

## DYNAMIC OPTIMAL TAXATION, RATIONAL EXPECTATIONS AND OPTIMAL CONTROL

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Within a rational expectations framework, policy has effect if it alters relative prices and policy evaluations are exercises in modern public finance theory. The time inconsistency of an optimal taxation plan precludes the use of standard control theory for its determination. In this article recursive methods are developed that overcome this difficulty. The technique is novel in that the constraint set as well as the value function are determined recursively. Even though there is little hope of the optimal plan being implemented – because of its time inconsistency – we think the exercise is of more than pedagogical interest. The optimal plan's return is a benchmark with which to compare the time consistent solution under alternative institutional constraints which society might choose to impose upon itself.

### 1. Introduction

The purpose of this article is to examine the problem of optimal policy selection within the rational expectations competitive framework. The natural context within which to discuss optimal policy selections is optimal taxation.<sup>1</sup> This allows us to be fairly explicit as to how the policymaker's objective is related to those of the economic agents. We assume that there is a representative consumer, and that there is no conflict between public and private objectives. The individual values public goods, but prefers not to pay taxes. The role of the policymaker is to provide public goods in such a way as to maximize the utility of the representative consumer subject to the constraint that goods are financed by proportional taxes upon labor and capital incomes.<sup>2</sup>

We first present a standard static taxation model, for which optimization is fairly straightforward. We then consider a multiple period problem with

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<sup>1</sup>For a review of the optimal taxation literature, see Sandmo (1976).

<sup>2</sup>Justifications for ruling out non-distortionary taxes of the lump sum variety are based upon heterogeneity of consumers, which is not an element of our models. Heterogeneity was not introduced because, even in static models, it greatly complicates the analysis [cf. Romer (1975)] and would obscure the points we wished to make.

capital and show that standard control-theory techniques are not applicable to the optimal taxation programming problem.<sup>3</sup> Bellman's (1957) principle of optimality fails and an optimal policy will in general be time inconsistent. That is, the optimal policy plan at time  $t'$  greater than  $t$  is not the continuation of the plan that was optimal at time  $t$ .<sup>4</sup> Not only does this severely complicate the determination of the optimal policy by precluding the applications of optimal control theory, but more importantly, the time inconsistency makes it doubtful whether such a policy would ever be implemented. Rather we would expect the time consistent solutions to be adopted, that is, the rule resulting if the policy action taken at each point in time is best taking into consideration both the current period outcome and a *correct* evaluation of the end of period position. Unlike games against nature, for which control theory was developed, the consistent solution is in general not optimal and as shown in Kydland and Prescott (1977) may be very suboptimal.

In spite of the time inconsistency problem, we do not think the determination of the optimal policy and the resulting return is without interest. The optimal return is a benchmark with which to compare the return of the time consistent policy under a particular set of institutional constraints. Possibly a constitutional amendment requiring the budget to be balanced in peacetime or an institutional arrangement which results in the process of policy change being long and protracted might result in the time consistent solution being nearly optimal. If so, it would provide strong justification for society establishing such arrangements.

## 2. A simple static optimal taxation model

Consider an economy with a large number of small economic agents. Assume that the preferences of the representative household can be represented by a utility function  $u(c, n, g)$ , where  $c$  is consumption,  $n$  is labor supply, and  $g$  is public expenditures. With a representative household, the social welfare function is well defined, namely to maximize the same utility function. The decision variables of the household are  $c$  and  $n$ , and, assuming a linear production function  $y = \omega n$ , the household is constrained by

$$0 \leq c \leq (1 - \tau)\omega n \quad \text{and} \quad 0 \leq n \leq \bar{n},$$

<sup>3</sup>An excellent overview of control theory with emphasis on its applicability to economics can be found in Chow (1975). Holly and Zarrop (1979), using non-recursive methods, have developed and applied an algorithm for computing optimal solutions for finite horizon, deterministic problems.

