Volume Title: Annals of Economic and Social Measurement, Volume 5, number 2

Volume Author/Editor: NBER

Volume Publisher:

Volume URL: http://www.nber.org/books/aesm76-2

Publication Date: April 1976

Chapter Title: Decentralized Stabilization Policies: Optimization and the Assignment Problem

Chapter Author: Finn Kydland

Chapter URL: http://www.nber.org/chapters/c10445

Chapter pages in book: (p. 249 - 261)

Annals of Economic and Social Measurement, 5/2, 1976

DECENTRALIZED STABILIZATION POLICIES: OPTIMIZATION AND THE ASSIGNMENT PROBLEM

BY FINN KYDLAND*

The main approach to the problem of decentralized macroeconomic policymaking in the literature so far has been the so-called assignment problem, which is concerned with how to pair economic instruments with targets so as to insure stability of the economy. We argue that a realistic model would be one in which the policymakers all care about the same target variables but, because of different political pressures, they assign relatively different weights to the various targets. We formulate a theory of decentralized macroeconomic policymaking as a dynamic game between the monetary and fiscal authorities and derive equilibrium solutions for these games. Noncooperative solutions are discussed, and we also consider the possibility that the fiscal authorities are dominant in the sense that they announce their decision first, thereby taking into account the reaction function of the monetary authorities. The theory is applied to a simple model of the U.S. economy, with particular attention to the question of whether the solutions are stable under various assumptions of the relative weights on the targets.

1. INTRODUCTION

In recent years there has been a lot of interest among economists in the problem of how to control policy instruments in an optimal way so as to achieve economic stabilization.[1] In almost all of the work on this subject it is assumed that there is only one decision maker, or at least that the preferences of the policymakers can be reflected by a single objective function. However, in many countries, the instruments of the public sector are under the control of different policymakers who may be under different political pressures and thus have conflicting views on target values or the relative importance of these targets. For instance, in the United States, it is unlikely that the fiscal and monetary authorities have the same views on what the targets of their policies should be. It is not clear either that much cooperation is taking place between them.

The main approach to the problem of decentralized policymaking in the literature so far has been what is commonly called the assignment problem.[2] In these models, which are mainly deterministic without lags in the structural equations, there are usually two policy controllers,[3] each of whom has control of a particular instrument. Each controller is to vary his instrument in response to changes in a single target variable which has been assigned to him. It is usually assumed that he will vary his policy instrument at a rate proportional to the deviation of the target variable from its target value. If the assignment is made according to the criterion that each instrument should be directed towards that target on which it has relatively the greatest impact, then it is shown that the target values will be approached from any initial point. With the wrong assignment the system becomes unstable in the sense of moving away from the target values.

* The Norwegian School of Economics and Business Administration.

[1] See for instance Chow [1, 2, 3, 4], Pindyck [13, 14], and Prescott [15].

[2] See Mundell [11]. A list of some of the literature on this topic can be found in Whitman [23], as well as in the recent article by Tsiang [22].

[3] See Lancaster [9] for a discussion of the problems involved in generalizing to the $n \times n$ case.

As pointed out above, in the United States, monetary and fiscal policies are in fact decentralized. Each policymaker clearly worries about more than one target variable, and he is unlikely to just blindly carry out some ad hoc policy rule without regard to what the other policymaker is doing. A more realistic model for such a situation seems to fall within a game-theoretic framework. In this paper we propose such a framework for the decentralized policy problem in dynamic linear economic models.

Each policy controller is assumed to minimize a loss function that may include all the target variables, but supposedly with different relative weights. Each policymaker forms expectations of what the others are going to do, and these expectations will in equilibrium be rational in the sense that the expected decisions turn out to be actual ones.

