the general idea was to incorporate the process of multiple bank credit expansion—the credit multiplier—in the format of mathematical programming of networks. Thus contact was established between one of the oldest areas of economic research and one of the most modern techniques of mathematical programming. A first result, which became immediately apparent, suggested that it was now possible to abandon the assumption of fixed asset-preference ratios of traditional multiplier formulas and to replace them by optimising behaviour at the nodes of the financial intermediaries [14]. The multiplier itself became part and parcel of the optimal solution of the network.

Inherent in these original developments was an ambivalence between the decentralised decision-making of banks (and other financial intermediaries) and the global optimisation procedures of network theory. These aspects of the problem were brought into the open in a recent study [13], where it was shown that the network problem in hand is actually a case of decomposition theory. There is one 'division' (to make decentralised decisions) for each node of the network; in addition there are 'couplings' between the nodes as defined by the Kirchhoff conditions. The problem of decentralisation is to find prices of capital instruments—and any other information which may be required—which when delegated down to divisions would cause them to select individual portfolios which would coincide with the solution to the global problem.

Just as the ordinary textbook credit multiplier exists in two versions, one static and one dynamic, it should be possible to exploit the idea of a flows-of-funds network both in a static context and in a dynamic context.

In the common presentation the dynamic element in the dynamic cre multiplier is a behaviour lag of the private (non-bank) sector. The private sec borrows money from the banks in one period; a certain fraction thereof leaves system as 'leakage' (desired cash holdings with the private sector) and the remain is then redeposited with the banks. A dynamic sequence of such 'rounds' is set The total amount of lending is obtained as a geometric series. It converges if fraction of total leakage in each round is less than unity. In equilibrium, act reserves of banks are equal to desired reserves.

Similarly, assume that a lag of behaviour for the private sector is introducec the flow-of-funds network. Funds flow into this sector in one period; after a cert fraction thereof has left the system as cash leakage, the remainder is fed back i the intermediaries in an ensuing time period. Again, a dynamic sequence of 'roun is set up. A dynamic credit network or a dynamic flow-of-funds network is t defined.

As the funds are being propagated through the network each finan intermediary will adjust its portfolio in a decentralised setting. Will the timepath these decisions be convergent?

From a methodological point of view our work can be seen as an attempt evolve a concept of dynamic portfolio process analysis. The idea of monet process analysis has a long history in economic theory and is usually formalisec terms of difference equations and in *ex ante–ex post* language. In each single peri of analysis we shall decompose the flow-of-funds network into a hierarchical ch of successive portfolio optimisations[1].

17-1 Flow-of-Funds as a Network

The flow-of-funds table is a matrix arrangement showing a cross-tabulation of t sources of funds and the uses of funds in an economy. We shall here demonstrat variation of such a display featuring explicitly the financial intermediaries via wh the funds are channelled before they reach their ultimate destinations.

Consider the simple network shown in Figure 17:1. The sources of funds and t uses of funds are introduced in the usual fashion [3]. In the simple prototy economy of this example there are two sectors of ultimate units: households a the business sector. The sources of funds are the financial saving of households a the financial saving of the business sector. There are four forms of uses of funds the ultimate units: holding of cash, household loans, business loans and real est loans.

The novelty, if any, in the network arrangement in Figure 17:1 is the expli introduction of the financial intermediaries [7]. Three bodies of financ

[1] General dynamic portfolio process analysis must of course include the dynamic formation interest rates and of prices on financial market paper. This feature will not be demonstra here; interest rates are supposed fixed and given throughout.