<sup>4</sup>The point was established in Kydland and Prescott (1977) and discussed at greater length in Prescott (1977). Calvo (1978) has demonstrated the time inconsistency of an optimal monetary policy. As inflation can be viewed as a tax on liquidity, his results can be viewed as being within the optimal taxation framework and therefore as being complementary to this analysis.

where  $\tau$  is the income tax rate and  $\bar{n}$  is the maximum amount of labor services that can be supplied. Note that  $\omega$  is the marginal product of labor and therefore in equilibrium the wage.

Assume now that  $\tau$  and  $g$  are set by a policymaker with the objective of maximizing the utility of the representative consumer, i.e.,

$$\max u(c, n, g), \quad (1)$$

where the aggregate variables are appropriately measured in per capita terms. One constraint is that there be sufficient revenue to finance government purchases; that is,

$$\tau\omega n \geq g. \quad (2)$$

The tax rate is constrained by  $0 \leq \tau \leq \bar{\tau} < 1$ .

The second set of constraints come from the maximization problem of the stand-in consumer. In order to insure a unique interior solution, we make the following fairly standard assumptions: The utility function is strictly increasing and strictly concave in consumption  $c$  and the negative of labor supplied  $-n$  and is continuously differentiable. In addition

$$(1 - \bar{\tau})\omega u_c(0, 0, g) > -u_n(0, 0, g),$$

$$(1 - \tau)\omega u_c((1 - \tau)\omega\bar{n}, \bar{n}, g) < -u_n((1 - \tau)\omega\bar{n}, \bar{n}, g),$$

for all  $0 \leq \tau \leq \bar{\tau}$  and  $0 \leq g \leq \bar{n}\omega$ , where  $u_c$  and  $u_n$  are the partial derivatives with respect to  $c$  and  $n$ , respectively. The consumer maximizes his utility over his decision variables  $c$  and  $n$  subject to his budget constraint which will be binding. Thus, his problem is

$$\max_{c, n} u(c, n, g),$$

subject to

$$c = (1 - \tau)\omega n.$$

With the above assumptions, the consumer is in equilibrium if and only if

$$u_n(c, n, g) = -(1 - \tau)\omega u_c(c, n, g). \quad (3)$$

Thus, the policy problem is to maximize (1) subject to constraints (2) and (3), the inequality constraints

$$\tau \leq \bar{\tau}, \quad n \leq \bar{n}, \quad c + g \leq \omega n,$$

and the non-negativity constraints. This is a well-defined programming problem. The compactness of the constraint set and continuity of the objective function guarantee the existence of a solution.

### 3. The dynamic certainty case

We now introduce capital which makes the problem dynamic. We shall consider the simplest type of model in which consumers own capital which they rent to firms. The firms use capital and labor as inputs to produce output which can either be consumed in the same period, be used as a public good, or be used to augment the capital stock. The decision variables of the consumer in period  $t$  are consumption, labor supply, and how much capital to carry over to the next period. Finally, public good expenditures are financed by taxes on labor and capital income.

For this problem we can define an equilibrium where the prices clear the markets for given present and future values of the government policy variables. Using the price of output as a numeraire, these equilibrium prices would be  $\{w_t^*\}_{t=0}^\infty$  and  $\{r_t^*\}_{t=0}^\infty$ , where  $w_t$  is the wage rate and  $r_t$  the rental rate of capital in period  $t$ . In what follows, we shall omit the stars and assume that we are referring to the equilibrium prices in stating the constraints imposed by the rational expectations maximizing assumptions.

We assume that the utility function of the representative consumer is time separable with discounting:

*Consumer's Problem:*

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t, n_t, g_t),$$

subject to

$$k_{t+1} + c_t \leq k_t + (1 - \theta_t)r_t k_t + (1 - \tau_t)w_t n_t,$$

$$c_t, n_t, k_{t+1} \geq 0, \quad n_t \leq \bar{n}, \quad t=0, 1, \dots, \quad k_0 \text{ given.}$$

Here  $0 \leq \theta_t \leq \bar{\theta} < 1$  and  $0 \leq \tau_t \leq \bar{\tau} < 1$  are the tax rates applicable to capital and labor income, respectively.

