

THE WORKWEEK OF CAPITAL AND ITS CYCLICAL IMPLICATIONS*

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The neoclassical growth model studied in the Kydland and Prescott 'Time to Build' paper is modified to permit the capital utilization rate to vary. The effect of this modification is to increase the amplitude of the aggregate fluctuations predicted by theory as the equilibrium response to technological shocks. If, following Solow, the changes in output not accounted for by changes in the labor and tangible capital inputs are interpreted as being the technology shocks, the statistical properties of the fluctuations in the postwar United States economy are close in magnitude and nature to those predicted by theory.

1. Introduction

In Kydland and Prescott (1982), the competitive neoclassical growth model with shocks to technology is studied. Leisure was introduced as an additional argument affecting preferences and labor services as an input to the aggregate production function. This artificial economy was calibrated so that its equilibrium would display the growth observations of Kuznets (1966). In particular, its average consumption and investment shares of output match those of the postwar U.S. economy as do the capital-output ratios and factor shares. Further, the allocation of time between market and nonmarket production activities is consistent with household studies and does not change secularly as technology advances.

Services from inventory stocks were introduced as a factor of production and a production period for new capital consistent with micro observations assumed. We found that the competitive equilibrium for this economy displays fluctuation in output having amplitude and serial correlation properties similar

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to those of the U.S. economy. The relative variability of both consumption and investment matched the data. Further, the majority of the variability in output was accounted for by variability in the labor input. This exercise led us to the conclusion that if, following Solow (1957), residuals in the aggregate production function are interpreted as shocks to technology, there would be a puzzle if industrial market economies did *not* display business cycle fluctuations.

For both the neoclassical growth model and postwar American economies capital stocks do not fluctuate with output. At the business cycle frequencies these variables are nearly orthogonal. But the services of capital need not be equal to the capital stock as for the model that we studied. The utilization of capital can and surely does fluctuate systematically with output and employment. A factory can be operated two shifts rather than one or operated six rather than five days a week. If hours of labor services are proportionate to the workweek of capital, increases in labor services do not reduce the ratio of capital services to labor services.

The question that is addressed in this paper is: how does the nature of the fluctuations predicted by the stochastic growth model change if the workweek of capital is permitted to vary? The principal finding is that fluctuations are larger and the stochastic growth model better mimics the behavior of the American economy.

In section 2, the economy that we study is specified. In section 3, we show that by defining workdays of different lengths to be different commodities, the economy can be put into the Debreu (1954) general equilibrium framework. The competitive equilibrium is found by supporting the Pareto optimum. The model is calibrated in section 4 and findings are reported in section 5. Section 6 contains a concluding comment.

2. The economy

2.1. Preferences

The stand-in household's preferences are ordered by

$$E \sum_{t=0}^{\infty} \beta^t u \left(c_t, \sum_{i=0}^{\infty} \alpha_i l_{t-i} \right),$$

where $c_t \geq 0$ is date t consumption, $l_t \geq 0$ is date t leisure, $0 < \beta < 1$ is the discount factor, and E is the expectation operator. The parameters α_i are constrained to sum to 1 and $\alpha_{i+1}/\alpha_i = 1 - \eta$ for $i = 1, 2, \dots$. Thus, $0 < \alpha_0 \leq 1$ and $0 \leq \eta \leq 1$ determine the values of the α_i . The case of $\alpha_0 = 1$ corresponds to a standard time-separable utility function. Theoretically, of course, α_0 could

be greater than one, implying that leisure choices in neighboring periods are complements, not substitutes. Justifications for so restricting α_0 to be less than one and for this distributed lag of leisure based upon household production theory with household capital can be found in Kydland (1983) and evidence based on panel data in Hotz, Kydland and Sedlacek (1985).

The period utility function is assumed to have the form

$$u(c_t, \alpha(L)l_t) = [c_t^\mu (\alpha(L)l_t)^{1-\mu}]^\gamma / \gamma,$$

where $\gamma < 1$ and $\gamma \neq 0$. To summarize, β , α_0 , η , γ and μ are the parameters of preferences. The unit elasticity of substitution between c_t and $\alpha(L)l_t$ was dictated by the requirement that there be no secular change in $\alpha(L)l_t$ associated with secular increase in productivity. The household endowment of time per period is normalized to be one.

The utility function can also be represented as follows:

$$u(c_t, \alpha(L)l_t) = u(c_t, \alpha_0 l_t + \eta(1 - \alpha_0)a_t),$$

where

$$a_{t+1} = (1 - \eta)a_t + l_t.$$

This alternative formulation permits the application of recursive methods.