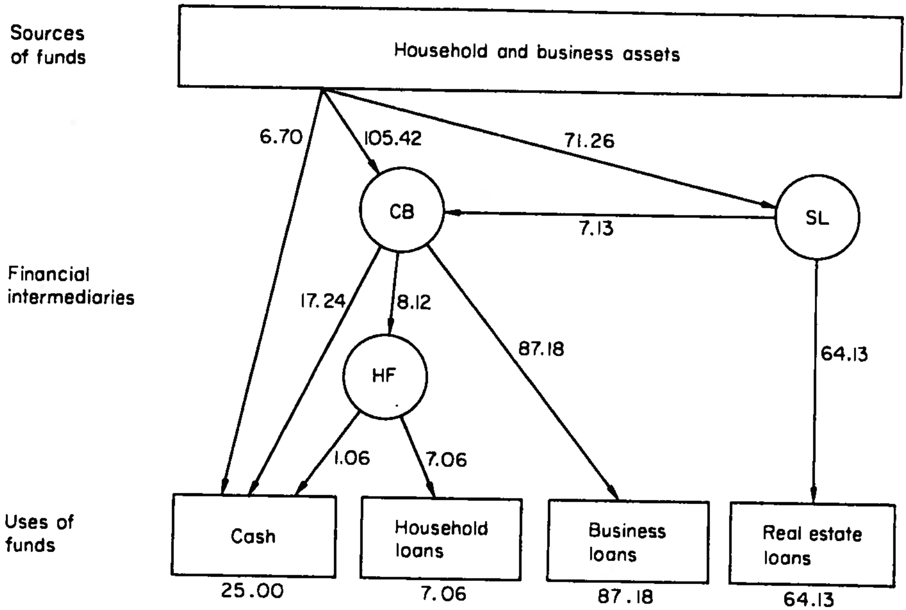

Figure 17:1 The network of flow of funds
The figures relate to the example in section 17-6

intermediaries are shown:

CB: commercial banks
SL: savings and loan associations, insurance companies
HF: household financing institutions

The intermediaries are depicted as *nodes* in the network; the *links* show the channelling of funds between the nodes. All links are *directed* (i.e. they show the direction of funds from the raising of funds to the placements of funds.) The *influx* into the network is identified as the sources of funds at the top of the figure; the flows enter the network, propagate via the intermediaries, and finally leave the network via the *efflux* channels (i.e. the uses of funds). Note that total influx = total efflux. The following links are shown in Figure 17:1:

1 *Placements of ultimate sectors*: cash, deposits with commercial banks, shares in savings and loan associations (ownership of accrued reserves in insurance companies)

2 *Placements of savings and loan associations*: demand deposits with commerc banks, real estate loans

3 *Placements of commercial banks*: reserves with the central bank, term loans household finance companies, business loans

4 *Placements of household finance companies*: cash, household loans

The network arrangement can be interpreted as a double-entry book-keep accounting system: each node represents a balance sheet and the links indicate t individual entries in the balance sheets. Each link is entered as a credit (on the as side) of the balance sheet of the originating node, and is entered as a debit (on t liabilities side) of the balance sheet of the destination node. The well-kno *Kirchhoff's law* must hold: the sum of all flows arriving into a node must equal t sum of all flows leaving the node. In other words: the balance sheet of flows at ea node must balance.

A well-known mathematical representation of all Kirchhoff conditions is sho in Figure 17:2. The totality of Kirchhoff conditions has the form of a system linear equations. For each node there is one Kirchhoff equation, the left side

Links (placements of . . .) / Nodes	. . . ultimate sectors			. . . savings and loan associations		. . . commercial banks			. . . household finance companies		
	Cash	Bank deposits	Shares in savings and loan associations	Bank deposits	Real estate loans	Reserves	Term loans to household finance	Business loans	Cash	Household loans	
Sources of funds	− 1	− 1	− 1								= − influx
Savings and loan associations			+ 1	− 1	− 1						= 0
Commercial banks		+ 1		+ 1		− 1	− 1	− 1			= 0
Household finance							+ 1		− 1	− 1	= 0
Uses of funds	+ 1				+ 1	+ 1		+ 1	+ 1	+ 1	= + efflu

Figure 17:2 Kirchhoff conditions—expressing book-keeping identities

which is presented in matrix form, with the variables (the links) on the top and the coefficients as entries in the matrix. The coefficient matrix can be recognised as the so called *incidence matrix*, which contains one row for each node and one column for each link in the network. The coefficients are entered as follows: for each link (each column) -1 is entered for the origin node of the link, and $+1$ is entered for the destination node of the link; all other entries in the column are 0 (suppressed in the figure). The right side of the system of equations is written in a somewhat analogous manner: all influxes into the network are entered with a minus sign, all effluxes with a plus sign; all other entries are 0.

