A DYNAMIC DOMINANT FIRM MODEL OF INDUSTRY STRUCTURE*

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Abstract

This paper presents characteristics of dynamic equilibria in a model of industry structure where one firm is dominant in the sense of taking into account how its rivals will react to its decisions. The model is consistent with observed stable differences in market shares, with the market share of the dominant firm declining slowly over time. The market share of the dominant firm was found to decrease when the elasticity of industry demand increased, and when adjustment costs increased relative to long-run unit cost. The issue of barriers to entry is also discussed.

I. Introduction

Some of the stylized facts of industry structure that an adequate theory should account for are:

1. Many manufacturing industries are characterized by highly unequal market shares. A common pattern is for the largest producer to be about twice the size of the second largest, who is substantially larger than the third.

2. The rankings of firms in industries according to their market shares, to the extent that these shares are significantly different, are fairly stable over time.

3. The market share of the dominant firm has typically declined over time.

Evidence of this, in particular on the last point, can be found in Burns (1936, pp. 77–140), who describes the histories of several industries for three or four decades. On page 142 he says: “It appears to be the common fate of leaders to suffer a decline in their proportion of the total business in the market.” Scherer (1970, pp. 217–218) also provides evidence to substantiate this claim with examples from industries such as ingots and castings, rayon, tin cans, corn products refining, farm implements, synthetic fibers, aluminum extrusions and, on the regional level, the gasoline industry.

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Nevertheless, in spite of the decline in market shares, a few firms have retained dominant positions in their markets for decades. Burns (1936) found substantial evidence of price-leadership behavior among some of these firms. Dominant-firm or price-leadership models did indeed attract a lot of attention prior to and during the fifties. As Cyert & March (1966) pointed out, however, the fact that the market share of the dominant firm has shown a steady downward trend is difficult to explain on the basis of the traditional price-leadership model. Worcester (1957), who, according to Scherer (1970, p. 216), provided the first complete and still definitive analysis of the dynamics of dominant-firm pricing, concluded that the dominant-firm case is a short-run phenomenon that will break down in the long run. If this is the case, it is hard to explain why this process would take three-fourths of a century or more as is indicated by the examples above.

This paper represents an attempt to re-examine the dominant-firm case in a model which is inherently dynamic in the sense that there are structural interconnections over time. This contrasts with the static models or sequences of static models which have dominated the industrial organization literature up to now.1 The main dynamic aspect has typically been the description of how expectations are formed, and even this is often presented in the most naive way.2 Focusing on how firms may behave out of equilibrium may say something about stability, but it is difficult to derive testable hypotheses of any interest from such analyses. Besides, even in equilibrium, firms are confronted with an inherently dynamic situation for which the static models may not be rich enough to provide the insights needed.

In our model, the capital stock at the start of each period uniquely determines output (and sales) in that period. We assume increasing cost of adjustment in changing capacity from one period to the next. This is a well-known theoretical explanation, within an optimization framework, of the empirical fact that firms do not immediately adjust their capital stock to the desired levels.3 We also introduce stochastic fluctuations in demand.

The basic behavioral assumption is noncooperative. The Nash equilibrium, however, is typically not consistent with the observed differences in market shares.4 We assume instead that one firm, for instance the original monopoly firm in the industry, is dominant in the sense that it takes into account how

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1 Exceptions among noncooperative models are Clemhout et al. (1971), Prescott (1973) and Flaherty (1976).
2 A typical example is the assumption of static expectations as in the Cournot model. More sophisticated reaction strategies are considered in Cyert & DeGroot (1970, 1971 and 1973), introducing learning over time, and in Friedman (1973 and 1976).
3 See Eisen & Strotz (1957), Lucas (1967) and others.
4 Various assumptions, such as different cost structures, will of course produce different market shares in a Nash equilibrium. Such an assumption is not convincing without an explanation of how these cost differences came about. Flaherty (1974) has recently used cost-reducing investment to explain cost differences. She was able to show that under certain assumptions stable noncooperative equilibria with unequal market shares are possible.