As an example we shall use a simple ad hoc model of the U.S. economy. It will be of a type normally used to illustrate the assignment problem. The fiscal authorities are assumed to control the net government deficit, while the Federal Reserve controls the interest rate. One possibility is to assume noncooperative behavior, although casual observation suggests that the relationship between the two policymakers is such that the fiscal authorities at certain intervals will announce their decision, while the Fed tries to do its best to meet its objectives, given the announced fiscal policy. In so doing, the fiscal authorities can take into account the reaction function of the Fed. If this is the case, the fiscal authorities can be considered a dominant player.

For this model we run simulations that show how the targets might be approached with two different assumptions of the relative weights that the two policymakers put on the targets. One of these examples compares with the wrong assignment in the assignment problem. Unlike what is the case there, our system is still stable in the sense that the targets are approached from any initial point, but speeds of adjustment towards the targets are significantly slower than for the "correct assignment." We also consider a different concept of stability, namely whether the decision rules will move towards the equilibrium rules given that each policymaker initially has incomplete information about the policy of the other policymaker.

2. A Noncooperative Model of Policymaking

The purpose of this section is to provide an equilibrium framework for the selection of policies under noncooperative behavior. For simplicity of notation we shall assume that the linear structural equations of the economy can be written as

$$y_t = g(y_{t-1}, x_{ft}, x_{mt}, \varepsilon_t),$$

where x_{ft} denotes the instrument(s) under the control of the fiscal authorities, and x_{mt} the instrument(s) controlled by the Federal Reserve. The disturbances ε_t are independently distributed over time with mean zero and finite variances. Also, assume that each policymaker evaluates alternative policies according to some preference (loss) function that can be approximated by a quadratic function

$$E\left\{\sum_{t=1}^{T} w_{it}(y_t, x_{ft}, x_{mt})\right\}, \qquad i = f, m. \tag{2.1}$$

These two functions reflect the fact that the policymakers will generally have different objectives. The objective functions are written so as to include possible dependence of the policymakers' preferences on the levels of the instruments or changes thereof. For instance, one may perceive a certain cost to maintaining the rate of interest away from some given level, or it may be thought costly in some sense to let the interest rate change a lot from one period to another.

We see that each policymaker has to know or assume something about the other policymaker's behavior in order to solve his optimization problem. We think of each player as selecting a sequence of policy rules[4] $x_i = \{x_{it}(y_{t-1})\}_{t=1}^{T}$, $i = f, m$, given the rules for the other player. In equilibrium we assume that each policymaker has rational expectations about the decisions of the other player. This leads to the concept of noncooperative solution[5] as a basis for a definition of equilibrium for our model.

An appropriate solution or equilibrium concept has been developed in Kydland [7]. An equilibrium solution is characterized by two sequences of functions, $x_f^0 = \{x_{ft}^0(y_{t-1})\}_{t=1}^{T}$ and $x_m^0 = \{x_{mt}^0(y_{t-1})\}_{t=1}^{T}$, where x_f^0 minimizes (2.1) for the fiscal authorities, given x_m^0, and x_m^0 minimizes (2.1) for the Fed, given x_f^0. We now outline how these solutions can be computed.[6]

Define the functions

$$v_{it}(y_{t-1}) = E\left\{ \sum_{s=t}^{T} w_{is}(y_s, x_{fs}^0, x_{ms}^0) \right\}, \qquad i = f, m;$$

that is, $v_{it}(y_{t-1})$ is the total value for policymaker i of the sequence of noncooperative solutions $x_s^0 = \{x_s^0(y_{s-1})\}_{s=t}^{T}$. Then we can write

$$V_{it}(y_{t-1}, x_{jt}) = \min_{x_{it}} E\{w_{it}(y_t, x_t) + v_{i,t+1}(y_t)\}$$

subject to

$$y_t = g(y_{t-1}, x_t, \varepsilon_t) \qquad y_{t-1}, x_{jt} \text{ given}.$$

Using the above notation, we can now define what we mean by equilibrium.