17-2 Micro Behaviour

In disaggregating the flows of funds we desire to break down book-keeping entities into concepts that can be related to the planning and decision-making of behavioural units. In this section we outline a prototype set of possible assumptions of underlying micro theory:

Behaviour of intermediaries

For each intermediary a small portfolio model of linear programming type is assumed. Figure 17:3 shows some fundamental sample relations for three LP formulations, one for each intermediary,

The portfolio model for savings and loan associations (insurance companies) is shown on lines (1) to (4) and (11). Given a certain expected inflow of savings capital (line 11), the associations are assumed to face the problem of allocating their assets optimally between high-liquid low-yield demand deposits with banks and low-liquid high-yield real estate loans to households and business. The sample constraints include a budget relation (line 2), a condition on minimum liquidity (line 3), and a condition on maximum lending risk (line 4).

The portfolio model for commercial banks is shown on lines (1), (5) to (7), (12) and (13). Given a certain expected inflow of deposits into the banks (lines 12 and 13), the banks are assumed to face the problem of allocating their remaining assets optimally between reserves, term loans to household finance companies, and business loans. The sample constraints include a budget relation (line 5), a condition on minimum liquidity (line 6), and a condition on maximum lending risk (line 7). The reserves consist of required reserves and excess reserves. The bank is assumed to want to hold excess reserves as a protection against deposit risk.

Finally, the portfolio model for household finance companies is shown on lines (1), (8) to (10) and (14). Given a certain expected availability of term loans that can be raised from banks (line 14), the household finance companies face the problem of making an optimal decision on the amount of household loans that they want to grant. Their cash holding has the nature of a reserve; it provides a cash buffer which can be drawn upon to meet variability in the demand for household loans.

	Savings and loan associations (insurance companies)			Commercial banks					Household finance companies			
	Bank deposits	Real estate loans	Shares (accrued reserves)	Reserves	Term loans to HF	Business loans	Deposit of ultimate sectors	Deposits of SL	Cash	Household loans	Term borrowing	
1 Profit	+ 4	+ 8	− 4	+ 3	+5	+8	0	−4	0	+9	−5	
Divisional constraints												
2	+1	+1	−1									= 0
3	−1		+0.10									≤ 0
4		+1										≤ 75
5				+1	+1	+1	−1	−1				= 0
6				−1			+0.15	+0.20				≤ 0
7					−1.5	+1						≤ 75
8									+1	+1	−1	= 0
9									−1	+0.15		≤ 0
10										+1		≤ 100
11			+1									= expectations
12							+1					= expectations
13								+1				= expectations
14											+1	= expectations

Figure 17:3 Linear-programming formulations for three intermediaries

The minimum liquidity protection is specified (line 9), as is the maximum lenc risk (line 10).

Behaviour of ultimate units

The propensity of ultimate units to hold assets is assumed to be related to manner in which the available finance was raised. More specifically, each do which flows into the portfolio of ultimate units is assumed to be allocated unique and characteristic proportions. Constant coefficients of *feed-back* to network of flows of funds as renewed financial saving are assumed as shown Figure 17:4.

One additional dollar of finance raised by ultimate units in the form of . . .	. . . is placed as follows: As cash	As bank deposits	As savings in and loan association (insurance companies)
Household loans	0.09	0.40	0.51
Business loans	0.09	0.76	0.15
Real estate loans	0.09	0.25	0.66
Private sector net worth (Government transfer *)	0.05	0.50	0.45

Figure 17:4 Placement feed-back ratios of ultimate units
* Note that in the present model private net worth = public debt = quantity of money

17-3 Dynamic Equilibrium

The prototype model outlined in the preceding section was constructed so that each variable (link in the network) is controlled by one and only one economic subject (one node). To the controlling unit this variable is a *control variable*. Seen from the point of view of any other unit it is a *parameter of expectations*.

Figure 17:5 illustrates by way of an arrow scheme (as in Tinbergen [15]) the logical structure of the model. Consider the two first columns of dots, denoting the values attached to links in period *t*, *ex ante* and *ex post* respectively. There are three systems of arrows in period *t*:

1 *The decision-making of savings and loan associations.* The control variables under the control of savings and loan associations are determined *ex post* on the basis of the relevant expectations (arrows from line (3) *ex ante* to lines (4) and (5) *ex post*).