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the other firms will react to its decisions. In the existing dominant-firm literature it has often been assumed that the other firms in the industry behaved competitively. Since the number of rivals is not necessarily very large, we assume that they behave noncooperatively among themselves. We also indicate how this structure can be generalized to allow for more general distributions of market shares.

Simple static models generally do not give a satisfactory explanation as to why there is only a limited number of firms in many industries. Several barriers to entry have been suggested, in particular in the empirical literature. Examples are limit pricing, which has received considerable theoretical attention, often combined with economies of scale, and capital requirements. Although limit pricing might be relevant for a dominant-firm model, we ignore the issue in this paper. We assume constant returns to scale in the long run in order to be consistent with the large body of evidence supporting this hypothesis in many industries.

This leaves the capital requirements barrier, which will turn out to be important in our dynamic model as a determinant of the number of firms in the industry. When entering an industry, a firm has to make initial investments which imply negative cash inflows for a number of periods. With a positive interest rate these initial outlays will count heavily when the total sum over the horizon of the net discounted cash inflows is computed. This sum will become small compared to the corresponding sum for a firm already in the industry, perhaps even negative, in which case it clearly does not pay to enter. Thus it becomes important to analyze not only stationary points, but also the equilibrium paths toward these points.

II. A Dynamic Model of Oligopoly

We assume that the industry produces a homogeneous commodity, and that the inverse demand curve can be written in the form

\[ p_t = a_t - b \sum_{j=1}^{n} y_{jt} \]

where \( p_t \) is the price, \( a_t \) is a stochastic demand shift variable, \( b \) is a fixed parameter, \( y_{jt} \) is output (equal to sales) by firm \( j \) in period \( t \), and \( n \) is the number of firms in the industry. The price can be thought of as measured net of any constant unit production cost.

Output per period is assumed to depend on the capital stock at the start of the period. Without loss of generality we can choose units so that maximum

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1 For an early discussion, see Stigler (1947).
2 See e.g. Bain (1956) or Hall & Weiss (1967).
3 See Kamien & Schwartz (1971) and references cited therein.

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output is equal to the capital stock. Assuming also that each firm always uses all of its capacity, we can write

\[ y_{t+1} = (1 - \delta) y_t + x_{it}, \quad i = 1, \ldots, n, \]

where \( x_{it} \) is investment by firm \( i \) in period \( t \), and \( \delta \) is the depreciation rate.

The unit cost of investment is assumed to be \( q \) as long as the capital stock is maintained. For deviations from this investment rate \( \delta y_t \) we assume a quadratic cost of adjustment, \( c(x_{it} - \delta y_t)^2 \). This insures that we have constant returns to scale in the long run. The fact that the cost of adjustment depends only on the individual firm's own change in capacity means that these costs essentially can be regarded as internal to the firm. For instance, it may require more resources to increase capacity rapidly in one period than if the expansion is spread across several periods. An alternative (or additional) assumption could easily be used whereby the cost of adjustment would depend on the deviation of total industry investment from what is needed to maintain the industry capital stock. In any case, the cost structures of the firms are assumed to be the same. The problem formulations of each firm are therefore symmetrical in some sense.

Each firm is assumed to maximize the expected sum of net discounted cash inflows over the horizon:

\[ E\left\{ \sum_{t=1}^{\infty} \beta^{t-1} w_t(y_t, \alpha_t, x_t) \right\}, \]

where

\[ w_t(y_t, \alpha_t, x_t) = \left( \alpha_t - \beta \sum_{t=1}^{\infty} x_t \right) y_t - qx_t - c(x_t - \delta y_t)^2, \]

and \( \beta = 1/(1 + r) \), where \( r \) is the interest rate.