Definition: An equilibrium for each time period $t = 1, \ldots, T$ is a pair of decision rules $x_{ft}^0 = x_{ft}^0(y_{t-1})$ and $x_{mt}^0 = x_{mt}^0(y_{t-1})$ such that

$$\min_{x_{it}} E\{w_{it}[g(y_{t-1}, x_{it}, x_{jt}^0, \varepsilon_t), x_{it}, x_{jt}^0] + v_{i,t+1}[g(y_{t-1}, x_{it}, x_{jt}^0, \varepsilon_t)]\}$$

$$= E\{w_{it}[g(y_{t-1}, x_{it}^0, x_{jt}^0, \varepsilon_t), x_{it}^0, x_{jt}^0] + v_{i,t+1}[g(y_{t-1}, x_{it}^0, x_{jt}^0, \varepsilon_t)]\}$$

$$i = f, m; \qquad j \neq i.$$

The equilibrium solution has the characteristic that no policymaker has any incentive to change his decision rule in any period, given the decision rules of the other policymaker.

[4] The state vector can be expanded in a well-known way so as to include, for instance, lagged decision variables.

[5] Also called Nash equilibrium [12]. Discussions of Nash equilibria in differential games can be found in Starr and Ho [19, 20].

[6] The computational details for the linear-quadratic case are given in [6] and [7], and computer programs are listed in [6], and can be provided upon request.

The equilibrium decision rules can be computed using backward induction. At time t the first-order conditions for a minimum[7] for each of the two policymakers, define two mappings

$$y_{t-1}, x_{jt} \rightarrow x_{it}, \qquad i=f, m; \qquad j \neq i.$$

In equilibrium these solutions can be written

$$x_t^0 = x_t^0(y_{t-1}).$$

Given these solutions we can now evaluate the value functions at time t as

$$v_{it}(y_{t-1}) = V_{it}[y_{t-1}, x_{jt}^0(y_{t-1})], \qquad i=f, m; \qquad j \neq i.$$

The solution concept outlined above is a feedback solution. However, unlike the case of only one decision maker, this solution is not the same as the open loop solution, where the decisions are sequences of functions of the initial y_0 and of all previously observed random variables, that is,

$$x_{it} = x_{it}(y_o, \varepsilon_1, \ldots, \varepsilon_{t-1}), \qquad t=1, \ldots, T; \qquad i=f, m.$$

This difference, which is explained in detail in [7], would occur even in the absence of uncertainty. Intuitively, the reason is the following. In making his decision, policymaker i knows that his decision will affect the state variables. A change in the state variables will change the other policymaker's decisions in the future and affect future losses for policymaker i. This fact is taken into account in the feedback solution when policymaker i makes his decision. Of course, both types of solutions represent equilibrium solutions in some sense. However, the feature described above seems to lend more realism to the feedback solution as a description of the movement of an economic system.

The argument for the feedback solution as the appropriate equilibrium concept seems even more convincing when the planning horizon is infinite. In this case we take the objective functions of the policymakers to be

$$E\left\{ \sum_{t=1}^{\infty} \beta_i^{t-1} w_i(y_t, x_{ft}, x_{mt}) \right\}, \qquad i=f, m, \tag{2.2}$$

where $0 < \beta_f, \beta_m < 1$ are discount factors. We are here looking for stationary solutions, that is, an equilibrium characterized by two functions $x_f^0 = x_f^0(y_{t-1})$ and $x_m^0 = x_m^0(y_{t-1})$. They are equilibrium decisions if x_f^0 minimizes (2.2) for the fiscal authorities, given x_m^0, while x_m^0 minimizes (2.2) for the Fed, given x_f^0. For the case in which $w_i(\cdot)$, $i=f, m$, are quadratic and the constraints linear with additive disturbances, the method of successive approximations has been found to work effectively in computing the solutions. One solves a T-period problem to determine $v_{f1}^{(T)}(y_0)$ and $v_{m1}^{(T)}(y_0)$, the first-period value functions for the truncated T-period problem. These value functions are then used to determine the same functions for the $T+1$ period problem. These functions have been found to converge quickly to some limiting functions $v_f(y_{-1})$ and $v_m(y_{-1})$ satisfying the respective functional equations for the two policymakers.[8] These functions, then, imply a pair of equilibrium solutions for the infinite-period game.