2 *The decision-making of commercial banks.* The control variables are determined

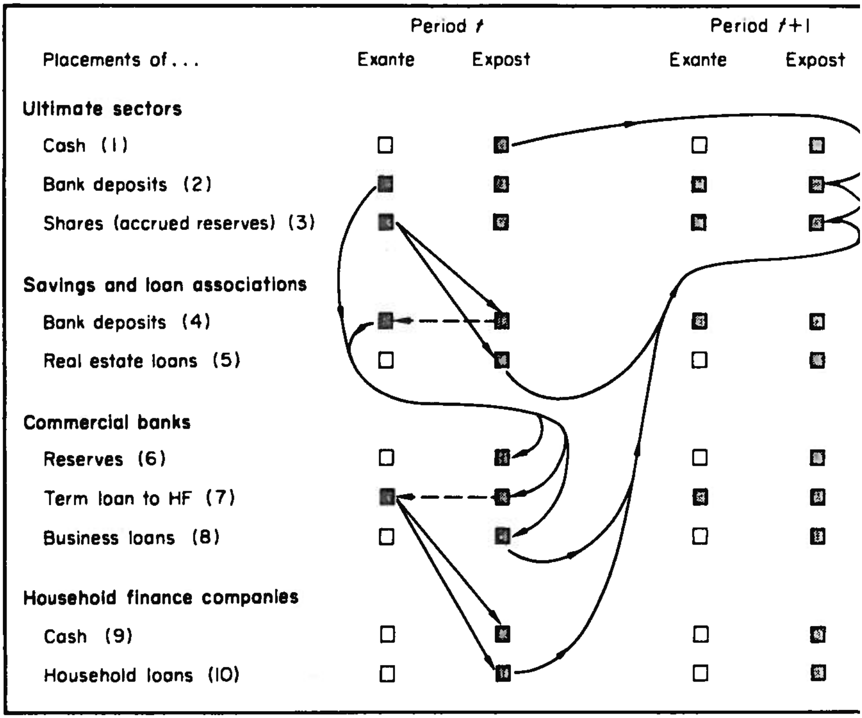

Figure 17:5 Arrow scheme illustrating logical structure of dynamic equilibrium

ex post on the basis of relevant expectations (arrows from lines (2), (4) *ex ant* lines (6), (7) and (8) *ex post*).

3 *The decision-making of household finance* companies (arrows from line (7 *ante* to lines (9) and (10) *ex post*).

Note how the joint specification of all relevant expectations (filled *ex ante* d uniquely determines all *ex post* values.

Next consider the dynamic features of the model, i.e. the relationships betw variables in periods t and $(t + 1)$. The postulated behaviour of the ultin sectors provides the dynamic feature of the model. In effect, the feed-back ratio Figure 17:4 create a dynamic dependence of variables in one period on the va attached to variables in the preceding period. Loans extended to the ultim sectors in one period will feed back to the network as new placements of t ultimate sectors in the subsequent period. The decision-making of the ultim sectors in one period will depend upon the lending *ex post* that was extended them during the past period (arrows from lines (1), (5), (8) and (10) in period t lines (2) and (3) in period $t + 1$).

It only remains to discuss the dynamic formation of expectations. A special ca of dynamic models which has been studied extensively in the literature *equilibrium*: all behaviour units make correct forecasts so that *ex ante* will alwa coincide exactly with *ex post*.[2] This assumption introduces a recursive feature in the model. A given variable may be controlled by one subject; the expectatio which other subjects may have formed concerning the outcome of this variable w then be determined (see stippled arrows orientated in reverse). The decision-makir problem of these other subjects will then be well-defined and decisions on the control variables can then be taken, and so on.

Turning to the mathematical representation of our model, the equations (11) t (14) in Figure 17:3 which specify the determination of expectations must now b replaced by corresponding equations in Figure 17:6, i.e. equations (11*a*) to (14 respectively. Note that equations (13*a*) and (14*a*) involve variables from more tha one 'division' at the same time. Thus these equations have the character of so calle 'coupling constraints'.