Randomness enters the model through the parameter \( \alpha_t \) of the demand function. We assume that \( \alpha_t \) is subject to a first-order autoregressive process given by

\[ \alpha_t = q\alpha_{t-1} + \mu + \varepsilon_t, \]

where \( -1 < q < 1, \mu > 0 \), and \( \varepsilon_t \) are random disturbances which are uncorrelated over time with mean zero and finite variance \( \sigma_\varepsilon^2 \).

In this paper we assume that the \( n \)-th firm is dominant, while the other \( n-1 \) firms behave noncooperatively, given what the dominant firm does. As is shown in Kydland (1978), the dominance of a single firm may be acceptable to the rivals in the sense of increasing their profits by providing an entry barrier. Many times, though, it is a fairly realistic institutional assumption that in each period the dominant firm publishes a price list, valid for the whole period, while the "small firms" then set their own prices on the basis of this price list.

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We now define an equilibrium in terms of policy or decision rules, that is, functions of the current state.

Definition: An equilibrium for each time period \( t, t = 1, \ldots, T \), is a set of decision rules \( x_{it} = x_{it}(y_{it}, a_{it-1}, x_{it-1}) \), \( i = 1, \ldots, n - 1 \), and \( x_{nt} = X_n(y_{nt}, a_{t-1}) \) such that

\[
\max_{x_{it}} E\left[w(y_{it}, a_{it}, x_{it}) + \beta v_{i,t+1}(y_{i,t+1}, a_{i,t+1}) \middle| X_{it}, j = 1, \ldots, n, j \neq i \right] \\
= E\left[w(y_{i,t}, a_{i,t}, x_{it}) + \beta v_{i,t+1}(y_{i,t+1}, a_{i,t+1}) \middle| X_{it}, j = 1, \ldots, n, i = 1, \ldots, n, \right]
\]

where

\[
v_{i,t+1}(y_{i,t+1}, a_{i,t+1}) = E \left[ \sum_{s=t+1}^{T} \beta^{t-s} w(y_{s}, a_{s}, x_{s}) \middle| x_{it} = x_{it}(y_{i,t}, a_{i,t-1}, x_{it-1}), i = 1, \ldots, n-1, \right]
\]

\[
x_{ns} = X_n(y_{n,t}, a_{t-1}), s = t+1, \ldots, T
\]

In the definition above, \( v_{i,t+1} \) denotes the value of firm \( i \) when all firms adhere to the equilibrium decision rules from period \( t+1 \) until the end of the horizon. In other words, each firm chooses the best decision rule for period \( t \), given the last observed state variables \( y_t \) and \( a_{t-1} \), the decision rules of the other firms, and that decisions will be similarly selected in periods \( t+1, \ldots, T \).

The equilibrium decision rules can be computed by backward induction as is explained in detail in Kydland (1977). In the case of infinite horizon the stationary decision rules would be of the form \( x_i = X_i(y, a_{-1}, x_n) \), \( i = 1, \ldots, n-1 \), and \( x_n = X_n(y, a_{-1}) \), which would satisfy a set of \( n \) functional equations, one for each firm. The same computational procedure could then be the basis for successive approximations by value iterations, where the solution for a \( T \)-period horizon would be used to compute the solution for a \( (T+1) \)-period horizon, and so on. An alternative computational procedure for obtaining the stationary decision rules would be policy iterations rather than value iterations.

For our model, in which the objective functions are quadratic and the constraints linear with additive disturbances, the equilibria are easily computable. The decision rules are all linear. The results reported in the next section were computed using \( T \) large enough to make the change in first-period decision rules very small when the horizon increases by one period.