Factor supply and product demand are determined from the following first-order conditions for  $t=0, 1, \dots$ :<sup>5</sup>

$$u_n(c_t, n_t, g_t) = -(1 - \tau_t)w_t u_c(c_t, n_t, g_t), \quad (4)$$

<sup>5</sup>Assumptions similar to those of the previous section concerning the function  $u(c, n, g)$  are easily developed that guarantee an interior solution for consumption and labor supply with uniformly bounded marginal utility for all feasible tax policies.

$$u_c(c_{t+1}, n_{t+1}, g_{t+1}) = \{1/\beta[1 + (1 - \theta_{t+1})r_{t+1}]\} u_c(c_t, n_t, g_t). \quad (5)$$

Since the consumer owns the capital, the firm's problem is static:

*Firm's Problem:*

$$\max[f(k_t, n_t) - r_t k_t - w_t n_t], \quad t = 0, 1, \dots,$$

where  $f(k, n)$  is a constant-returns-to-scale production function. In equilibrium, profits are zero and therefore they need not be included in the budget constraint of the consumer. The profit-maximizing conditions are simply

$$r_t = f_k(k_t, n_t), \quad (6)$$

and

$$w_t = f_n(k_t, n_t). \quad (7)$$

We assume that  $f(0, 1) = \omega > 0$  and that  $f(k, n)$  is strictly concave in  $k$ , strictly increasing in both its arguments, and positive. These assumptions guarantee that in equilibrium (6) and (7) must be satisfied. We also assume there is a  $\bar{k}$  such that  $\bar{k} = f(\bar{k}, \bar{n})$  and that  $k_0 \leq \bar{k}$ . Element  $\bar{k}$  is the maximal sustainable capital stock.

The *optimal taxation problem* is then to choose  $\{\pi_t = (g_t, \theta_t, \tau_t)\}_{t=0}^{\infty}$  so as to maximize

$$\sum_{t=0}^{\infty} \beta^t u(c_t, n_t, g_t),$$

subject to

$$g_t \leq \theta_t r_t k_t + \tau_t w_t n_t, \quad t = 0, 1, \dots,$$

and constraints (4)–(7), which are imposed by the rational expectations equilibrium assumption.

The labor supply-consumption decisions,  $n_t$  and  $c_t$ , of the consumer depend not only upon the current state of the economy,  $k_t$ , and current tax rates,  $(\theta_t, \tau_t)$ , but also upon future tax rates. *Until the sequences of future tax rates are specified the equilibrium current decisions of the consumers cannot be determined.*

Suppose the optimal policy sequence  $\{\pi_t^0 = (g_t^0, \theta_t^0, \tau_t^0)\}_{t=0}^{\infty}$  exists and is unique. This optimal policy will be time inconsistent in the sense that at time  $s > 0$  the policy  $\{\pi_t^0\}_{t=s}^{\infty}$  will not be optimal at time  $s$ . The reason it is not optimal is because current equilibrium decisions of the consumer are

functions of the current state, current policy decisions, and anticipated *future* policy actions.

Before proceeding further, a definition of *state variable* is needed.

### 3.1. *Definition of state variable*

A decision problem is Markovian if after any number of periods, say  $t$ , the effect of decisions of the current and subsequent periods upon the total return depends only upon the state of the system at the beginning of the  $t$ th period and subsequent decisions. A preference-information-technology-wealth distribution structure is Markovian if each agent's decision problem is Markovian in the assumed state variable vector, given that other agents' decision problems are Markovian in that state vector.

The state variable must, among other things, reflect the effect of past decisions upon the subsequent production possibility set. This is typically accomplished by including capital stocks and inventory stock variables as components of the state vector. When preferences are time separable, no state variables are needed to index the effect of past decisions upon the individual's ordering of subsequent consumption paths (or distributions of subsequent consumption paths if there is uncertainty). For some analyses, the current distribution of money, bonds, and stocks must be specified by the state vector. Knowing the current state is sufficient for determining the current and subsequent competitive allocations and equilibrium prices (or process governing them if there is uncertainty). With this definition, *prices and current-period decision variables are not state variables*. Current equilibrium prices and decisions are functions of the state variables.