[7] We assume that the second-order conditions are satisfied as well.

[8] Conditions have not yet been established insuring uniqueness of the solution.

In the open loop formulation we could also let T go to infinity, in which case the first-period decision rules would settle down to some stationary rules as functions of the initial state variables. It is interesting to note that if one of the players were to take this open loop decision rule for the other player as given and solve his one-player infinite horizon problem (which would be a standard control problem), his optimal stationary decision rule would not be the same as the one given by the open loop solution. On the other hand, it is obvious that, if instead he takes the feedback rule of the other player as given and solves his own one-player problem, he will get back his feedback rule as the optimal one for his one-player problem. These comments suggest that players groping for equilibrium decision rules which imply no incentive to change their rules, are likely to end up with feedback rules instead of open loop solutions.

3. A Dominant Player Model of Policymaking

In this section we stress only the main departures from the analysis of Section 2. We assume that the fiscal authorities are dominant in that they can make their decision first, thereby taking into account the reaction function of the monetary authorities.[9] An equilibrium solution is then characterized by two sequences of functions, $x_f^0 = \{x_{ft}^0(y_{t-1})\}_{t=1}^T$ and $x_m^0 = \{x_{mt}^0(y_{t-1}, x_{ft})\}_{t=1}^T$.

As before we define value functions $v_{ft}(y_{t-1})$ and $v_{mt}(y_{t-1})$ that represent the total values for the two policymakers of the two sequences of equilibrium solutions $\{x_{fs}^0(y_{s-1})\}_{s=t}^T$ and $\{x_{ms}^0(y_{s-1}, x_{fs}^0(y_{s-1}))\}_{s=t}^T$. These value functions will now be helpful in indicating how the equilibrium solutions can be determined by backward induction.

Define for period t

$$V_{mt}(y_{t-1}, x_{ft}) = \min_{x_{mt}} E\{w_{mt}(y_t, x_t) + v_{m,t+1}(y_t)\} \tag{3.1}$$

subject to

$$y_t = g(y_{t-1}, x_t, \varepsilon_t), \qquad y_{t-1}, x_{ft} \text{ given.}$$

The solution for the Federal Reserve is of the form

$$x_{mt}^0 = x_{mt}^0(y_{t-1}, x_{ft}).$$

Taking account of this, the problem for the fiscal authorities can now be written

$$v_{ft}(y_{t-1}) = \min_{x_{ft}} E\{w_{ft}(y_t, x_{ft}, x_{mt}) + v_{f,t+1}(y_t)\}$$

subject to

$$y_t = g_t(y_{t-1}, x_{ft}, x_{mt}, \varepsilon_t),$$

$$x_{mt} = x_{mt}^0(y_{t-1}, x_{ft}), \qquad y_{t-1} \text{ given.}$$

[9] Solutions of models with a dominant player are often called Stackelberg solutions [18]. Such solutions have been considered in the context of differential games by Simaan and Cruz [16, 17].

The equilibrium solution is of the form

$$x_{ft}^0 = x_{ft}^0(y_{t-1}),$$

which, when substituted into (3.1) gives

$$v_{mt}(y_{t-1}) = V_{mt}[y_{t-1}, x_{ft}^0(y_{t-1})].$$

The solution just described is the feedback solution. As in the noncooperative case this solution is not the same as the open loop solution which can be written on the form

$$x_{mt} = x_{mt}(y_0, x_{f1}, \ldots, x_{fT}, \varepsilon_1, \ldots, \varepsilon_{t-1}),$$
$$x_{ft} = x_{ft}(y_0, \varepsilon_1, \ldots, \varepsilon_{t-1}), \qquad t = 1, \ldots, T.$$