2.2. Technology

Aggregate output is constrained by

$$f(z_t, h_t, k_t, y_t) = [(z_t h_t^\psi k_t^\theta)^{-\nu} + \sigma y_t^{-\nu}]^{-1/\nu},$$

where k_t is the productive stock of physical capital other than inventories, y_t the stock of inventories, and z_t the date t technology shock. Restrictions on the parameters are $0 < \theta < 1$, $\sigma > 0$, $\nu > -1$ and $1 - \theta < \psi < 1$. Because $\psi + \theta > 1$, there are apparent increasing returns. With $\psi = 1 - \theta$, the only margin over which hours can be varied is the number of workers operating a given quantity of capital. With $\psi = 1$, the only margin is the length of the workday of capital and labor.

We think this captures in a reasonable way the option to vary the capital utilization rate. One way to increase output is to operate a given plant more hours per week, say, by operating it six rather than five days per week or two rather than one shift per day. Another way is to put into operation a marginal plant or machine. Such adjustments on average are probably associated with

some decrease in the instantaneous capital-output ratio as variations in output are typically born by the less capital-intensive plants. The third margin is to increase the number of workers per plant. Ex post substitution opportunities are surely smaller than ex ante opportunities but they are not zero. These considerations are what led us to select this technology.

The production function differs from the one we used previously in another important respect. For the above CES production function, capital and labor are combined to form a composite input and that composite input is combined with inventories to produce goods and services. Previously, k_t and y_t were nested to form a composite capital service which in turn was combined with the labor input to produce output. We think these assumptions made here match better with micro observations. They also result in inventories being a procyclical variable, rather than moving countercyclically, in conformity with observations.

Total output is allocated to noninventory investment i_t , inventory investment $y_{t+1} - y_t$ and consumption c_t :

$$i_t + c_t + y_{t+1} - y_t \leq f(z_t, h_t, k_t, y_t).$$

New productive capacity is not built in a period, which we take to be a quarter of a year. The construction period is J quarters, with ϕ_{j+1-j} being the fraction of the value put in place in the j th period of construction for $j = 1, \dots, J$. Let s_j for $j = 1, \dots, J$ be the total value of projects (that is, the sum of value put into place at each of the j stages) that are j periods from completion. Then

$$i_t = \sum_{j=1}^J \phi_j s_{jt}.$$

Note that the ϕ_j sum to one and are positive as they are fractions.

The capital stocks that index the effect of past production decisions upon subsequent production possibilities are s_1, \dots, s_{J-1} , k_t and y_t . Productive capital is assumed to depreciate geometrically at rate $0 < \delta < 1$, so

$$k_{t+1} = (1 - \delta)k_t + s_{1t}.$$

Further,

$$s_{j,t+1} = s_{j+1,t} \quad \text{for } j = 1, \dots, J-1.$$

The latter set of constraints is that projects $j+1$ stages from completion are j stages from completion next period.

The shock to technology z_t is the sum of a persistent component z_{1t} and a transitory component z_{2t} ,

$$z_t = z_{1t} + z_{2t} + \bar{z}.$$

The process on z_t is first-order autoregressive with

$$z_{1,t+1} = \rho z_{1t} + \xi_{1,t+1}, \quad 0 < \rho < 1,$$

while

$$z_{2,t+1} = \xi_{2,t+1}.$$

Further, z_t is observed only with error at the beginning of the period. Observed is

$$\pi_t = z_t + \xi_{3t}.$$

Decisions on employment h_t and new capital projects s_{jt} are made contingent on π_t and the previous history of the z_t . Output is then observed and z_t deduced. Then the consumption-inventory carry-over decision is made contingent upon z_t as well as the history of shocks. The vectors ξ_t are iid normal with mean zero and diagonal covariance. With this assumption, the conditional expectations of z_{1t} and z_{2t} based upon available information are sufficient, relative to the history, for decision making. Further, the conditional expectations are linear functions of their previous value and the latest observation whether it be π_t or z_t . For further details, see Kydland and Prescott (1982).

3. Supporting the Pareto optimum as a competitive equilibrium

The technology appears to display increasing returns to scale and does, if the obvious commodity point is employed – namely, the one for which hours and capital are inputs. Because competitive equilibria fail to exist with increasing returns, an alternative commodity point is needed if we are to use competitive analysis. Our approach is to assume that capital is specific to the household; that is, only workers can effectively operate their own equipment. Then we treat hours-capital pairs as being different commodities. A commodity of type (k, h) provides

$$zh^\psi k^\theta$$

units of a composite input. As there is a continuum of commodities, measures are needed to specify the quantities of all these different commodities. See Mas-Colell (1975) and Jones (1984) for competitive theory with measures as commodity points.