An equilibrium should be required to be stable in the sense that there is a tendency for the decision rules to move toward the equilibrium decisions. It seems reasonable, in particular in a stochastic environment, for an equilibrium to be characterized by solutions in policy or decision rule space rather than sequence space. A property which makes the feedback solution above a reasonable candidate for an equilibrium is that if any one firm perfectly foresee the rival firms' decision rules and solves the control problem resulting when these rules are considered as constraints along with the relations (1) and (2), then its equilibrium decision rules turn out to be the optimal ones for this problem.
Moreover, if the decision rules start out away from equilibrium, but are modified, for example according to an adaptive process as more is learned about the other firms' behavior, then, under reasonable assumptions, this process will converge toward the equilibrium solution.\footnote{Similar processes were used in Kydland (1976) to investigate stability of a dynamic noncooperative model.}

An alternative solution would be obtained by using the variational approach where the decisions can be viewed as a sequence of points rather than decision rules. This solution, however, is inconsistent under replanning when there is a dominant firm,\footnote{See Kydland (1977) for a more detailed discussion of this solution concept and of a solution, characterized by policy rules, which is also inconsistent under replanning. A simple illustration may be useful. Assume that the horizon consists of two periods, $t-1$ and $t$, and that firm 2, the dominant firm, wishes to maximize $u_i(x_{i,t-1}, x_{i,t-1}, x_{2,t}, x_{1,t})$ subject to $x_{i,t-1} = X_{i,t-1}(x_{i,t-1}, x_{2,t})$ and $x_{2,t} = X_{2}(x_{i,t-1}, x_{2,t})$. Taking $x_{i,t-1}$ as given and differentiating with respect to $x_{2,t}$, we get:}

$$\frac{\partial u_i}{\partial x_{2,t}} + \frac{\partial u_i}{\partial x_{2,t}} \frac{\partial X_{2}}{\partial x_{2,t}} = 0.$$ \hfill (1)

However, taking account of the effect of $x_{2,t}$ on firm 1's decision in period $t-1$, we get:

$$\frac{\partial u_i}{\partial x_{2,t}} + \frac{\partial u_i}{\partial x_{2,t}} \frac{\partial x_{i,t-1}}{\partial x_{2,t}} = 0.$$ \hfill (2)

Only if firm 2's decision in period $t$ has no effect on firm 1's decision in period $t-1$ will the solution implied by the first equation be optimal for the two-period horizon as a whole.

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taking into account the reaction functions of the firms on lower levels, and
taking as given the decisions of the higher-level firms. An interesting special
case would have one firm on each hierarchical level of decreasing dominance.
These firms would all end up with different market shares in the stationary
equilibrium. Thus our model could be consistent with the more general distribu-
tions of market shares that are observed in many industries.

III. Some Comparisons of Dynamic Equilibrium Solutions

In this section we compute some equilibrium solutions and check the consistency of the results with empirical facts of industry structure. We also deal with the question of what determines the number of firms in the industry and look at the characteristics of the solutions when certain parameters change.

Some simplifying assumptions are made for these examples. We abstract from foreseen growth in demand, but sometimes investigate the effects of unforeseen jumps in demand which are then perceived by the firms as permanent. We also assume that the firms in an industry with a given number of firms do not take the threat of entry into account when making their decisions. This assumption may be somewhat unrealistic, but is certainly less unsatisfactory, the more firms there are in the industry. If a new firm actually does enter, the decision rules are then modified to the new equilibrium rules for an industry with one more firm.

To some extent we are interested in points which are stationary in the stochastic sense that the variables are at these points on the average while fluctuating around them as demand fluctuates. Sometimes, however, the equilibrium path toward the stationary point will also be important, especially when a new firm enters, or when the demand shifts permanently. This is the main reason why we rely on examples enabling us to compute these paths.

The basic example will have the following values for the parameters: \( q = 2 \),
\( c = 5 \), \( \delta = 0.1 \), \( r = 0.1 \), \( \mu = 0.2 \), \( \sigma = 0.8 \) and \( \sigma_z = 0.05 \). The values for \( \mu \), \( q \) and \( \sigma_z \) imply an average for \( a \), of one and a standard deviation of 0.083. The relative values of the per unit investment cost \( q \) and the cost of adjustment factor \( c \) should be reasonable. For instance, assume that a firm with capital stock of 0.2 wants to increase its capacity by 5% in one period. While the cost per unit of maintaining its capital stock is two, we now have to add the amount 0.0005 to the total investment expenditures for this period. If we make this out in per unit terms, the total per unit cost is 2.017, or less than a 1% increase over the normal cost of maintaining the capital stock. This does not seem overly much considering the sizeable increase in capacity. With a 10% increase in capacity the per unit cost increase over normal cost would be 2.5%.