As has been shown in [7] and [17] the open loop solution in general is such that it is not optimal for the players to carry through with their plans. This means that if the players make the decisions given by x_{i1}, $i = f, m$, then the original plan $x_{ft}(y_0, \varepsilon_1, \ldots, \varepsilon_{t-1})$, $t = 2, \ldots, T$, for the fiscal authorities is no longer optimal for the remaining $T-1$ periods of the horizon. This would be the case even in the absence of uncertainty. In equilibrium one must assume that the players will foresee this, and the feedback solution will then be the appropriate equilibrium concept.[10]

The extension to an infinite horizon problem goes along the same lines as indicated in Section 2. Both for the noncooperative and dominant player case one can compute the covariance matrix for the stationary solution if the covariance matrix for the disturbances is known. The computations are similar for both cases and are outlined in [7].

4. A Model of the U.S. Economy

To illustrate the theory of the previous two sections we shall use a simple model adapted from one reported in McFadden [10]. It is similar to some of the models used in the literature on the assignment problem, except that we are introducing lags to make the model dynamic.

The variables we use are the following:

Y = domestic U.S. production,
X = U.S. aggregate expenditure,
C = U.S. aggregate consumption,
S = U.S. aggregate saving,
I = U.S. domestic investment,
M = U.S. imports of foreign goods and services,
K = net capital outflows from the U.S.,
T = taxes net of transfers,
G = U.S. government expenditures for goods and services,
B = U.S. surplus in international balance of payments,
E = U.S. export of goods and services, assumed constant,
$D = G - T$ = net government deficit.

[10] A more thorough discussion of this issue can be found in Kydland [7, 8].

All the variables above are annual rates, and are measured in billions of dollars, deflated to a uniform price level. In addition we have

r = U.S. domestic interest rate, measured in percent.

The following identities link the variables:

$$Y=C+S+T,$$
$$X=C+I+G+K,$$
$$B=E-M-K,$$
$$B=Y-X.$$

The assumed behavioral relations are[11]:

$$S=0.5Y-0.25Y_{-1}-40,$$
$$M=0.091Y+0.3M_{-1}-31.5,$$
$$I=0.12Y-1.75r-2r_{-1}+47,$$
$$K=-0.76r+13.7$$

The reduced form of this model is

$$\begin{bmatrix} B \\ Y \end{bmatrix} = \begin{bmatrix} 0.242 & -0.048 \\ 0.637 & 0.531 \end{bmatrix} \begin{bmatrix} B_{-1} \\ Y_{-1} \end{bmatrix} + \begin{bmatrix} 1.098 & -0.193 \\ -3.716 & 2.123 \end{bmatrix} \begin{bmatrix} r \\ D \end{bmatrix} + \begin{bmatrix} 0.202 \\ -4.730 \end{bmatrix} r_{-1} + \begin{bmatrix} 16.29 \\ 307.9 \end{bmatrix}.$$

5. Numerical Results

To complete our example we have to make some assumptions about the loss functions of the two policymakers. The Federal Reserve, having control over the interest rate, is assumed to put relatively more weight on balance of payments equilibrium,[12] while the main target of the fiscal authorities is full employment GNP. We also assume that each policymaker perceives an increasing cost to changing the instruments from one period to another.

With these assumptions, the one-period loss functions are initially taken to be:

$$w_1=10B^2+0.01(Y-580)^2+1.7(r-r_{-1})^2+0.02(D-D_{-1})^2,$$
$$w_2=0.5B^2+0.02(Y-580)^2+0.3(r-r_{-1})^2+0.1(D-D_{-1})^2.$$

Without any difficulty we could have included cross-product terms for the targets, and we could also have assumed that the desired target values were different for each controller. As it is, each policymaker would prefer $B=0$ and $Y=580$. We also note that, although each policymaker has no direct control over the other policymaker's instrument, he still perceives some loss associated with changes in that instrument. Of course, each policymaker can only affect the other

[11] The relations of the original model by McFadden [10] were adapted from an econometric model and modified to approximate the 1963 U.S. national accounts. Some of the figures for that year were: $E=32$, $I=83$, $C=353$, $M=26.5$, $G=109$, $T=100$, and $S=98$.