The naive application of optimal control is likely to lead to a consistent solution for reasons given in Kydland and Prescott (1977). The solution is consistent in the sense that it is best, given the current state variables and that policies will be similarly selected in the future. This consistent solution will be suboptimal, however, because the effects of the policy for any future period  $t$  on agents' behavior in earlier periods are not taken into account. For the optimal taxation example the consistent solution for the current period is to first tax away all the capital income because the capital is already there and does not enter directly into the utility function, or, in other words, it is supplied inelastically. Anticipating this, agents will save little and the capital stock will be small.<sup>6</sup>

A superior alternative would be to restrict policymakers to use a well-understood policy rule with good operating characteristics. Even if the best one among such rules could be determined, however, the time inconsistency

<sup>6</sup>In his paper on optimal taxation, Ramsey (1927, p. 59) briefly considers the dynamic problem, but because it is 'considerably more difficult' essentially assumes the dynamics away.

of this policy makes it a doubtful question whether policymakers would continue to use it in future periods. If not, expectations of economic agents would clearly be affected, leading to a change in their behavior. It is still of interest, however, to study optimal policy over time, even though it is time inconsistent. For example, when a new tax system is introduced, this is often a relevant restriction; more generally, one might imagine large costs associated with changes in policy.

### 3.2. Determining optimal policy

The time inconsistency severely complicates the computation of the optimal policy. Standard recursive methods are no longer applicable. In what follows we outline a possible computational procedure, and point out the difficulties involved.

To obtain restrictions imposed by the rational expectations equilibrium assumption, we formulate the Lagrangean for the consumer

$$L = \sum_{t=0}^{\infty} \beta^t \{ u(c_t, n_t, g_t) + \lambda_t [(1 - \tau_t)w_t n_t + (1 - \theta_t)r_t k_t + k_t - k_{t+1} - c_t] \}.$$

The first-order conditions are<sup>7</sup>

$$u_c(c_t, n_t, g_t) = \lambda_t, \tag{8}$$

$$u_n(c_t, n_t, g_t) + \lambda_t(1 - \tau_t)w_t = 0, \tag{9}$$

$$\beta[1 + (1 - \theta_{t+1})r_{t+1}]\lambda_{t+1} - \lambda_t = 0, \quad t = 0, 1, \dots \tag{10}$$

In addition, we have the profit-maximizing conditions (6) and (7).

Using (6)–(9) along with the budget constraint of the consumer, we can write

$$x_t = d(k_t, \pi_t, \lambda_t), \quad t = 0, 1, \dots, \tag{11}$$

where  $x_t = (c_t, n_t, k_{t+1})$  are the decision variables of the consumer. Constraint (10), using (6), becomes

$$\beta[1 + (1 - \theta_{t+1})f_k(k_{t+1}, n_{t+1})]\lambda_{t+1} - \lambda_t = 0. \tag{12}$$

But, from (11),  $n_{t+1}$  is a function of  $k_{t+1}$ ,  $\pi_{t+1}$  and  $\lambda_{t+1}$ , and (12) can

<sup>7</sup>These equations are equivalent to eqs. (4) and (5), and, in fact, represent a derivation of (4) and (5).

therefore be written as

$$\lambda_t = h(\pi_t, k_t, \lambda_{t-1}), \quad t = 1, 2, \dots, \quad (13)$$

This constraint is unusual in that it goes backwards in time. It says that the ratio of next period's to this period's marginal utilities with respect to consumption must equal the after-tax rate of transformation between consumer goods next period and consumer goods this period. The introduction of a pseudo-state variable  $\lambda_{t-1}$ , which is a shadow price, is necessary because of the time inconsistency of the optimal policy.