The use of measures permits us to apply competitive equilibrium theory to the study of this economy. A person who works h hours utilizing k units of capital is a type (h, k) worker. Measure $m(A)$ specifies the measure (number)

of type (h, k) workers in the measurable set A . The output of the economy at date t is

$$\left[\left(z \int h^\psi k^\theta dm \right)^{-\nu} + \sigma y^{-\nu} \right]^{-1/\nu}.$$

If the set of possible values of (h, k) pairs is discrete, output is

$$\left[\left(z \sum_h \sum_k h^\psi k^\theta m_{hk} \right)^{-\nu} + \sigma y^{-\nu} \right]^{-1/\nu},$$

where m_{hk} is the number of people who work h hours using k units of capital. This is a constant returns to scale production function with inputs m_{hk} and y . This result does not depend upon the set of possible (h, k) pairs being finite.

Let $x = (k, s, y, a)$ denote the vector of the state variables affected by the decisions of agents. Let $R(x, x_{t+1}, z_t)$ be the maximal obtainable current utility given the technology constraints. This entails maximizing $u(c_t, \alpha_0 l_t + \eta(1 - \alpha_0)a_t)$ over measures m with the property that their total mass is one and they place mass upon a single point (h, k) . Note $h = 1 - l_t$ and $k \leq k_t$. In other words, hours worked is the time endowment less leisure while capital services supplied, k , are constrained by the existing stock of capital.

Let value functions v_n be defined recursively as follows:

$$v_{n+1}(x, z) = \max_{x' \in \Gamma(x, z)} \left\{ R(x, x', z) + \beta \int v_n(x', z') \pi(z, dz') \right\},$$

given $v_0(x, z) = 0$. Here $\pi(z, A)$ is the probability $z_{t+1} \in A$ given $z_t = z$ and the convex set $\Gamma(x, z)$ specifies the set of feasible x' given x and z . Our computational procedure requires that for each n the maximization problem be concave in x' and $v_{n+1}(x, z)$ be concave in x . Otherwise, it may not be optimal for the ex ante identical agents to remain identical.

To find the competitive equilibrium allocation, the Pareto optimal allocation which maximizes the average utility of the identical agents is computed. A candidate price system is found from the marginal conditions. As this candidate assigns finite value to all points in the commodity space, it is a valid price system.

4. Calibrating the economy and computing equilibrium

To determine, for a set of parameter values, the stochastic difference equations describing the equilibrium for this model, we exploit the result that the equilibrium allocation can be found by solving the stand-in utility maxi-

zation problem described in the previous sections. The aim is to determine the business cycle variance-covariance properties of the model and to contrast them with data for the postwar U.S. economy. Due to the form of the utility function and the nonlinearity of the resource constraint, this is infeasible for the original formulation of the model. Instead, we study the approximate economy which is found by making a quadratic approximation around the steady state of the exact model. The steady state is also used for the purpose of calibrating the model in the sense of making it consistent with average relations from national income and product accounts and with various micro observations. For these reasons, we first turn to the determination of the steady state.

4.1. Steady state

We let variables without time subscripts denote the steady state. It is found by setting z_t equal to its unconditional mean, which is denoted by z , in every period. The parameter z is a scale parameter and is chosen so that the steady state output equals one.

The parameters that play a role for the steady state are β , γ , μ , α_0 and η from the utility function and δ , z , θ , ψ , σ , ϕ_1, \dots, ϕ_J , ρ and ν from technology. The variables of the model are consumption c , investment plans s which in the steady state are equal to actual investment, inventories y , capital stock k , hours h and the real interest rate r . For each set of parameter values, a steady state results. Thus, if we denote by ω the vector of parameters and let S be the vector of variables, then one can determine a mapping $S(\omega)$.

For our purpose, the inverse mapping is of particular interest, that is, the mappings from part of the vector S or functions thereof to a subset of the parameters. The state vector typically consists of variables for which there are long-run relations in the data that have been essentially constant throughout the sample period. Examples are consumption-output, inventory-output and capital-output ratios and households' average allocation of time to market activity.