Some results for the numerical example are presented in Table 1. In addition to output (equal to capital stock) and price for stationary solutions, the expected present values of the firms are computed for two alternative starting
points. The first assumes that the firms have already reached the stationary point (thus starting with capital stocks 0.2044 and 0.2348 in the case of duopoly), while the other computation is for the case of one firm just entering the industry (starting at 0 and 0.3 in the case of duopoly).

Bearing in mind the simple static model presented in the previous section, yielding the stationary point of the variational approach as a solution, at least two aspects of these results are striking. While the static model predicts the output of the dominant firm to remain the same regardless of the number of firms entering and the market share to approach 50%, we now see that the dominant firm output at the stationary level has decreased from 0.3 in a monopoly to 0.1473 when five additional firms have entered, and the market share has decreased to slightly more than one-fourth. This is even more striking since, as we pointed out, the cost of adjustment is quite small. We also see that the present value of what a firm can earn as it is about to enter the industry is much less than the present value after the stationary level of capital stock has been reached. The relative difference between the two present values becomes larger, the larger the number of firms already in the industry.

For the sixth firm the present value is only 0.0218 as compared to 0.2187 at the stationary level. Note that we have assumed that the investment made in any one period does not yield any productive services until the next period. In some industries longer lags are probably realistic, in which case these differences are likely to become even more dramatic.

The description of several industries over three or four decades in Burns (1936) gives the impression that a typical pattern of development is one where the output of dominant firms increases with increasing demand while at the same time a steady decline in market share is experienced. We shall see that such a development is possible without the dominant firm losing its dominance as we have defined it.

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Table 2

<table>
<thead>
<tr>
<th>Number of firms in industry</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stationary solution</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output of dominant firm</td>
<td>.2288</td>
<td>.2025</td>
<td>.1845</td>
<td>.1718</td>
</tr>
<tr>
<td>Output per rival firm</td>
<td>.1763</td>
<td>.1386</td>
<td>.1186</td>
<td>.0961</td>
</tr>
<tr>
<td>Market share of dominant firm</td>
<td>.3945</td>
<td>.3276</td>
<td>.2888</td>
<td>.2335</td>
</tr>
<tr>
<td>Price of output</td>
<td>.5176</td>
<td>.4818</td>
<td>.4610</td>
<td>.4478</td>
</tr>
<tr>
<td>Path toward stationary solution</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected value of entering firm</td>
<td>.1450</td>
<td>.0762</td>
<td>.0454</td>
<td>.0295</td>
</tr>
</tbody>
</table>

For simplicity we assume that the situation described in Table 1 is disturbed by an unexpected jump in demand, which the firms then correctly perceive as permanent. Thus, the average intercept $\bar{a}$ with the price axis increases from 1 to 1.1, keeping the slope of the demand curve constant. Some results for this case are given in Table 2.

If the number of firms were to remain unchanged in the new situation, at say 5, all the firms' stationary outputs would have increased, with market shares being the same as before the demand shift. However, in the new situation the value of a sixth firm entering has increased from 0.0218 to 0.0295. This may now encourage a new firm to enter, in which case the new stationary equilibrium is characterized by the dominant firm producing 0.1718 compared to 0.1582 before the demand shift, although its market share has dropped from 0.2888 to 0.2635. We also see that at first the price will increase, and then eventually drop below its original level.