The problem is not a Markov decision problem [see Bellman (1957)] because there is another constraint which must be determined recursively. Let  $\Omega$  be the set of  $(k_t, \lambda_{t-1})$  for which there exists a policy sequence with an equilibrium. That is, there exists sequence  $\{x_s, \pi_s, \lambda_s\}_{s=t}^{\infty}$  such that constraints (11) and (13) are satisfied for periods  $s \geq t$ , where  $t > 0$ . For  $\lambda_{t-1}$  sufficiently small, and therefore consumption in the previous period sufficiently high, no policy sequence for which there is an equilibrium will exist. This necessitates the addition of constraints

$$(k_{t+1}, \lambda_t) \in \Omega, \quad t = 0, 1, 2, \dots$$

We define the elements  $\lambda_{\min}$  and  $\lambda_{\max}$  to be the minimum and maximum, respectively, of  $u_c(c, n, g)$  over the set of  $(c, n, g)$  for which  $c, n, g \geq 0$ ,  $c + g \leq f(\bar{k}, n)$ , and  $n \leq \bar{n}$ .

Dropping the time subscript and using the prime to denote next period value and the minus one subscript last period value, we define the following mapping  $\Phi$  of closed subsets of  $Z \equiv [0, \bar{k}] \times [\lambda_{\min}, \lambda_{\max}]$  into the same space:

$$\Phi(\Omega) = \{(k, \lambda_{-1}) \in Z: \text{there exists } (\pi, x, \lambda)$$

satisfying the constraints below\},

$$x \equiv (c, n, k') = d(k, \pi, \lambda),$$

$$\lambda = h(\pi, k, \lambda_{-1}),$$

$$(k', \lambda) \in \Omega.$$

In addition there are the non-negativity constraints and maximum constraints on tax rates and labor supplied. We seek the largest set which is a fixed point of  $\Phi$ . The mapping  $\Phi$  is monotonic in the sense that  $\Omega' \subseteq \Omega$  implies  $\Phi(\Omega') \subseteq \Phi(\Omega)$ . Consider the decreasing sequence of sets

$$\Omega_{i+1} = \Phi(\Omega_i) \quad \text{where} \quad \Omega_1 = Z.$$

The limit of this sequence is a greatest fixed point of  $\Phi$ , and is the recursively

determined constraint set  $\Omega$ . This set is non-empty, for given any  $k \in [0, \bar{k}]$ , there is an equilibrium path for policy  $\theta_t = \tau_t = g_t = 0$ . This implies in addition that the projection of  $\Omega$  on  $[0, \bar{k}]$  is  $[0, \bar{k}]$ .

Let  $v(k, \lambda_{t-1})$  be the maximum present value at time  $t$  over all feasible current and future policies given the constraints implied by the rational expectations equilibrium concept for current and future private decisions, and given the current capital stock  $k_t$  and last period's marginal utility  $\lambda_{t-1}$ , which provides the link to the past. Then part of the policy problem is to solve the functional equation

$$v(k, \lambda_{-1}) = \max_{\pi, x, \lambda} [u(c, n, g) + \beta v(k', \lambda)],$$

subject to the constraints

$$x = (c, n, k') = d(k, \pi, \lambda),$$

$$\lambda = h(\pi, k, \lambda_{-1}),$$

$$g \leq \theta f_k(k, n)k + \tau f_n(k, n)n,$$

$$x, \pi \geq 0,$$

$$\tau \leq \bar{\tau}, \quad \theta \leq \bar{\theta}$$

and

$$(k', \lambda) \in \Omega \quad \text{where} \quad (k, \lambda_{-1}) \in \Omega.$$

The optimal policy is of the form  $\pi = \pi(k, \lambda_{-1})$ . This means that at time  $t$ , given  $k_t$  and  $\lambda_{t-1}$  (which define admissible combinations of  $\pi_t$  and  $\lambda_t$ ), we obtain  $\pi_t = \pi(k_t, \lambda_{t-1})$ , which determines  $\lambda_t$ , and therefore  $c_t$ ,  $n_t$ , and  $k_{t+1}$ .