The steady state interest rate will, in general, have a close relationship to β and the rate of growth of consumption. Abstracting from growth, however, we have

$$r = (1 - \beta)/\beta. \quad (1)$$

From the condition $f_y = r$ and the fact that steady state output is one, it follows that

$$\sigma = ry^{\nu+1}. \quad (2)$$

Letting consumption be the numeraire, the steady state relative price of

services from capital is

$$q = (r + \delta) \sum_{j=1}^J (1+r)^{j-1} \phi_j.$$

Steady state capital stock then follows from the relation

$$\theta = qk / (1 - \sigma y^{-\nu}), \quad (3)$$

where the capital-output ratios are the averages for the postwar U.S. economy. Given the depreciation rate, we have $s = \delta k$. In the steady state, there is no inventory accumulation. Thus, when s is determined, steady state consumption is total output minus this quantity. We can now also determine from the production function the value of z that will make steady state output equal one as follows:

$$z = [(1 - \sigma y^{-\nu})^{1/\nu} h^\psi k^\theta]^{-1}.$$

Turning to the consumer's problem, recall that we already implicitly know c and all that is left to be determined from utility maximization is h . Without the dependence of current utility on past leisure choices, its value would have followed from the condition $u_l/u_c = f_h$. In the present case, however, the relevant first-order condition can be written as

$$\mu f_h^t / c_t = (1 - \mu) \sum_{j=0}^{\infty} \beta^j \alpha_j [\alpha(L) l_{t+j}]^{-1} \quad \text{for all } t,$$

where

$$f_h^t = \psi (z_t k_t^\theta)^{-\nu} h_t^{-\psi\nu-1}.$$

In the steady state, $c_t = c$, $l_t = l$ and $f_h^t = f_h$ for all t . Using the facts that α_j sum to one, that $\sum_{j=0}^{\infty} \beta^j \alpha_j = (\alpha_0 r + \eta) / (r + \eta)$ and that $l = 1 - h$, this expression can be written as

$$(1 - \mu)(\alpha_0 r + \eta)c / (r + \eta) = (1 - h)f_h. \quad (4)$$

4.2. Calibration

The parameters can be divided into two groups, those that affect the steady state vector a great deal and those that do not. The growth observations are used to narrowly restrict the first set of parameters. The parameters that affect

one or more variables in the steady state a lot are μ , θ , σ and β . In particular, as is evident from relations (1)–(4) above, there are close relationships between the parameter–variable combinations β and r , σ and y , θ and k (and therefore s) and μ and h .

Some of the remaining parameters are determined from micro studies of firm and household behavior. The quarterly depreciation rate δ is set equal to 0.025 on the basis of micro studies. The resource allocation to new capital is assumed to be uniform over the construction period, and so all the ϕ_i 's are set equal to $1/J$. The value of J is set equal to three. This is one quarter less than what was assumed in Kydland and Prescott (1982). Given that equipment and consumer durables are considered part of the capital stock, we think this represents better calibration as the construction periods for these durables are significantly less than a year.

Previous studies imply a narrow range of possible values for the curvature or risk-aversion parameter γ in the utility function. We use -0.5 , which implies slightly more curvature than for the logarithmic utility function. In Kydland and Prescott (1982), the values $\alpha_0 = 0.5$ and $\eta = 0.1$ were used and are used here. The selection of these values has recently received support from micro studies.¹

The parameter ν , which determines elasticity of substitution between inventories and the composite capital–labor input, is not narrowly restricted. We did not find good measures of this parameter in the literature. Observations at the firm technology level, however, suggest limited substitution opportunities, and consequently we restrict this parameter to be large. Admittedly, if good measurement found that this parameter is in fact smaller, the implications of theory would change some.

The parameter ψ , which determines to what extent the workweek is used as the margin over which to vary hours, is crucial to this model. Without variability in the workweek, this parameter is of course restricted to $1 - \theta = 0.64$. We have no good measure of this parameter for the U.S. economy, but clearly both margins are operative to a considerable extent. Thus, we chose the value of 0.82, which is half way between the two extremes of 0.64 and 1.²

The magnitude of the variance of the exogenous technology shocks and their serial correlation properties are measured by examining the statistical properties of changes in aggregate output not accounted for by changes in the inputs. The autoregressive coefficient ρ in the law of motion for the permanent part of z_t is fixed at 0.95. The values for the standard deviations of the permanent, transitory and indicator shocks, respectively, measured as percentages of the mean of z_t , are 0.0076, 0.0015 and 0.0076. The implied variance of the

¹See Hotz, Kydland and Sedlacek (1985).

²The value of 0.82 is also close to the largest one for which our computational methods were applicable when the nonseparable utility function is used.

technology shock is approximately equal to that obtained by Prescott (1986) using Solow growth accounting to obtain a z_t sequence.³ It is worth noting that permitting the workweek of capital to vary systematically with hours of employment affects hardly at all the estimate of the variance of the technology shock. Prescott (1986) finds that its value is reduced from 0.763 percent to only 0.759 percent when ψ is increased from 0.64 to 0.85. With $\psi = 0.85$, the estimated standard deviation of the shock is at its smallest for any ψ .