We have now arrived at a set of interrelated mathematical programmin problems. We have formulated three distinct LP problems, one for each division but they are coupled together via certain coupling conditions, which the solution to the three problems must simultaneously fulfil.

The concept of *order* is important. We have assumed a certain *recursive causa ordering* of the problem. In each period the following hierarchical ranking i introduced:

first: savings and loan associations
second: commercial banks
third: household finance companies

The divisions are assumed to optimise their behaviour in each period in this order and on the basis of full information about all earlier decision taken by division in this causal chain. Thus, first the SL associations will act and they will make their decisions for the period without paying notice to the coupling conditions at all. In this manner *inter alia* their bank deposits in Figure 17:5 line (4) will be determined. The stage is then set for the commercial banks to go. To them the meaning of the coupling equation (13*a*) is now settled. Otherwise they will optimize their behaviour without regard to any other coupling constraints. The burden of obeying equation (14*a*) is shifted down along the hierarchy of causal ordering and over to

[2] Other terms that have been used to describe such a situation is moving equilibrium, dynamic equilibrium, perfect foresight, etc. See Hayek [8].

	Savings and loan associations (insurance companies)			Commercial banks					Household finance companies			
	Bank deposits	Real estate loans	Shares (accrued reserves)	Reserves	Term loans to HF	Business loans	Deposits of ultimate sectors	Deposits of SL	Cash	Household loans	Term borrowing	
(11a)			+1									= } Determined by events in last period, see Fig 4
(12a)							+1					= }
(13a)	−1							+1				= 0
(14a)					−1						+1	= 0

Figure 17:6 Determination of expectations in equilibrium network

the household finance companies. In this manner term loans to the househ finance companies in Figure 17:5 line (7) are determined. Finally, it is time for household finance companies to have their say. The coupling condition (14*a* now defined and they have to optimise their portfolio under the condition c given availability of term loans.

The assumptions of causal order [11] have the nature of *extraneous* or *ad* information which has to be added to the system of mathematical programm formats in equations (1) to (10) and (11*a*) to (14*a*), in order to make e divisional problem well defined.

17-4 Imbedding and Decomposition

We shall now show how it is possible to imbed the three divisional problems insi larger master problem, and that the optimal solution to the master problem r coincide with the optimal solutions to the three divisional problems.

This situation is somewhat analogous to the one encountered in decomposit theory in mathematical programming [4, 5], except that we here start from divisional problems and then construct the master problem as an artifice solution.

In order to bring out the principles of imbedding, the problem is now presente with a more general notation. Let the master problem be

$$\left.\begin{array}{llllll} \text{Max} & c_1x_1 & +c_2x_2 & +c_3x_3 & & \\ \text{subject to} & B_1x_1 & & & \leqslant b_1 \\ & & B_2x_2 & & \leqslant b_2 \\ & & & B_3x_3 & \leqslant b_3 \\ & C_{11}x_1 & +C_{12}x_2 & & = d_1 \\ & C_{21}x_1 & +C_{22}x_2 & +C_{23}x_3 & = d_2 \end{array}\right\} \text{(M)}$$

where x_1, x_2, x_3 are the decision variables for the three divisional problems, $(x_1,x_2,x_3 \geqslant 0)$

In the terminology of decomposition theory the master problem or the globa problem (M) encompasses three divisions $i = 1, 2, 3$, and consists of three divisiona constraints $B_ix_i \leqslant b_i$ as well as coupling constraints (in the two last lines). The ide: of decentralisation is connected with the splitting-up of problem (M) into three separate divisional problems, containing the divisional constraint . The *problem o coherent decentralisation* [2] is to determine such information which, wher delegated to divisions, will cause them to arrive at solutions which togethei constitute an optimum solution to the global problem.

In our present application the situation is precisely the reverse. Three divisional problems are given, for savings and loan associations, for commercial banks and for household finance companies (Figure 17-3, lines 1-10). In addition, a set of couplings must be adhered to (Figure 17-6). By *coherent imbedding* one implies a procedure which allows the formulation of a corresponding master problem (M), which can be decomposed by coherent decentralisation into the original divisional problems.

If such an imbedding procedure can be found, the method of solution of the given set of divisional problems is as follows: (i) perform the imbedding, (ii) solve the master problem, (iii) the divisional parts of the master solution constitute optima to the divisional problems.