Note that constraint (13) puts no restrictions on  $\lambda_0$ . Given  $k_0$ , the policymaker can choose  $\pi_0$ ,  $x_0$ , and  $\lambda_0$  which maximize

$$u(c_0, n_0, g_0) + \beta v(k_1, \lambda_0),$$

subject to

$$x_0 = d(k_0, \pi_0, \lambda_0),$$

$$g_0 \leq \theta_0 f_k(k_0, n_0)k_0 + \tau_0 f_n(k_0, n_0)n_0,$$

$$(k_1, \lambda_0) \in \Omega.$$

This final part of the policy problem of choosing the initial  $\lambda_0$  determines the optimal policy for the entire future. Thus the pseudo-state variable  $\lambda_{t-1}$  is a

device to ensure that the effects of future policies on agents' behavior in earlier periods are taken into account.<sup>8</sup>

#### 4. An extension to uncertainty

In this section we outline the extension of our analysis to environments with uncertainty. Suppose preferences and technology are affected by random shocks. For example, some exogenous random event may affect the demand for national defense, or a new scientific discovery may affect the production possibility set. In this section we assume the utility function of the representative individual is state contingent depending upon the shock  $s$ . For example, the value of national defense may vary over time depending upon the world political situation. The consumer maximizes

$$E \sum_{t=0}^{\infty} \beta^t u(c_t, n_t, g_t, s_t),$$

subject to his budget constraint. From his point of view, government expenditure  $g_t$  and the tax-rate parameters  $(\theta_t, \tau_t)$ , which affect his budget constraint, are exogenous as is the shock  $s_t$ . For simplicity, the process governing  $s_t$  is assumed to be a Markov chain with  $m$  possible states. The transition probabilities  $\Pr(j|i)$  are the probabilities that  $s_{t+1} = j$  given  $s_t = i$  for  $i, j = 1, \dots, m$ .

To simplify notation, the time subscript has been omitted in the subsequent development. Prime variables denote next-period variables, and the subscript  $-1$  last period. Let  $v(s_{-1}, k, \lambda_{-1})$  be the maximal obtainable expected utility for the representative individual given that the beginning-of-period capital stock was  $k$  and that  $\lambda_{-1}$  and the consumer equilibrium decisions this period are consistent.

The proposed dynamic program is

$$v(s_{-1}, k, \lambda_{-1}) = \max_{s=1}^m \Pr(s|s_{-1}) [u(c_s, n_s, g_s, s) + \beta v(s, k'_s, \lambda'_s)], \quad (14)$$

subject to constraints for  $s = 1, \dots, m$ ,

$$\begin{aligned} k'_s + c_s &= k + (1 - \tau_s) f_n(k, n_s) + (1 - \theta_s) f_k(k, n_s), \\ (1 - \tau_s) f_n(k, n_s) u_c(c_s, n_s, g_s, s) &= -u_n(c_s, n_s, g_s, s), \\ \lambda'_s &= u_c(c_s, n_s, g_s, s), \\ \tau_s n_s f_n(k, n_s) + \theta_s k f_k(k, n_s) &\geq g_s, \\ (k'_s, \lambda'_s) &\in \Omega(s), \end{aligned}$$

<sup>8</sup>This argument was partly motivated by comments in Bryant (1977).

and constraint

$$\lambda_{-1} = \beta \sum_{s=1}^m \Pr(s|s_{-1}) u_c(c_s, n_s, g_s, s) [1 + (1 - \theta_s) f_k(k, n_s)]. \quad (15)$$

The decision variables are individual decision  $X \equiv \{c_s, n_s, k'_s\}_{s=1}^m$ , policy decision  $\Pi \equiv \{g_s, \theta_s, \tau_s\}_{s=1}^m$  and  $\Lambda \equiv \{\lambda_s\}_{s=1}^m$ . The constraints that  $(k'_s, \lambda_s)$  belong to set  $\Omega(s)$  are feasibility constraints. The sets  $\Omega(s)$  must be such that there is a solution to the right-hand side of (14) if and only if  $(k, \lambda_{-1}) \in \Omega(s_{-1})$  for  $s_{-1} = 1, \dots, m$ . Thus, like the value functions  $v$ , the constraint sets are determined recursively. One can, at least in theory, find a solution which is computable as the limit of finite  $T$ -period problems as  $T$  becomes infinite.