[12] Comments made in Tobin [21] regarding macroeconomic policymaking in the 1960s seem to confirm the realism of this assumption.

policymaker's instrument through the effect of his own decision rule on the other's decision rule.

We shall first compute the deterministic time paths of the state variables and instruments assuming that they initially have the values[13] $B_0 = -2$, $Y_0 = 551$, $r_0 = 8$, and $D_0 = 9$. The horizon is assumed to be long enough so that increasing it by one period does not change any of the coefficients of the first-period decision rules to the sixth significant digit. Typically this would mean a horizon of about 20 periods, and these decision rules should be very close to the stationary ones for the infinite horizon model. The discount factor is 0.95 for both policymakers. For easy reference we refer to noncooperative solutions by NC and dominant player solutions by DP. The solutions for the objective functions above will thus be referred to by NC1 and DP1.

The time paths of B and Y for the noncooperative solutions are shown in Figure 1. As an illustration we also show the time path for Y in the dominant player solution.

The values of the losses for the whole horizon are slightly lower for DP1 than NC1 for both policymakers. The reduction is about 3.5 percent for the fiscal authorities. It may seem surprising, then, that the curve for Y^{DP1} is strictly below the curve for Y^{NC1}, even though the fiscal authorities, who have Y as their main target, are dominant and manage to reduce their loss compared to the noncooperative solution. The explanation cannot be seen in the figures, but can easily be understood by looking at the paths for D. By being dominant the fiscal authorities manage to use their instrument more effectively and the reduced changes in D required in each period more than make up for the longer distance of Y from the target value in every period.

According to the criterion for assigning instruments to targets the interest rate r should be assigned to B, because that is where it has relatively the stronger impact. Similarly, D should be assigned to Y. If the opposite assignment is made, then the system will be unstable. In the model just described the instruments are assigned according to the assignment criterion in the sense that each policymaker puts relatively more weight on the target on which his instrument has relatively the stronger impact.

We shall now see what happens if this is not the case. Specifically, we assume that, relative to the targets, the Fed now has the loss function the fiscal authorities had previously, while the fiscal authorities have the one the Fed had. The new loss functions are then:

$$w_1 = 0.5B^2 + 0.02(Y-580)^2 + 1.7(r-r_{-1})^2 + 0.02(D-D_{-1})^2,$$

$$w_2 = 10B^2 + 0.01(Y-580)^2 + 0.3(r-r_{-1})^2 + 0.1(D-D_{-1})^2.$$

The new equilibrium solutions are referred to as NC2 and DP2. In order that comparisons with NC1 and DP1 be meaningful, care was taken when choosing the weights of the original objective functions to insure that the total losses for NC1 and DP1 would be approximately equal for both the monetary and fiscal

[13] These are approximately the actual values for 1963.

authorities, and that the contribution to the losses due to changes in the control variables amounted to approximately the same percentage of the total loss for each policymaker.

The time paths of the solutions from NC2, starting from the same point as before, are shown in Figure 1. While in the assignment problem the wrong assignment leads to instability, this is not so for the comparable interchanging of the loss functions in our model. However, it does lead to significantly slower speeds of adjustment.