In ordinary decomposition theory the prices c_i of the master problem are taken as given and fixed; during the process of decomposition there may be a need to 'correct' these prices, so that when delegated to divisions they ensure coherent decentralisation.

In the present case we start from a micro situation and proceed to form a corresponding master problem. Consider the prices entered on line (1) in Figure 17:3. In any empirical application the prices to be used here are of course quoted market prices. No 'correction' has yet been undertaken. No guarantee can be given that coherent decentralisation can be ensured. We realise that in fact these prices do not belong to the divisional problems but to a corresponding master problem.

The three formulated micro problems must therefore be *corrected*. The prices must be corrected so that coherent action hopefully is ensured.

The procedure of delegation of information from the master problem divisional problems differs somewhat from the standard pattern in the literatur addition to delegating corrected prices we impose a hierarchial order. ' decomposition procedure we propose to call 'hierarchical decomposition.'

In our model the following causal train of delegation of information wil followed in each period:

1 To the first division: delegate prices corrected for any couplings with second or the third division.
2 To the second division: delegate information about the first division's opti choice, and delegate prices corrected for any additional couplings with the t division.
3 To the third division: delegate information about the optimal choices of t the first and the second division.

We may write the dual to the master problem (M) as follows:

$$\left.\begin{array}{llllll} \text{Min} & u_1 b_1 + u_2 b_2 + u_3 b_3 & + \nu_1 d_1 & + \nu_2 d_2 & & \\ \text{subject to } u_1 B_1 & & + \nu_1 C_{11} & + \nu_2 C_{21} & \geqslant c_1 & \\ & u_2 B_2 & + \nu_1 C_{12} & + \nu_2 C_{22} & \geqslant c_2 & \\ & u_3 B_3 & & + \nu_2 C_{23} & \geqslant c_3 & \end{array}\right\} \quad (D)$$

$$u_1, u_2, u_3 \geqslant 0$$

If we denote the optimal solution to (D) as $(\bar{u}_1, \bar{u}_2, \bar{u}_3, \bar{\nu}_1, \bar{\nu}_2)$, the divisional prob for division 1 (to be solved *first*) becomes:

$$\left.\begin{array}{l} \text{Max } (c_1 - \bar{\nu}_1 C_{11} - \bar{\nu}_2 C_{21}) x_1 \\ \text{subject to } B_1 x_1 \leqslant b_1 \\ \qquad x_1 \geqslant 0 \end{array}\right\} \quad (P1)$$

For division 2 (to be solved *after* the solution x^*_1 to the first division has b obtained), the problem becomes

$$\left.\begin{array}{l} \text{Max } (c_2 - \bar{\nu}_2 C_{22}) x_2 \\ \text{subject to } B_2 x_2 \leqslant b_2 \\ \qquad C_{12} x_2 = d_1 - C_{11} x^*_1 \\ \qquad x_2 \geqslant 0 \end{array}\right\} \quad (P2)$$

The problem (P3) for division 3 may be formulated similarly.

Our main result, the proof of which may be found in our 1970 TIMS confere paper, is then as follows. Denote the solution to the master problem $(\bar{x}_1, \bar{x}_2,$

Then assuming that $\bar{x}_i$ is feasible for the divisional problem (Pi) for all i, it w
follow that $\bar{x}_i$ are optimal solutions for the divisions.

Conversely, solve the divisional problems (Pi) and calculate the correspondi
divisional duals (u_1*), (u_2*, v_1*), (u_3*, v_2*). Then if these duals togeth
constitute a feasible solution to the master dual (D), it must follow that it is also
optimal solution to it, and the solutions to the direct divisional problems constitu
an optimal solution to the direct master problem (M).

17-5 Conditions for Convergence

So far we have studied a single time period. We are now ready to turn to the centr problem of the paper, namely: what happens to the system as we move from peric to period. This problem is already touched upon in the Tinbergen arrow diagra (Figure 17:5) and in the feedback matrix in Figure 17:4, where changes in the us of funds lead to changes in the assets of the ultimate sectors. It seems reasonable t assume that the changes take place with one period lag.