The optimality eq. (14) is of interest for the maximization over elements specifying actions contingent upon  $s$  – that is, prior to observing  $s$ . Consequently,  $s$  is not a state variable in the sense of dynamic programming. This is necessary because of the form of constraint (15). It jointly depends upon decisions contingent upon all  $s$  and not just the realized  $s$ .

As for the deterministic case, constraint (15) is not applicable in the initial period and choosing  $\lambda$  for period zero is part of the optimization problem. This choice determines the optimal shock-contingent policy for the entire future. Let the optimal decision rules be

$$\Pi = \Pi(s_{-1}, k, \lambda_{-1}), \quad X = X(s_{-1}, k, \lambda_{-1}), \quad \Lambda = \Lambda(s_{-1}, k, \lambda_{-1}).$$

The actual policy selected depends upon the realized shock. If the shock is  $s$ , the policy action is the  $s$ th component of  $\Pi$ , and the equilibrium decision of the representative consumer the  $s$ th component of  $X$ . The state variables for the subsequent period are the realized shock  $s$ , the consumer's equilibrium  $k'$ , and the  $s$ th component of  $\Lambda$ .

To summarize, in the abstract at least, even the stochastic case can be formulated as a recursive problem using the pseudo-state variable  $\lambda_{-1}$  along with recursively determined constraint sets  $\Omega(s)$ . This formulation leads to unusual constraints, however, and the problem of actually computing an optimal policy would appear quite formidable even for relatively simple parametric structures.

## 5. Concluding remarks

We have not argued that optimization and quantitative methods should not be used for policy evaluation and selection within self-fulfilling and rational expectations environments. Rather we have argued that naive

application of optimal control theory methods will result in time-consistent but suboptimal policies. As was demonstrated in Kydland and Prescott (1977) and Prescott (1977), the consistent solution can be very suboptimal. We think the determination of optimal policy, and more importantly its return, is important. It provides a standard with which to compare time consistent policy under alternative institutional constraints. In environments where policy *rules* can be changed only after an extended period of public deliberation, the time consistent rule may be nearly optimal.

We do not think discounting is the crux of the consistency problem. For our optimal taxation problem, the results were insensitive to the discount factor  $\beta$  and the inconsistency problem did not disappear as  $\beta$  approached one. If  $\beta$  equals one, there are problems with existence of competitive equilibria because the infinitely-lived representative household's utility is infinite. We suspect this problem can be circumvented by introducing a Ramsey 'deviation from bliss' preference ordering. With such an analysis the inconsistency problem will remain.

Finally, we emphasize that the fixed-rule procedure we advocate does not necessarily imply constant values or constant growth rates of the policy instruments. Quite possibly a feedback rule with the tax parameters varying systematically with economic conditions may dominate any policy of constant tax parameters. A policy rule, however, is needed before one can predict what equilibrium process will govern the economy and an implication of dynamic maximization theory is that a policy-rule invariant autoregressive model will not exist.

It should be noted that our equilibrium framework led to decision rules that depended not only on the state and tax rates, but also on the unobservable shadow price  $\lambda$ . This causes an estimation problem. To the extent, however, that everything in eq. (12), except  $\lambda_t$  and  $\lambda_{t+1}$ , can be observed, including the discount factor  $\beta$ , at least the ratios of next period's to this period's shadow price could be determined, and these prices can clearly be scaled any way we want. Such relationships might therefore conceivably be estimated.<sup>9</sup> But even if the relationships can be identified and estimated, we have not yet found practical methods for computing the optimal policy when there are stochastic constraints because of the high dimensionality of the dynamic program.

<sup>9</sup>Identified, policy-invariant structures specifying current decisions as functions of current state, current prices, and expectations of future prices and policy parameters exist. Huntzinger (1979) and Taylor (1979) have estimated such structures and Wallis (1979) has developed the econometrics much further. An alternative approach is that of Hansen and Sargent (1980) who use maximum likelihood methods to estimate parameters of preferences, technology, and exogenous processes directly.

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