Since the cost of adjustment associated with changes in capacity is the basic feature which makes our model dynamic, it may be of interest to ascertain how sensitive our results are to changes in the cost-of-adjustment factor $c$. It turns out that if the cost of adjustment increases, ceteris paribus, the market share of the dominant firm becomes smaller. If, for instance, the parameter $c$ is doubled, the stationary output of the dominant firm in a five-firm industry is 0.1430 instead of 0.1582, while the output of each of the four other firms is 0.1010 instead of 0.0974. We also found that higher costs of adjustment made the present values of the stationary solution for the nondominant firms higher. However, at the same time it became less profitable for a new firm to enter. With an increase in $c$ from 5 to 10, the present values of the stationary solutions increased from 0.2708 to 0.2816 in a five-firm industry, while the present value of the entering firm decreased from 0.0335 to 0.0286. This shows that it is important to know something about the equilibrium path toward

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1 This result appears to be consistent with the empirical finding of Sherman (1971) that firms tend to be more equal in size when entry barriers are high.

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the stationary level of capital stock when evaluating the capital requirement barrier to entry in the presence of cost of adjustment.

It has been suggested that industry demand typically becomes more elastic over time; see e.g. Scherer (1970, pp. 213–216). In the present model, in which the demand function is approximated by a linear curve, a reasonable way to investigate this possibility is to select the stationary point on the demand curve for a given number of firms and then reduce both the slope parameter $b$ and the average of the demand shift variable $a$, so as to make the new demand curve go through the same point on the average. This was done with a reduction in $b$ from 1 to 0.9 for a number of combinations of the remaining parameters, and unlike the static model in the previous section, the result was always a decrease in the market share of the dominant firm.

For our model, the expected present value of each firm is higher, the larger the variance in demand, although the contribution of this factor is very small in the examples above. The possibility, however, of unfavorable drawings from the distribution of $\varepsilon_1$ just after entry, resulting in losses which can be difficult to make up for later due to the positive interest rate, may be a deterrent to prospective entrants. For example, the sixth firm entering the industry, the results of which are reported in Table 1, has an expected value of 0.0218. But the probability of a negative value is approximately 0.05. In general, even though the higher variance of demand increases the expected value of the entering firm, there is at the same time a higher probability of a negative value.

IV. Concluding Comments

In this paper we have provided an equilibrium framework for industry structure consistent with certain observed persistent differences in market shares within industries, with the market share of the dominant firm typically declining slowly over time. Some essential features of the model are the assumption that one firm is dominant in the sense of taking into account the rival firms’ reactions to its decisions, and the assumption of increasing costs associated with changes in capacity, thus introducing structural interconnections over time which make the model inherently dynamic. The cost structure was assumed to be the same for all firms. The predictions of this model turn out to be quite different from those of the corresponding static model. While that model would predict the dominant firm output to remain constant as more firms enter the industry and the market share to approach 50%, we found that the equilibrium steady-state output for the dominant firm declined substantially as more rivals entered the industry. We also found the market share of the dominant firm to decline when the cost of adjustment increased relative

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1 As was pointed out, the static model can be viewed as the steady-state version of the dynamic model when variational methods are used.

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to the long-run unit cost of investment, and when the elasticity of demand increased.

Determining an appropriate equilibrium concept for a dynamic dominant
firm model involves some rather unusual problems. An important considera-
tion is the stability of the equilibrium in the sense that there is a tendency
in the economy toward this equilibrium—also when the equilibrium is char-
acterized by dynamic decision rules as is the case in the present paper. We
argued that our equilibria are likely to be stable in this sense, given that there
is a dominant firm. In Kydland (1978) we have also shown that the very
existence of a firm which is dominant rather than behaving noncooperatively
along with the others, may provide a barrier to entry, thus making its domin-
ance acceptable to the rivals. These results provide a justification for the
dominant firm equilibrium concept used in this paper. As we have indicated,
our results are quite consistent with the stylized facts of many industries. The
next step is to subject the model to more careful empirical testing.

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