It can be argued that we have a mathematical programming problem for eac period (i.e. a sequence of problems over the periods which have to be solved) an the question is whether the sequence converges towards an equilibrium.

Without going into mathematical detail, it should be clear that as long as there i no change of basis in the master problem (M) from one period to the next, th master solution can be obtained from a linear system of difference equations. Th conditions for such a system to converge towards a stable solution can be easil expressed in the characteristic roots of the coefficient matrix [1, 6, 9].

If the master problem (M) changes basis in some period, a sufficient conditio for convergence is that both the old and the new systems of difference equations are stable.

Whereas the convergence of the simple textbook credit multiplier typically requires that the sum of all coefficients of 'leakage' (increased cash holdings by households and firms, and increased reserves by the intermediaries) be less than one[3], we have here a much more complicated criterion, which does not immediately allow such a straightforward interpretation.

17-6 Numerical Example

The example is set up as follows:

1 Define first an initial situation of equilibrium. We have chosen the example displayed in Figure 17:1. The example was constructed by solving the programming model in Figure 17:3. For the expected placements of the ultimate sectors we put

[3] See e.g. Sheppard and Barrett [10].

shares = 71.26 and deposits = 105.42. For the two remaining variables of e ations we used equations (13*a*) and (14*a*). The initial figures just mentioned chosen so that the total initial money supply in the model would equal 25. that in this initial situation of equilibrium desired cash holdings equal actua holdings everywhere.

2 Introduce an injection of base money of amount 10 into the systen instance in the form of the government issuing short-term debt placed in the c bank. We shall now observe how this 'pulse' is propagated through the system.

3 The injection reaches the private sector, and 5% 'leaks out' as increased holdings. 45% reaches the SL associations and 50% the commercial ban increased deposits. Total deposits with these institutions are thus stepped up tc

71.26 + 4.5 = 75.76

and

105.42 + 5 = 110.42

respectively.

4 This increased flow of funds into the financial institutions adds in the instance to their reserves. There will thus arise a discrepancy between their a cash-holding and the desired level. This discrepancy is the actual drive of dynamic network. In order to find the adjustments of the financial institutio the first round, the right-hand sides of equations (11) and (12) are set equal tc deposit levels computed above. The new solution is shown as period 1 in Fi 17:8.

5 Consider Figure 17:4. The loan increases affect deposits one period (shown in Figure 17:7 under 'period 1,' lines (5) and (6)). The deposits ir associations and commercial banks are stepped up to:

75.76 + 3.91 = 79.66
110.42 + 3.76 = 114.17

respectively. We then change the right-hand sides of equations (11) and (12) to amounts above and solve. The new increases in the loans are shown in Figure 1 under 'period 2,' lines (1)–(3).

The dynamic pattern thus continues. In Figure 17:8 we show the solutions twenty periods and demonstrate how they converge.[4] In the new equilibrium ac cash again equals desired cash. The economic subjects of the network now desir hold thirty-five units of cash.

The numerical example is given in terms of the master problem format. We however correct the prices of the divisional problems and solve the problems in hierarchical order described earlier in the paper. The divisional solutions will tl

[4] In period 4, real estate loans hit their upper limit and the master problem changes basis. solution to the master problem from then on enters a path of convergent growth. We h checked that the mathematical requirements for convergence are satisfied.

converge towards the same solution as in Figure 17:8. The corrected prices a found by using the dual variables for equations (13*a*) and (14*a*) in Figure 17: These are 0.0294 and 0.0283, respectively. We therefore correct the followin prices

Bank deposits	$0.04 - (-1) \times 0.0294 = 0.0694$
Term loans to HF	$0.05 - (-1) \times 0.0283 = 0.0783$

The remaining prices are unchanged.

17-7 Conclusion

A flow-of-funds table is a cross tabulation of sources of funds and uses of funds o the financial sector of the economy. A flow-of-funds network is a pictographica extension of this concept, showing explicitly the network of financial portfolio participating in the intermediation of funds, connecting ultimate suppliers an ultimate users of funds.