We now return to the parameters determined by growth observations. With a quarterly real interest rate of 1 percent, a value for β of 0.99 is implied. The average value for h is 0.3 based upon the micro finding that households allocate about 30 percent of their productive time allocation to market production.⁴ The implied value of the parameter μ varies somewhat depending on the model version. For the case of intertemporally nonseparable utility in leisure, for example, the value of μ is 0.325 when the production function parameter $\psi = 1 - \theta = 0.64$, while its value becomes 0.274 in the example with $\psi = 0.82$. The average inventory-output ratio is approximately 1 when output is measured quarterly. This yields a value of σ of 0.01.

To determine capital's factor share, we added depreciation of capital to rental, corporate and interest income. These national income account numbers were augmented by estimates of depreciation and implicit rental income on durables owned by the household. The results of these calculations for 1977, which is a typical year, was a factor share for capital of 36 percent. Besides implying $\theta = 0.36$, this share implies 10 for the quarterly capital-output ratio.

4.3. Computational issues

Given the steady state, the remaining steps in the computation of equilibrium are similar to those described in Kydland and Prescott (1982). To deal with nonlinearity in the resource constraint, we substitute for c_t from the resource constraint in the utility function and then make a quadratic approximation around the model's rest point. As the remaining constraints are linear, it is then fairly straightforward to determine recursively the decision rules that describe the equilibrium behavior of the approximate economy. The decision variables are h_t , s_{jt} , y_{t+1} and c_t , of which c_t is determined from the resource constraint. The equilibrium can be represented by a linear stochastic system of difference equations consisting of constraints determining next period's capital stocks, the decision rules which are functions of a state vector that includes conditional expectations of the unobserved permanent and

³ Nelson and Plosser (1982, p. 153, fn. 15) report that a random walk process mimics well in a formal statistical sense the technology shocks that Solow (1957) obtains.

⁴ For males, the figure is close to 40 percent. See Ghez and Becker (1975, p. 95).

Table 1

Cyclical behavior of the U.S. economy: Deviations from trend of key variables, 1954:1-1982:4.^a

Variables x	Std. dev.	Cross-correlation of output with				
		$x(t-2)$	$x(t-1)$	$x(t)$	$x(t+1)$	$x(t+2)$
Gross national product	1.8%	0.57	0.82	1.00	0.82	0.57
Personal consumption expenditures						
Services	0.6	0.53	0.66	0.72	0.61	0.52
Nondurable goods	1.2	0.54	0.71	0.76	0.59	0.35
Durable goods	5.0	0.60	0.75	0.81	0.61	0.34
Fixed investment expenditures	5.3	0.59	0.78	0.89	0.78	0.56
Nonresidential investment	5.2	0.28	0.54	0.79	0.86	0.79
Structures	4.6	0.23	0.42	0.62	0.70	0.68
Equipment	6.0	0.27	0.56	0.82	0.87	0.78
Capital stocks						
Total nonfarm inventories	1.7	-0.13	0.15	0.48	0.68	0.74
Nonresidential structures	0.4	-0.36	-0.20	-0.03	0.16	0.32
Nonresidential equipment	1.0	-0.26	-0.03	0.23	0.41	0.54
Labor input						
Nonfarm hours	1.7	0.25	0.57	0.85	0.89	0.75
Productivity (GNP/hours)	1.0	0.60	0.51	0.34	-0.04	-0.28

^aSource: Prescott (1983) who used the Citibank Data Base.

transitory technology shocks, and the linear relations for updating these conditional expectations from the most recent observations.⁵

5. Findings

Given a set of parameter values and variances of the three shocks, sample series can be generated from the model. These series are decomposed into a cyclical and a growth component using the same method as for the data.⁶ In both cases, several statistics are computed and used for comparison. They are autocorrelations of output of order up to 5, standard deviations of the cyclical components of all the variables and their correlations with cyclical output.⁷ The U.S. statistics, using quarterly data for the 116-quarter period 1954-82, are reported in table 1. Using the model, 50 repeated samples of length 116 quarters are drawn, and averages and standard deviations are computed for each of the statistics.

⁵Note that investment fluctuations are never large enough to make gross investment zero, so that it is not an issue whether to permit conversion of existing capital back to consumption goods.

⁶See Kydland and Prescott (1982, fn. 15).

⁷Available upon request is the complete set of statistics examined for the model economy as well as for studied variants of it. The appendix reports the economy's equilibrium set of stochastic difference equations.