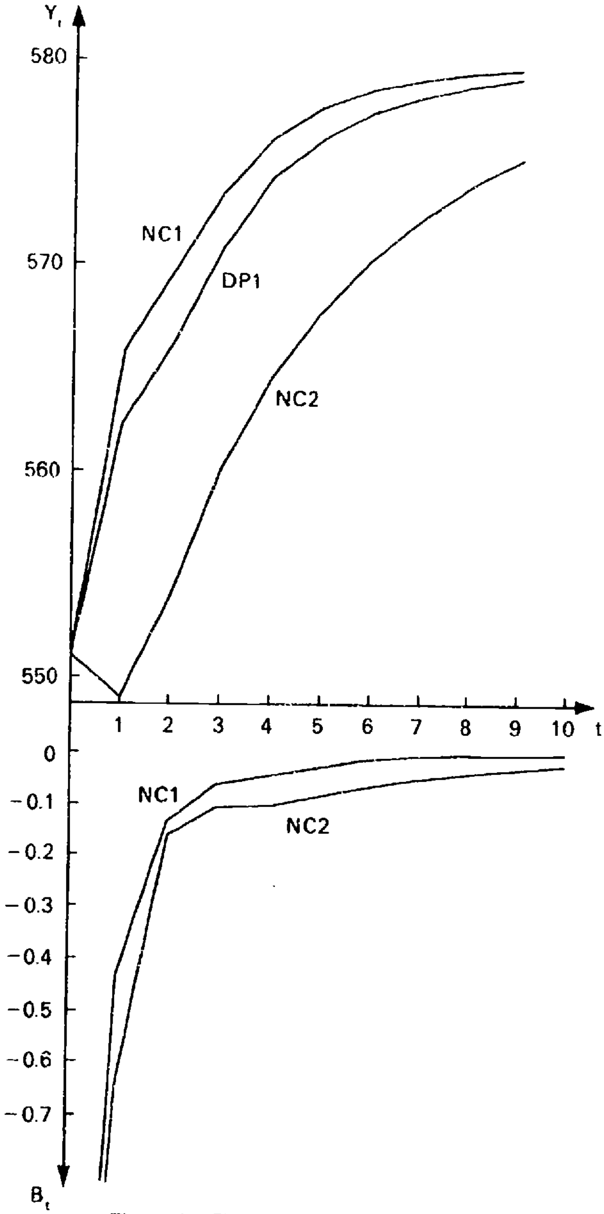

Figure 1 Time paths of target variables

So far we have assumed that the model is deterministic, and we studied how the targets might be approached under two different assumptions about the loss functions. However, it is clearly more realistic to think of the behavioral relations as including stochastic errors, which will make the target values attainable only on the average. An interesting exercise, then, is to compute the covariance matrices of the stationary solutions. We shall compare NC1 with NC2, although the results of a comparison between DP1 and DP2 are similar.

Assume for simplicity that we have found the covariance matrix for the errors of the reduced form to be

$$\sigma^2\begin{bmatrix} 0.25 & 0 \\ 0 & 1 \end{bmatrix}.$$

We denote the variances for B and Y in the stationary solutions by σ_B^2 and σ_Y^2, respectively. Given the assumed covariance matrix for the error terms, we find that NC1 results in $\sigma_B^2=0.252\sigma^2$ and $\sigma_Y^2=1.261\sigma^2$, while for NC2 we get $\sigma_B^2=0.254\sigma^2$ and $\sigma_Y^2=2.817\sigma^2$. We thus see that the variance of Y has increased substantially when the policymakers put relatively more weight on the "wrong" targets. One should also note that the increased variability of Y is not compensated by a lower variability of D as was the case when comparing NC1 with DP1. Here we also get a substantial increase of σ_D^2, from 0.158 to 0.276.

6. A Further Look at Stability

In the previous section we talked about stability in the sense in which it is normally used in the literature on the assignment problem, namely referring to whether the target variables will move towards the target values or not. However, a more interesting concept of stability is the question of whether the decision rules will have a tendency to move towards the equilibrium decision rules.

For the noncooperative case, assume that the monetary authorities initially think that the fiscal authorities are following some decision rule $d_{f,0}^e(y_{-1})$. Taking this as given, they derive their own optimal decision rule, say $d_{m,1}(y_{-1})$. However, the monetary authorities will soon realize that the fiscal authorities use some other policy rule, say $d_{f,1}(y_{-1})$, and revise their expectations for the future. Similarly, as the monetary authorities revise their decision rules, the fiscal authorities will revise their expectations.