The flow-of-funds network offers a convenient format for the study of th propagation of credit expansion processes in commercial banks, savings banks an other financial intermediaries. In effect, such expansion processes develop as th result of interaction between ultimate suppliers and users of funds (shown as th sources and the sinks of the network) and the financial intermediaries (appearing a the nodes of the network). Just as the well-known textbook credit multiplier exists in the two versions, one static and one dynamic, similarly the more general flow-of-funds network processes can be thought of either in a static or a dynamic context. The dynamic version would arise when there are lags of behaviour either in the ultimate sectors, or the intermediaries, or both.

The present paper outlines a programming format, which can be employed to generate very general types of such dynamic processes of propagation of funds. In each period the mathematical program takes the form of a series of interconnected individual portfolio problems, one for each intermediary. It was shown that they could be imbedded in a corresponding master program of decomposition type, and a new decomposition procedure (called hierarchical decomposition) was introduced to obtain coherency between the master optimum and the individual (divisional) optima. Between periods a behaviour lag between funds raised and funds placed for ultimate sectors was introduced.

Under general conditions the dynamic process set up may converge. As long as the master problem in any one given application does not change basis, the process will behave as a system of linear difference equations. But if the master problem changes basis during the course of the credit expansion the path of the solution will move on to follow another set of such linear difference equations.

The dynamic flow-of-funds network can be viewed as an instance of dynamic portfolio process analysis. It provides a format for the study of causal sequences of the formation of expectations, of successive portfolio optimisations, and of the

	Period 0	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8
1 Increase in household loans	–	1.60	1.22	0.97	1.35	1.07	0.77	0.55	0.40
2 Increase in business loans	–	2.77	2.10	1.67	2.33	1.84	1.32	0.95	0.69
3 Increase in real estate loans	–	4.05	3.52	2.93	0.37	0	0	0	0
4 Increase in net worth	10	0	0	0	0	0	0	0	0
5 Increase in bank deposits 0.40 x (1) + 0.76 x (2) + 0.25 x (3) + 0.50 x (4)	5	3.76	2.97	2.39	2.40	1.82	1.31	0.95	0.68
6 Increase in S & L shares 0.51 x (1) + 0.15 x (2) + 0.66 x (3) + 0.45 x (4)	4.5	3.91	3.26	2.68	1.28	0.82	0.59	0.42	0.31

Figure 17:7 **Propagation in successive time periods of an initial injection of cash into flow-of-funds network**

	Initial solution	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8		Period 20		Final solution
S & L association:													
Bank deposits	7.13	7.58	7.97	8.29	10.60	11.88	12.70	13.29	14.02	...	14.78	...	14.80
Real estate loans	64.13	68.18	71.70	74.63	75.00	75.00	75.00	75.00	75.00	...	75.00	...	75.00
Shares	71.26	75.76	79.66	82.92	85.60	86.88	87.70	88.29	88.71	...	89.79	...	89.80
Commercial banks:													
Reserves	17.24	18.08	18.72	19.23	20.05	20.67	21.10	21.42	21.64	...	22.22	...	22.23
Term loans to HF	8.12	9.97	11.37	12.48	14.03	15.26	16.14	16.77	17.23	...	18.38	...	18.41
Business loans	87.12	89.95	92.05	93.72	96.05	97.89	99.21	100 16	100.85	...	102.57	...	102.61
Deposits of ultimate sectors	105.42	110.42	114.17	117.14	119.53	121.93	123.75	125.06	126.01	...	128.39	...	128.43
Deposits of S & L	7.13	7.58	7.97	8.29	10.60	11.88	12.70	13.29	13.71	...	14.78	...	14.80
Household finance:													
Cash	1.06	1.30	1.48	1.63	1.83	1.99	2.11	2.19	2.25	...	2.40	...	2.40
Household loans	7.06	8.67	9.89	10.85	12.20	13.27	14.03	14.59	14.98	...	15.99	...	16.01
Term borrowing	8.12	9.97	11.37	12.48	14.03	15.26	16.14	16.78	17.23	...	18.38	...	18.41

Figure 17:8 Dynamic solution of flow-of-funds network

propagation of monetary pulses from one portfolio to another. There seems t scope for considerable further developments in this direction. If lags and dela portfolio responses are indeed essential features of the working of monetary financial mechanisms, such extensions should add to our knowledge of economic world.

17-6 References

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