Table 2
Cyclical behavior of the Kydland–Prescott economy with fixed workweek.^a

Variables x	Std. dev.	Cross-correlation of output with				
		$x(t-2)$	$x(t-1)$	$x(t)$	$x(t+1)$	$x(t+2)$
Output	1.45% (0.16)	0.45 (0.09)	0.71 (0.06)	1.00 (—)	0.71 (0.06)	0.45 (0.09)
Consumption	0.48 (0.06)	0.30 (0.08)	0.60 (0.05)	0.92 (0.02)	0.77 (0.05)	0.60 (0.09)
Investment	4.38 (0.46)	0.39 (0.10)	0.59 (0.08)	0.88 (0.03)	0.83 (0.02)	0.51 (0.07)
Inventory stock	1.13 (0.09)	0.31 (0.09)	0.50 (0.08)	0.79 (0.04)	0.67 (0.05)	0.22 (0.11)
Capital stock	0.40 (0.06)	-0.34 (0.10)	-0.21 (0.08)	-0.07 (0.07)	0.17 (0.04)	0.53 (0.03)
Hours	0.86 (0.09)	0.42 (0.10)	0.63 (0.07)	0.93 (0.01)	0.69 (0.04)	0.38 (0.08)
Productivity (output/hours)	0.72 (0.06)	0.38 (0.08)	0.68 (0.05)	0.90 (0.02)	0.60 (0.08)	0.45 (0.10)
Real interest rate (annual)	0.17 (0.02)	0.47 (0.10)	0.69 (0.07)	0.85 (0.09)	0.58 (0.12)	0.42 (0.08)

^a These are the means of 50 simulations, each of which was 116 periods long. The numbers in parentheses are standard errors.

We examine the statistical properties of the model economy and compare them with those of the postwar U.S. economy which are presented in table 1. In particular, we want to know how much of the variability of real GNP and other key aggregate time series is accounted for by technological shocks whose magnitudes are of the same magnitude as the residuals of the aggregate production function. We also report statistics measuring comovements with the cycle, both contemporaneous and lead-lag relationships.

In table 2, the statistics for the model economy with $\psi = 0.64$, that is, with no variability in the workweek, are presented. This is the version of the current model which is close to the one in Kydland and Prescott (1982) with the main difference being the way in which inventories enter the production technology. With technology shocks calibrated to correspond to measured technology shocks, this economy accounts for roughly 80 percent of the variability of output. In table 3, the case of $\psi = 0.82$ is presented. For it, the capital utilization rate varies directly with the labor utilization rate.

5.1. The major finding

Introducing a variable workweek of capital results in significantly larger output fluctuations in response to fluctuations in the Solow technology param-

Table 3
Cyclical behavior of the modified Kydland–Prescott economy with variable workweek.^a

Variables x	Std. dev.	Cross-correlation of output with				
		$x(t-2)$	$x(t-1)$	$x(t)$	$x(t+1)$	$x(t+2)$
Output	1.79% (0.19)	0.39 (0.10)	0.65 (0.07)	1.00 (—)	0.65 (0.07)	0.39 (0.10)
Consumption	0.56 (0.07)	0.26 (0.08)	0.57 (0.05)	0.90 (0.03)	0.74 (0.06)	0.59 (0.09)
Investment	5.27 (0.56)	0.41 (0.10)	0.57 (0.08)	0.89 (0.03)	0.80 (0.03)	0.43 (0.08)
Inventory stock	2.02 (0.34)	0.25 (0.06)	0.18 (0.14)	0.60 (0.08)	0.52 (0.05)	0.00 (0.13)
Capital stock	0.57 (0.08)	-0.33 (0.08)	-0.06 (0.10)	-0.10 (0.06)	0.08 (0.07)	0.56 (0.03)
Hours	1.12 (0.12)	0.36 (0.11)	0.56 (0.09)	0.95 (0.01)	0.59 (0.07)	0.29 (0.09)
Productivity (output/hours)	0.81 (0.07)	0.35 (0.09)	0.66 (0.05)	0.90 (0.02)	0.62 (0.08)	0.46 (0.10)
Real interest rate (annual)	0.25 (0.05)	0.38 (0.11)	0.64 (0.09)	0.64 (0.20)	0.43 (0.16)	0.36 (0.08)

^a These are the means of 50 simulations, each of which was 116 periods long. The numbers in parentheses are standard errors.

eter. It is true that when the model economy with a variable workweek is calibrated, the variance of the technology shock is slightly smaller than for the economy without this feature. But this is more than offset by the larger fluctuations induced by a shock of a given magnitude. The volatility measure of output is 1.79 for the calibrated variable-workweek economy while it is only 1.45 for the fixed-workweek economy. The value of the corresponding statistic for the postwar U.S. economy is 1.8.