Define one iteration to include one successive modification by each policymaker. Assume that the changes in expectations from iteration n to $n+1$ follow the following adaptive schemes:

$$d_{i,n+1}^e(y_{-1})=d_{i,n}^e(y_{-1})+\lambda_i[d_{i,n}(y_{-1})-d_{i,n}^e(y_{-1})], \qquad i=f, m,$$

where $0<\lambda_i\leq 1$. The special case of $\lambda=1$ corresponds to static expectations in the sense that each policymaker believes that the other one will continue to behave in the future according to the most recent decision rule.

In Tables 1 and 2 some computational results are shown with $\lambda=1$ for NC1 and NC2, respectively. Only the monetary policies, which are of the form

$$r_t=d_1B_{t-1}+d_2Y_{t-1}+d_3r_{t-1}+d_4D_{t-1}+d_0,$$

are shown, although similar results are obtained for the fiscal rules. The first monetary policy rule is obtained assuming that the fiscal rule is simply $D_t = D_{t-1}$. The fiscal rule for the first iteration is then obtained, taking as given the monetary rule from the first iteration, and so on. The equilibrium decision rules are listed at the end of each table. The decision rules for NC1 converge quickly to these equilibrium rules, indicating that the equilibrium decisions of Section 5 are quite stable. The rules from NC2 also converge, indicating that they are stable as well, but the rate of convergence is substantially lower.

TABLE 1

MONETARY POLICIES FOR NC1

Iteration No.	Coefficient of B_{-1}	Y_{-1}	r_{-1}	D_{-1}	constant
1	−0.1503	0.03613	−0.07937	0.1692	−11.64
2	−0.2066	0.02752	0.06160	0.1014	−5.739
3	−0.2218	0.02532	0.09793	0.08199	−4.137
4	−0.2256	0.02490	0.1057	0.07628	−3.752
5	−0.2265	0.02484	0.1071	0.07460	−3.663
∞	−0.2268	0.02485	0.1072	0.07389	−3.635

TABLE 2

MONETARY POLICIES FOR NC2

Iteration No.	Coefficient of B_{-1}	Y_{-1}	r_{-1}	D_{-1}	constant
1	−0.00382	0.02239	0.1483	0.1864	−8.093
2	0.05575	0.01350	0.3565	0.06117	−0.4265
3	0.07223	0.00990	0.5419	0.04374	−0.4841
4	0.06070	0.00902	0.5686	0.03222	0.1382
5	0.05849	0.00830	0.6030	0.02815	0.1976
∞	0.05361	0.00749	0.6214	0.02451	0.5405

Similar results could have been obtained for the dominant player solutions. The only difference would have been that the monetary policy at each iteration had the form $d_m(y_{-1}, x_f)$, where x_f denotes the fiscal control variable.

7. Concluding Comments

In summary, we have proposed a new game-theoretic approach to the problem of decentralized policymaking. We have presented a positive theory for how the policymakers may act optimally, given expectations of what the other policymakers will do. The possibility of one policymaker being dominant was studied. Among the potential applications we here chose to formulate a model related to the assignment problem, but based on what we think are more realistic assumptions. Our conclusions turn out to be somewhat different, in particular with regard to stability.

Alternatively, the framework presented could have aimed at studying decentralized policies in a model where the policymakers put different weights on such targets as inflation and unemployment. Most models presented so far, which are mostly normative, assume a single preference function and perfect coordination of the instruments.[14] Our framework might also be used to shed some light on the controversy over rules versus discretion, in particular with regard to monetary policy. The advantage of our framework is that we can take account of the fact that if fiscal policymakers behave rationally, their decision rules will not remain stable when the monetary rule changes.[15] We do make the assumption that the decision rules of the rest of the economy remain stable. This is done in order to enable us to concentrate on the interaction between the two policymakers. However, in other models one may also wish to take into account the fact that economic agents, if they behave rationally, will change their decision rules if certain policy rules change.

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[14] For a recent example, see Pindyck [14].

[15] Goldfeld and Blinder [5] have discussed the possible econometric implications of failing to take account of this type of endogeneity of policy rules.

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