5.2. Behavior of inventories

Inventory investment is a highly volatile component of output and contributes disproportionately to total output variability. In making comparisons between inventory statistics for the model and the U.S. economy it is important that inventories be measured in a similar way for the two economies. The U.S. data on inventories clearly include a significant portion of what in our model is physical capital under construction as our measure of capital includes consumer durables, such as automobiles, and equipment as well as structures. To measure inventories in a way closer to the actual data, some fraction of these goods in process should be added to inventories. We picked

one-half, which implies in steady state that goods in process are one-eighth of all inventories.

With this definition of inventories, the effect of a variable workweek is to increase the volatility of inventories from 1.13 to 2.02, which is even higher than the 1.7 figure for the U.S. data. Even though on average goods in process are only one-eighth of total inventory stocks, this component matters a lot. Without this component, volatility of inventories for the model economy with a variable workweek would be only 0.67 rather than 2.02. It is important to note that our use of the expression *goods in process* does not correspond to reported goods in process. With reported statistics, finished goods of an independent auto part manufacturer are not included in goods in process while they are for parts divisions of auto manufacturers.

As can be seen from the tables, the cross-correlations between output and the inventory stock are much closer to the U.S. data for the variable-workweek economy than for the fixed-workweek economy. With the workweek variable inventory stocks, like in the data, lag output fluctuations.

5.3. Hours behavior

Hours variability for the model is 1.12 which is substantially less than the 1.7 figure for the U.S. data. Another way to express this discrepancy is in terms of the output-labor elasticity emphasized by Lucas (1970), who estimated it to be near one. A puzzle at the time was the magnitude of the difference between this empirical elasticity and the one predicted by standard production-function specifications to equal labor's share. In our model, we can obtain the elasticity of cyclical output with respect to cyclical labor input by simply dividing the standard deviation of output by that of hours and multiplying by the correlation between hours and output. Doing that for each of the 116-period samples from the model on which table 3 is based gives us an average empirical elasticity of 1.52 with a standard deviation of 0.05. This is, of course, much larger than the labor share, which illustrates the importance of technology shocks for our model.

In terms of the output-labor elasticity, we are faced with the opposite problem of Lucas, that is, we need to explain why the empirical elasticity for the U.S. is so low relative to the model. This issue is, of course, related to the above-mentioned discrepancy in hours variability. A possible explanation is measurement errors. Hansen's (1986) division of the population into six demographic groups according to age and sex indicates that actual cyclical hours should be multiplied by a factor of 0.94 in order to obtain hours in quality-adjusted units. In Kydland (1984), prime-age males were further divided into skill groups using years of education as the criterion. It was found that hours varied substantially more over the cycle for individuals with less education. Adding this element would reduce the variability of the quality-

adjusted labor input significantly more. Furthermore, aggregate hours are difficult to measure accurately as evidenced by the discrepancy in figures obtained from establishment data and household data.⁸ The introduction of these factors reduces the empirical elasticity of output with respect to the labor input.

For example, if the cyclical variabilities of the quality-adjusted labor input is 90 percent of that of the unadjusted labor input and if there is an orthogonal error with standard deviation of 0.8 percent in measuring the labor input,⁹ then the variability of hours is 1.47 and productivity is 1.06. The corresponding numbers for the postwar U.S. economy are 1.7 and 1.0, respectively. The contemporaneous correlation between productivity and GNP falls to 0.56 and the empirical output-labor elasticity becomes 0.98.

Hours are somewhat more variable for the U.S. economy than for the calibrated model economy even when reasonable measurement errors are introduced. This discrepancy should not cast doubt upon our estimate of the contribution of technology shocks to aggregate fluctuations. There are surely other factors contributing to fluctuations such as terms-of-trade shocks, public finance shocks, preference shocks and monetary shocks. All such sources of fluctuations, provided that their induced fluctuations are not strongly and positively correlated with those induced by the technology shocks, contribute to reducing the empirical labor-output elasticity. That the empirical labor-output elasticity is so large is strong evidence in support of the finding that technology shocks are the primary source of aggregate fluctuations in the postwar U.S. economy. The fact that this elasticity is larger for the model economy than for the U.S. economy is evidence that other factors are of some importance.

5.4. Other findings

One may wonder to what extent larger values of ψ are a substitute for the intertemporally nonseparable utility function in terms of increasing the variability of hours and output and reducing the output-labor elasticity. For the economy with time-separable preferences, that is, $\alpha_0 = 1$, the finding is that as ψ increases from 0.64 to 0.82 to 1.0, the magnitude of output fluctuation increases from 1.30 percent to 1.46 percent to 1.68 percent when in fact it was 1.80 percent. Thus, the finding that fluctuations in response to technology shocks are larger if the workweek varies appears robust to the specification of preferences.

Imposing time-separable utility reduces output variability from 1.79 to 1.46, with reductions of similar proportions in its components. Two variables for

⁸See Prescott (1986).

⁹This percentage is consistent with Prescott's (1986) findings.

which the reduction in variability is particularly large are hours (to 0.67) and inventory stock (to 1.09). In the case of productivity, the variability rises to 0.85, and the persistence of output movements is larger, with first- and second-order autocorrelations of 0.72 and 0.46.

With no time to build, the main differences are in the statistics for the capital stock. One change is a reduction in the variability of the capital stock in spite of a slight increase in the variability of output. Another change with time to build is that capital lags output to a greater extent and moves more in conformity with the data. A final important change is that the variability of the inventory stock is reduced from 2.02 to 0.57. This occurs because there are no goods in process, which are the volatile component of the inventory stock.

6. A concluding comment

Our analysis should not be interpreted to mean that fluctuations are optimal and that there is no role for stabilization policy. Our view is that public finance considerations are not the principal factor driving the cycle and that abstracting from them at this stage is warranted. Only when we have considerable confidence in a theory of business cycle fluctuations would the application of public finance theory to the question of stabilization be warranted.¹⁰ Such an extension is straightforward in theory, though in all likelihood carrying it out will be difficult and will require ingenuity.

Appendix: Equilibrium process for the model

A.1. Laws of motion

$$a_{t+1} = 0.9a_t + h_t,$$

$$k_{t+1} = 0.975k_t + s_{1t},$$

$$s_{1,t+1} = s_{2t},$$

$$s_{2,t+1} = s_{3t},$$

$$z_{1,t+1} = 0.95z_{1t} + 0.05917 + \xi_{1,t+1},$$

$$z_{2,t+1} = \xi_{2,t+1},$$

$$z_t = z_{1t} + z_{2t},$$

$$\pi_t = z_t + \xi_{3t}.$$

¹⁰See Shoven and Whalley (1984) for a review of applications of general equilibrium analysis to issues in public finance and trade.

A.2. Forecasting rules

Let m_{0t} be the expectation of $(z_{1t}, z_{2t})'$, conditionally on the previous history of z_τ , $\tau < t$. Then the expectation of $(z_{1t}, z_{2t})'$, given π_t , is

$$m_{1t} = \begin{bmatrix} 0.50129 & -0.49871 \\ -0.019659 & 0.98034 \end{bmatrix} m_{0t} + \begin{bmatrix} 0.49871 \\ 0.019659 \end{bmatrix} \pi_t.$$

The expectation of $(z_{1t}, z_{2t})'$, given z_t , is

$$m_{2t} = \begin{bmatrix} 0.37924 & -0.96208 \\ -0.037924 & 0.96208 \end{bmatrix} m_{1t} + \begin{bmatrix} 0.96208 \\ 0.037924 \end{bmatrix} z_t.$$

A.3. Decision rules

State variables	Decision variables		
	h_t	s_{Jt}	y_{t+1}
k_t	-0.0093319	-0.25156	0.00050171
y_t	-0.0067304	2.2512	0.94847
a_t	-0.036587	-0.18985	-0.0059959
s_{1t}	-0.022726	-1.1728	-0.34657
s_{2t}	-0.010831	-1.0872	-0.33468
Const.	0.15672	-1.4633	-1.0833
m_{11t}	0.30548	2.6288	
m_{12t}	0.58741	5.3222	
m_{21t}			0.57511
m_{22t}			0.78731
h_t			2.3943
s_{Jt}			-0.32354

A.4. Parameter values

Preference

$$\beta = 0.990 \quad \mu = 0.274$$

$$\alpha_0 = 0.500 \quad \gamma = -0.500$$

$$\eta = 0.100$$

Technology

$\psi = 0.820$	$\rho = 0.950$
$\theta = 0.357$	$\sigma = 0.010$
$J = 3$	$\nu = 3.000$
$\phi_1 = 0.333$	$\delta = 0.025$
$\phi_2 = 0.333$	
$\phi_3 = 0.333$	

A.5. Covariance matrix of shocks¹¹

$$\Sigma_{\xi} = 1.183^2 \begin{bmatrix} 0.00760^2 & 0 & 0 \\ 0 & 0.00154^2 & 0 \\ 0 & 0 & 0.00760^2 \end{bmatrix}$$

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¹¹The 1.183 is the unconditional mean of the z_t process.